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VICTOR DANILOVICH MAZUROV
(ON HIS 70-TH BIRTHDAY)

This year marks 70-th birthday of the corresponding member of the Russian Academy of Sciences Victor Danilovich Mazurov. His parents, Daniil Petrovich and Evstoliya Ivanovna, met and married during their study at the Teachers Institute of the town of Zlatoust of Chelyabinsk region. After graduation from the institute in 1939, the young family moved from place to place over the Zlatoust district with a loggers' settlement. Their son Victor was born on 31 January 1943 in a small village of Yurak. In 1950 the family moved to the village of Kuvashi, where Victor went to school. After finishing seven-year school in Kuvashi, Victor moved to Zlatoust for completing his school education. In 1960, after finishing school with gold medal, Victor left for the city of Sverdlovsk, where he entered the Mathematics-Mechanics

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Faculty of the Ural University, following his brother Vladimir, 3 years older than himself.

In 1961 young university lecturer Victor Mikhaĭlovich Busarkin organized a student seminar on group theory, the work of which the second year student Victor Mazurov joined in. From this moment the principal scientific affection of Victor Danilovich was determined — group theory, a difficult and beautiful area of modern algebra, in which he obtained results of highest level. After half a year Victor remained the only participant of the seminar, and by the end of the year he solved the problem posed for him: he constructed an example of a Frobenius group with insoluble complement. A bit later Victor found in the university library a paper of the German mathematician Zassenhaus, in which this problem had already been solved in general setting. This taught him to follow what was going on in scientific world and showed him the importance of studying the theory of group representations, without mastering which substantial progress in the direction chosen by him was impossible. Since then he was spending a lot of time in the library studying the subject by himself. In February of 1965 at the tourist base "Kourovka" near Sverdlovsk, the first all-USSR symposium on group theory was organized, due to the efforts of Mikhail Ivanovich Kargapolov and Al'bert Ivanovich Starostin, two mathematicians, whom Victor Danilovich considers, along with Busarkin, to be his teachers. It is at this symposium that "Kourovka Notebook" was born, which became a world-wide known collection of unsolved problems of group theory. At the symposium, Victor Mazurov, a 5th-year student, gave three lectures on the theory of modular representations of finite groups. At the same time he received an invitation from Kargapolov to move after graduation of the university to Novosibirsk, to the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of USSR. The same year V. D. Mazurov graduated from the Ural University, defending his Diploma "On finite groups with a given Sylow 2-subgroup", the result of which was published in 1966 in "Doklady of the Academy of Sciences of USSR" by presentation of academician A. I. Mal'cev. In order to appreciate the meaning of this paper, we need a little excursus in history. In the middle of 1960s the theory of finite groups experienced a period of flourishing due to remarkable achievements of Western mathematicians: the Brauer–Fowler theorem on finiteness of the number of simple groups with a given centralizer of an involution (1955), the Feit–Thompson theorem on solubility of groups of odd order (1963), and theorems of Chevalley, Steinberg, Suzuki, and Ree on simple groups of Lie type. The hope arose of obtaining a complete classification of finite simple groups. The idea was to proceed by induction, departing from the structure of centralizers of involutions and Sylow 2-subgroups in known simple groups. Here the question of the structure of simple groups with a metacyclic Sylow 2-subgroup was among the top-priority ones. Mazurov's paper gave a characterization of finite simple groups with a semidihedral Sylow 2-subgroup — one of the most nontrivial cases in the solution of this problem. The proof was based on a subtle use of the theory of modular characters, its ideas subsequently led to new deep results. In fact, this was the first significant result in this area obtained in USSR. It is no coincidence that in the famous textbook "Finite groups" of 1968 by D. Gorenstein, the American mathematician who became head of the international project on solving the problem of classification of finite simple groups, the only reference to papers by Soviet mathematicians is the reference to the aforementioned paper by Mazurov. Another important event

happened in the life of Victor Danilovich during his university studies: in 1963 he married same-year student Nadezhda Pavlovna Khomenko, and subsequently became a father of three remarkable children Oleg, Dar'ya, and Vladimir, and a grandfather of six grandchildren.

At the beginning of 1966 Victor Danilovich was conscripted to the Soviet Army. Thanks to the efforts of A. I. Starostin, who wrote a letter to the Commander-in-Chief of the Leningrad military district, private Mazurov took part in the work of the International Congress of Mathematicians that took place in August of 1966 in Moscow, where he gave a sectional talk. The legendary story of the trip of Victor Danilovich to this congress is too amusing for being told in a scientific journal, we only mention that a documentary film about this trip for several years was demonstrated in cinemas before feature films. In December of 1966, shortly before discharge from the army, Mazurov received a telegram saying that he was appointed to a position of junior research fellow of the Algebra Department of the Institute of Mathematics of Siberian Branch of the Academy of Sciences. The persistence of Kargapolov has played a decisive role in the difficult choice between Novosibirsk and Sverdlovsk, where Starostin was waiting for Mazurov, — at the end of December of 1966 Victor Danilovich came to Akademgorodok and forever linked his life with this place.

In 1967 Mazurov published two more papers on simple groups with a metacyclic Sylow 2-subgroup and defended his Candidate (PhD) dissertation on this subject. In 1968 Mazurov constructed a series of examples refuting Thompson's conjecture on commutativity of 2-signalizers in finite simple groups. Note that in 2003 Mazurov jointly with A. S. Kondrat'ev obtained a complete description of 2-signalizers in finite simple groups. In 1969 he obtained another remarkable result: an affirmative solution of the Restricted Burnside Problem for groups of exponent 30 (which is the smallest nontrivial exponent modulo the papers of Hall–Higman and Kostrikin). In 1971–72 Mazurov published two large papers, which contained a classification of finite simple groups all of whose soluble subgroups have 2-length 1. This result superseded recent results of M. Suzuki and D. Gorenstein and was noted in our country and in the world. In 1973 Mazurov defended his Doctor of Sciences dissertation ('Habilitation'). Mazurov was invited to give a talk at the International Congress of Mathematicians in Vancouver in 1974.

Since 1967 Mazurov combines a research position at the Institute of Mathematics with teaching at the Department of Algebra and Mathematical Logic of the Novosibirsk University, where he became a professor in 1975. From the very beginning of his teaching activity he attracted talented students. The first of them, Sergeĭ Aleksandrovich Syskin, defended his Candidate dissertation already in 1972. In total Mazurov supervised 17 Candidates and 8 Doctors of Sciences. More than half of the current members of staff of the Laboratory of Group Theory of the Institute of Mathematics are his pupils and pupils of his pupils. Victor Danilovich substantially influenced the contents of the algebra courses at Mechanics-Mathematics Faculty, which he was lecturing for more than thirty years. Victor Danilovich devoted a lot of time and effort to working with talented schoolchildren: he traveled over Siberia to mathematical olympiads, conducted interviews, delivered popular-scientific lectures, participated in preparation and conduct of entrance examinations in mathematics in Novosibirsk Univerity. In 1975 he became Head of the Committee at

Presidium of Siberian Branch of the Academy of Sciences on conducting olympiads for schoolchildren and remains in this post at present.

In 1970s and beginning of 1980s Mazurov obtained quite a few more first-class results on classification of finite simple groups: description of groups in which a Sylow 2-subgroup contains an extra-special subgroup of index 2 (jointly with S. A. Syskin and V. V. Kabanov), description of simple groups in which the rank of intersections of Sylow 2-subgroups is at most 2 (jointly with S. A. Syskin), characterization of the Rudvalis sporadic group by the centralizer of its noncentral involution. In 1979 the famous Summer Institute on Finite Groups took place in Santa-Cruz, USA, the outcome of which was an announcement of completion of the classification of finite simple groups. Victor Danilovich also participated in this conference and gave a talk there. In the proof of the theorem of classification of finite simple groups, one of the most impressive mathematical results of the XX-th century, there is a considerable contribution of V. D. Mazurov.

In 1986 Mazurov became Head of the Laboratory of Group Theory of the Institute of Mathematics. At that time his research interests were focused on comprehension of the role of the classification theorem in the theory of finite groups. He posed and by the middle of 1990s solved, jointly with his pupils and colleagues, the problem of description of minimal permutation representations of finite simple groups. With the use of the classification he proved the theorem of existence in a simple group of a pair of conjugate Sylow p-subgroups with trivial intersection for every prime number p (jointly with V. I. Zenkov, 1996), completed the above-mentioned description of 2-signalizers in simple groups. Mazurov and his pupils E. P. Vdovin and D. O. Revin proved theorems of Sylow type for Hall subgroups of finite groups, jointly with E. I. Khukhro he obtained structure results on finite groups admitting an automorphism of prime order whose centralizer has bounded rank. In the middle of 1990s Mazurov became interested in the possibility of characterization of a finite simple group by its arithmetic parameters. Research in this area, which year after year becomes more and more popular among specialists all over the world, at that time was in embryonic state. Having begun with nontrivial examples (in particular, the well-known Praeger–Shi conjecture was refuted) and proofs of basic assertions, Victor Danilovich subsequently brought the research in this area to a qualitatively new level. First of all we point out deep papers of Victor Danilovich with his pupil A. V. Zavarnitsin on element orders in coverings of symmetric and linear groups. In 2009 Mazurov jointly with his pupils A. V. Vasil'ev and M. A. Grechkoseeva established the following fundamental fact: every finite simple group is uniquely characterized in the class of all finite groups by its order and set of element orders.

Although the main attention of Victor Danilovich was devoted to theoretical aspects of group theory proper, he was always interested in its applications to other questions of algebra and combinatorics. In this connection we can mention the following results: classification of cyclic automorphisms of abelian topological groups (jointly with M. I. Kabenyuk, 1974), description of finite groups of automorphisms of cubic graphs (jointly with D. G. Khramtsov, 1989), estimate for the number of q-ary words of length n with bounded number of one-letter subwords (jointly with A. V. Kostochka and L. Ya. Savel'ev, 1998).

Since the beginning of 1990s in the research activity of Victor Danilovich one more direction was formed, which lately is becoming a main one: studying infinite

groups from the viewpoint of possibility of transferring to them theorems on finite groups. In 1990s he described infinite 2-transitive permutation groups with abelian stabilizer of a point and, in particular, obtained a negative answer to the question of Ya. P. Sysak on existence of such groups with cyclic stabilizer. He also found sufficient conditions for the existence of a regular normal subgroup in a sharply 2-transitive infinite group. Possibly the most important result in this direction was obtained by Mazurov in 2000: he described periodic groups with elementary abelian centralizers of involutions. This theorem implies a lot of interesting corollaries, among which we can point out the existence of an infinite periodic group that is uniquely characterized by the orders of its elements in the class of all groups. Adjacent to these investigations there are joint results with A. Kh. Zhurtov and V. A. Churkin on periodic groups generated by quadratic pairs and acting freely on an abelian group, the proof of local finiteness of groups of exponent 24 without elements of order 6, as well as generalizations of theorems of Sanov and M. Hall obtained quite recently in co-authorship with D. V. Lytkina and A. S. Mamontov.

Perhaps the brightest illustration of the approach of Victor Danilovich to studying infinite groups can be given by the following example. Recall that in 1962 second-year student Mazurov constructed an example of a Frobenius group with insoluble complement. Namely, he showed that the special linear group of matrices $SL_2(5)$ of dimension 2 over the field of 5 elements can act freely on an abelian group A, that is, in such a way that every element acts on A without fixed points. In fact, this result followed from the general classification of Frobenius complements obtained by Hans Zassenhaus back in 1935. The central part of Zassenhaus' paper was the proof of the following assertion: the group $SL_2(5)$ is the only finite perfect (coinciding with its derived subgroup) group that can act freely on an abelian group. This proof was based on character theory and was notable for extreme technical difficulty. In 2003 Mazurov published a short note in "Journal of Algebra", in which he gave an elementary proof of Zassenhaus' theorem, which did not use character theory and therefore admitted generalization to infinite groups. In 2005, by applying this approach, Mazurov obtained a characterization of locally finite alternating groups in terms of the properties of their 3-cycles — one of his strongest results on infinite groups.

A substantial part of time and attention of Victor Danilovich was always occupied by scientific-organizational activities. He was a member of programme and organizing committees of dozens of international conferences and symposia, he is a member of several dissertation and academic councils and of editorial boards of several mathematical journals. From 2003 to 2012 Mazurov was Head of the Department of Algebra and Mathematical Logic of Novosibirsk State University. In 1975, starting from 5th edition, Victor Danilovich became one of the editors of "Kourovka Notebook", since 1990 the last seven edition were prepared by him together with his pupil E. I. Khukhro. This collection of unsolved problems, the authors of which are more than 300 specialists from all over the world, has long since acquired international recognition. It is difficult to overestimate the influence that "Kourovka Notebook" exerted and continues to exert on the development of modern group theory.

Scientific and pedagogical work of many years of Victor Danilovich received recognition both in our country and abroad. In 2003 he was elected a corresponding member of the Russian Academy of Sciences. In 2012 the American Mathematical

Society, a member of which V. D. Mazurov is since 1966, instituted the programme "Fellows of AMS", the aims of which were indicated as "to recognize members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics". The inaugural list of members of this programme, published on 1 November 2012, includes Professor Mazurov. Victor Danilovich was awarded high government honours of the Russian Federation.

As his colleagues and friends point out, personal qualities of Victor Danilovich form a rare blend, in which a prominent scientist combines with an exceptionally modest and mild person, sensible and balanced outlook on life realities with kindness and subtle sense of humour, adherence to scientific principles with tactfulness and attention to people around, wisdom and responsibility of a leader with optimism and cheerfulness. For his pupils, Victor Danilovich is an example of a real man and a great scientist.

We wish Victor Danilovich new scientific achievements, successes in his multifaceted activities, happiness and health to him and his remarkable family!

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