ON FINITE GROUPS ISOSPECTRAL TO FINITE SIMPLE UNITARY GROUPS OVER FIELDS OF CHARACTERISTIC 2

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Abstract. For every group $U = PSU_n(2^k)$ with $n \geq 5$, we find the number of finite groups having the same element orders as $U$.

Keywords: recognition by spectrum, unitary group, field automorphism.

Dedicated to 70th anniversary of V.D. Mazurov

1. Introduction

The spectrum $\omega(G)$ of a group $G$ is the set of its element orders. Two groups are said to be isospectral if their spectra coincide. A finite group $L$ is recognizable by spectrum if every finite group $G$ with $\omega(G) = \omega(L)$ is isomorphic to $L$. Denoting by $h(L)$ the number of pairwise non-isomorphic finite groups isospectral to $L$ we can write the property of $L$ to be recognizable as $h(L) = 1$. And $L$ is said to be almost recognizable if $h(L)$ is finite, and irreducible otherwise. The recognition by spectrum problem for a group $L$ is to determine whether $L$ is recognizable, almost recognizable or irreducible, and a stronger version of this problem is to find $h(L)$. The most recent survey on this subject can be found in [1–3].

This article is concerned with recognition of simple unitary groups $PSU_n(2^k)$. The group $PSU_3(2)$ is soluble and the groups $PSU_4(2), PSU_5(2)$ are irreducible [4, 5]. The groups $PSU_3(2^k)$ with $k \geq 2$, $PSU_4(2^k)$ with $k \geq 2$, and $PSU_6(2)$ are recognizable [6–9]. The rest of the groups $PSU_6(2^k)$ are known to be almost recognizable, but the precise numbers of isospectral groups are still unknown (see

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Lemma 5). The aim of the article is to find these numbers and to describe the isospectral groups.

**Theorem.** Let \( U = PSU_n(q) \), where \( n \geq 5 \), \( q = 2^k \), and \( U \neq PSU_5(2) \). Let \( d = (n, q+1) \) and \( f \) be the \( d \)-part of \(((q+1)/d, k)\).

(i) If \( n-1 \) is a power of 2 then \( h(U) = 1 \).

(ii) If \( n-1 \) is not a power of 2 then \( h(U) \) is equal to the number of divisors of \( f \). Furthermore, a finite group \( G \) satisfies \( \omega(G) = \omega(U) \) if and only if \( G \) is isomorphic to the natural extension of \( U \) by a field automorphism of order dividing \( f \).

In particular, \( U \) is recognizable by spectrum if and only if \(((q+1)/d, k) = 1 \) or \( n-1 \) is a power of 2.

In the above theorem and below, \( (m_1, m_2, \ldots, m_s) \) denotes the greatest common divisor of numbers \( m_1, m_2, \ldots, m_s \), and for natural numbers \( r \) and \( m \), the \( r \)-part of \( m \) is the smallest divisor \( t \) of \( m \) such that \( (m/t, r) = 1 \).

### 2. Notation and Preliminary Results

Let \( G \) be a finite group. Divisibility relation endows the set \( \omega(G) \) by a partial order, and the subset of elements maximal under this order is denoted by \( \mu(G) \).

Given a prime \( r \), we refer to the highest power of \( r \) lying in \( \omega(G) \) as to the \( r \)-exponent of \( G \).

The set of prime divisors of a natural number \( m \) is denoted by \( \pi(m) \). For a finite group \( G \), we define \( \pi(G) = \pi(|G|) \). Given natural numbers \( m \) and \( r \), we write \( m_r \) to denote the \( r \)-part of \( m \), and \( m_{r'} \) to denote the number \( m/m_r \). As usual, \( [m_1, m_2, \ldots, m_s] \) denotes the least common multiple of numbers \( m_1, m_2, \ldots, m_s \).

Given an integer \( q \) and an odd prime \( r \) such that \( (q, r) = 1 \), we write \( e(r, q) \) for the multiplicative order of \( q \) modulo \( r \), that is, the smallest natural number \( m \) satisfying \( q^m \equiv 1 \pmod{r} \). For an odd \( q \), we put \( e(2, q) = 1 \) if \( q \equiv 1 \pmod{4} \), and \( e(2, q) = 2 \) otherwise.

**Lemma 1** (Zsigmondy [10]). Let \( q \) be an integer whose absolute value is larger than 1. For every natural number \( k \), there is a prime \( r \) such that \( e(r, q) = k \) except the cases where the pair \((q, k)\) is in \( \{(2, 1), (3, 1), (2, 6), (-2, 2), (-2, 3), (-3, 2)\} \).

A prime \( r \) satisfying \( e(r, q) = k \) is said to be a primitive prime divisor of \( q^k - 1 \), and the set of all primitive prime divisors of \( q^k - 1 \) is denoted by \( R_k(q) \). It is easy to verify that \( R_k(q) \subseteq R_k(q^n) \) for coprime \( m \) and \( k \). The notation \( r_k(q) \) is used to denote an element of \( R_k(q) \), provided that this set is not empty.

**Lemma 2** ([11, Corollary 3]). Let \( n \geq 3, q \) be a power of a prime \( p, d = (n, q+1) \), and \( \varepsilon = -1 \). Then \( \omega(PSU_n(q)) \) consists of all divisors of the following numbers:

(i) \( \frac{q^n-\varepsilon^n}{d(q+1)} \);

(ii) \( \frac{(q^n-\varepsilon^{n_1}q^{n_2}-\cdots-\varepsilon^{n_s})}{n(n,n_2, n_3, \ldots, n_s+1)} \), where \( n_1, n_2 > 0 \) and \( n_1 + n_2 = n \);

(iii) \( q^{n_1} - \varepsilon^{n_1}q^{n_2} - \varepsilon^{n_2} \cdots - \varepsilon^{n_s} \), where \( s \geq 3, n_1, n_2, \ldots, n_s > 0 \) and \( n_1 + n_2 + \cdots + n_s = n \);

(iv) \( p^lq_{n_1-1}^{n_1-1} \), where \( l, n_1 > 0, p^l-1 + 1 + n_1 = n \);

(v) \( p^m[q_{n_1-1}^{n_1-1} \cdots - \varepsilon^{n_s}], \) where \( s \geq 2, m, n_1, \ldots, n_s > 0 \) and \( p^m-1 + 1 + n_1 + \cdots + n_s = n \);

(vi) \( p^l \), provided that \( p^{l-1} + 1 = n \) for \( l > 0 \).
Lemma 3 ([12, Corollary 1]). Let $m$ be an odd number and $\psi$ a field automorphism of $PSU_n(q^m)$ of order $m$. Then

$$\omega(PSU_n(q^m)(\psi)) = \bigcup_{r|m} r \cdot \omega(PSU_n(q^{m/r})).$$

Lemma 4. Let $q$ and $l$ be natural numbers and $q > 1$.

(i) If $r$ is an odd prime and $r$ divides $q + 1$ or $r = 2$ and $q + 1$ is divisible by 4, then $q^r - (-1)^r = t_r(q + 1) r$.

(ii) If $r$ and $l$ are coprime, then $2^{q^l-(-1)^l}$ divides $q^r - (-1)^r$, and $\frac{q^r - (-1)^l}{q - (-1)^l}$ divides $\frac{q^r - (-1)^l}{(n,q+1)}$ for every natural $n$.

Proof. See [13, Lemma 6] and [14, Lemma 8].

Lemma 5. Let $U = PSU_n(q)$, where $n \geq 5$, $q = 2^k$, and $U \neq PSU_5(2)$. If $G$ is a finite group such that $\omega(G) = \omega(U)$ then $U \leq G \leq \text{Aut } U$.

Proof. By the main result of [15], we may assume that $U \leq G/K \leq \text{Aut } U$, where $K$ is the soluble radical of $G$. The full preimage of $U$ in $G$ has the same spectrum as $U$, and [16, Corollary 1] implies that $K$ is identity.

3. Proof of the theorem

We begin with specifying a matrix representation of unitary groups. We identify $GU_n(q)$ with the subgroup of $GL_n(q^2)$ consisting of matrices $A = (a_{ij})$ such that $(a_{ij}^q)^2 = (a_{ij})^{-1}$. Then $SU_n(q)$ is the subgroup of matrices of determinant 1 in $GU_n(q)$; and $PGU_n(q)$ and $PSU_n(q)$ are the images of $GU_n(q)$ and $SU_n(q)$ in the projective group $PGL_n(q^2)$ respectively.

Let $U = PSU_n(q)$, where $q = p^k$ and $n \geq 5$. The group $PGU_n(q)$ acts on $U$ by conjugation, and we identify $U$ with $\text{Inn } U$ and $PGU_n(q)$ with the group of inner-diagonal automorphisms of $U$. Denote by $\delta$ the image of the unitary matrix $\text{diag}(1, \ldots, 1, \lambda)$, where $\lambda$ is a primitive $(q+1)$th root of unity in $GF(q^2)$, in $PGU_n(q)$. Denote by $\varphi$ the field automorphism of $U$ induced by Frobenius map $(a_{ij}) \mapsto (a_{ij}^p)$ of $GU_n(q)$. Then

$$\text{Aut } U = PGU_n(q) \rtimes \langle \varphi \rangle = \langle U, \delta \rangle \rtimes \langle \varphi \rangle$$

with $\delta^\varphi = \delta^\varphi, |(U, \delta)| = (n, q + 1)$ and $|\varphi| = 2k$.

Now we are ready to prove the theorem. Let $G$ be a finite group such that $\omega(G) = \omega(U)$. By Lemma 5, we have $U \leq G \leq \text{Aut } U$. Suppose that the index $|G : U|$ is even. By hypothesis, $q$ is even, so we may assume that $G$ contains $\gamma = \varphi^k$. The centralizer $C_U(\gamma)$ is a group of type $C_{n/2}(q)$ or $C_{(n-1)/2}(q)$ according as $n$ is even or odd [17, Proposition 4.9.2 (b)]. Hence one of the numbers $r(-q)$ and $r_{n-1}(-q)$ lies in $\pi(C_U(\gamma))$. On the other hand, $2r_n(-q), 2r_{n-1}(-q) \notin \omega(U)$. Thus the index $|G : U|$ is odd. The remaining part of the proof does not depend on the characteristic $p$ being 2, so in fact we can prove the following assertion, thus proving the theorem.

Proposition 6. Let $U = PSU_n(q)$, where $q = p^k$, $n \geq 5$, and $d = (n, q + 1)$. Suppose that $G$ is a finite group with $U < G \leq \text{Aut } U$ and $|G/U|$ being odd. Then $\omega(G) = \omega(U)$ if and only if $n - 1$ is not a power of $p$ and $G$ is conjugate in $\text{Aut } U$ to a subgroup of the group $U \rtimes \langle \delta \rangle$, where $\delta \in \langle \varphi \rangle$ and $|\delta| = ((q + 1)/d, k)_d$. 


Proof. Suppose that \( \omega(G) = \omega(U) \). Given an element \( g \in \Aut U \), we write \( \hat{g} \) to denote its image in \( \Out U \).

If \( G \cap \PGU_n(q) > U \) then the intersection of \( G \) and a maximal torus of \( \PGU_n(q) \) of order \( (q^n - (-1)^n)/(q + 1) \) is a cyclic group of order \( (q^n - (-1)^n)/c(q + 1) \) for some \( c < d \). However, Lemma 2 implies that \( (q^n - (-1)^n)/d(q + 1) \in \mu(U) \). Hence \( G \cap \PGU_n(q) = U \) and \( G/U \) is a cyclic group. Let \( \alpha = \beta \phi \), where \( \beta \in \langle \delta \rangle \) and \( \phi \in \langle \varphi \rangle \), generates \( G \) modulo \( U \) and let \( |\phi| = m \). Then \( \alpha^n \in G \cap \PGU_n(q) \), and therefore \( |G/U| = |\alpha| = m \).

We claim that \( \pi(m) \subseteq \pi(d) \). Otherwise, \( G \) contains a field automorphism of odd prime order \( r \) with \( r \) not dividing \( d \). Denote \( q^{1/r} \) by \( q_0 \). By Lemma 3, the spectrum of \( G \) includes the set \( r \omega(\PSU_n(q_0)) \). If \( r = p \) then the \( p \)-exponent of \( G \) exceeds that of \( U \). Hence \( (r, p) = 1 \). Denote \( e(r, -q) \) by \( s \). Below we will use information on spectra of simple groups from [18] (see also corrections to this article in [19] and [20]).

Suppose that \( s = 1 \), that is, \( r \) divides \( q + 1 \). Then \( r \) does not divide \( n \), and therefore \( r \omega(-q_0) \in R_n(-q) \). Thus by [18, Proposition 4.2], it follows that \( r \omega(-q_0) \notin \omega(U) \).

On the other hand, \( r \omega(-q_0) \in \omega(U) \). For convenience, we may assume that \( \phi = \omega^2(m-1)/m \) so as to have \( \delta^{-1} = \delta^{-1} \), where \( q_0 = q^{1/m} \). Then

\[
|\beta| = \frac{(q + 1)_r}{(q_0^2 - 1, d)_r},
\]

and we deduce that \( \beta \notin \langle \delta^{-1} \rangle \), or equivalently, \( \beta \notin \langle \delta^{-1}, d \rangle \). Hence \( |\beta| \) does not divide \( |\delta^{-1}, d \rangle \), and so there is a prime \( r \) such that

\[
|\beta|_r > \frac{(q + 1)_r}{(q_0^2 - 1, d)_r}.
\]

In particular, \( (q_0^2 - 1, d)_r > 1 \) and \( |\beta|_r > (q + 1)_r/d_r \). Since \( r \) divides \( q_0^2 - 1 \), by Lemma 4, we have \( (q^2 - 1)_r/(q_0^2 - 1)_r = m_r \). Now the equalities

\[
\alpha^m = (\beta \phi)^m = \beta^{e_1} \phi^{e_m} = \beta_1 q_0^{e_1} \ldots \beta^{e_m} q_0^{e_m} = \beta^{q_0^{e_1} \ldots q_0^{e_m}} = \beta^{q_0^{e_1} \ldots q_0^{e_m}}
\]

show that \( |\alpha|_r = |\beta|_r \). Moreover, since \( \alpha^m \in U \), we derive that \( \beta^{q_0^{e_1} \ldots q_0^{e_m}} \in \delta^d \), which implies

\[
|\beta|_r > \frac{(q + 1)_r}{d_r}, \quad m_r.
\]

Hence \( r \) divides \( m \). In particular, \( r \) is odd and \( r \) divides \( q_0 + 1 \). Since \( \alpha \) centralizes a subgroup of \( U \) isomorphic to \( SU_{n-1}(q_0) \), there is an element of order \( r_{n-1}(-q_0)|\alpha|_r \) in \( G \). The numbers \( m \) and \( n - 1 \) are coprime, so \( r_{n-1}(-q_0) \in R_{n-1}(-q) \). Then it follows from Lemma 2 that the number \( r_{n-1}(-q_0)|\alpha|_r \) can lie in \( \omega(U) \) only if it
divides \((q^{n-1} - (-1)^{n-1})/d\). However, \((q^{n-1} - (-1)^{n-1})_{r/d_r} = (n-1)_r(q+1)_r/d_r = (q+1)_r/d_r < |\alpha|_r\).

Thus \(\check{a}\) is conjugate to \(\check{v}\), as required. Moreover, arguing as above, it is easy to show that \(m\) divides \((q+1)/d\) (otherwise, there is a prime divisor \(r\) of \(m\) such that \(m_r r_{n-1}(-q_0) \in \omega(G) \setminus \omega(U)\)). It remains to establish that \(n = 1\) is not a power of \(p\). If \(n = p^l + 1\) for some \(l\) then by Lemma 2 we have that \(p^{l+1} \in \mu(U)\). But the same lemma asserts that \(p^{l+1} \in \omega(PSU_n(q^{1/m}))\), and now Lemma 3 implies that \(mp^{l+1} \in \omega(G)\); a contradiction.

Now we prove the converse implication. It suffices to consider the maximal case where \(G = U \times \langle \phi \rangle\) with \(|\phi| = ((q+1)/d, k)_d\). Notice that the number \(((q+1)/d, k)_d\) is odd. By Lemma 3, the spectrum of \(G\) is the union of the sets \(m \omega(PSU_n(q^{1/m}))\) over all divisors \(m\) of \(((q+1)/d, k)_d\).

Fix a number \(m\) dividing \(((q+1)/d, k)_d\) and let \(q_0 = q^{1/m}\). First we check that for every \(r \in \pi(m)\) and every natural number \(l\)

\[(q^l - (-1)^l)_{r} \geq m_r(q_0^l - (-1)^l)_{r},\]

and if \((l, m) = 1\) then for every divisor \(h\) of \(n\)

\[(q^l - (-1)^l)_{r}/(h, q - 1)_{r} \geq m_r(q_0^l - 1)_{r}/(h, q_0 - 1)_{r}.\]

Using Lemma 4, we derive that

\[(q^l - (-1)^l)_{r} = m_r(q_0^{m_r l} - (-1)^{m_r l})_{r}.\]

This implies (1) and also the equality \((q + 1)_r = m_r(q_0^{m_r} + 1)_{r}\). By hypothesis, \(m\) divides \((q + 1)/d\), so \((q + 1)_r \geq m_r (q_0^{m_r} + 1)_{r} \geq d_r\). This means that for every divisor \(h\) of \(n\) we have

\[(h, q + 1)_r = h_r = (h, q_0^{m_r} + 1)_r.\]

Now (3) and (4) show that

\[(q^l - (-1)^l)_{r}/(h, q + 1)_r = m_r(q_0^{m_r l} - (-1)^{m_r l})_{r}/(h, q_0^{m_r} + 1)_r.\]

By Lemma 4, if \((l, m) = 1\) then \(m_r(q_0^l - (-1)^l)_{r}/(h, q_0 + 1)_r\) divides the right-hand side of (5), so we have (2).

Now we are ready to verify that for every \(a \in \omega(PSU_n(q_0))\), there is \(b \in \omega(U)\) such that \(m a\) divides \(b\). By hypothesis, \(n = 1\) is not a power of \(p\), so we may assume that \(a\) is one of the numbers \(f(q_0)\) listed in (i)–(v) of Lemma 2. For convenience, we write \(\varepsilon\) in place of \(-1\).

Let

\[f(q_0) = \frac{q_0^n - \varepsilon^n}{(q_0 + 1)(q_0 + 1)}.\]

Denote by \(r\) the smallest prime in \(\pi(m)\). Then \((m, r - 1) = 1\). Hence \(r\) divides \(q_0 + 1 = (q_0^m + 1, q_0^{r-1} - 1)\). There is an element of order \(q^{n/r} - \varepsilon^{n/r}\) in \(U\). This order is divisible by \(f(q_0)\). Furthermore, by Lemma 4 and (1), we see that

\[(q^{n/r} - \varepsilon^{n/r})_r = \frac{(q^n - \varepsilon^n)_r}{r} \geq m_r(q_0^n - \varepsilon^n)_r \geq m_r(q_0^n - \varepsilon^n)_r \geq m_r f(q_0)_r\]

and

\[(q^{n/r} - \varepsilon^{n/r})_u = (n/r)_u(q^{n+1})_u = n_u(q^{n+1})_u = (q^n - \varepsilon^n)_u \geq m_u(q_0^n - \varepsilon^n)_u \geq m_u f(q_0)_u\]

for every \(u \in \pi(m), u \neq r\). Thus \(m f(q_0)\) divides \(q^{n/r} - \varepsilon^{n/r}\).
Thus in both cases in $U$

Denote the smallest of those by $\epsilon^{(n_1)}_m, q^{(n_2)}_0 - \epsilon^{(n_2)}$. There is an element of order $[q^{(n_1)/r} - \epsilon^{(n_1)/r}, q^{(n_2)}_0 - \epsilon^{(n_2)}]$ in $U$. This order is divisible by $f(q_0)$. Applying Lemma 4 and (1), we deduce that

$$[q^{(n_1)/r} - \epsilon^{(n_1)/r}, q^{(n_2)}_0 - \epsilon^{(n_2)}]_r \geq [q^{(n_1)}_0 - \epsilon^{(n_1)}, m(q^{(n_2)}_0 - \epsilon^{(n_2)})]_r =$$

$$= m_r[q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_2)}_0 - \epsilon^{(n_2)}]_r \geq m_r f(q_0)_r$$

and

$$[q^{(n_1)/r} - \epsilon^{(n_1)/r}, q^{(n_2)}_0 - \epsilon^{(n_2)}]_u \geq [m(q^{(n_1)}_0 - \epsilon^{(n_1)}, m(q^{(n_2)}_0 - \epsilon^{(n_2)})]_u =$$

$$= m_u[q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_2)}_0 - \epsilon^{(n_2)}]_u \geq m_u f(q_0)_u$$

for every $u \in \pi(m), u \neq r$.

Suppose now that $(n_1, m) = 1$. Then $(n_2, m) = 1$ as well. Hence by Lemma 4, we see that $f(q_0)$ divides $f(q)$. Moreover, (2) implies

$$f(q)_m \equiv \left[\frac{[q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_2)}_0 - \epsilon^{(n_2)}]}{(n,q + 1)_m} \geq \frac{m[q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_2)}_0 - \epsilon^{(n_2)}]}{(n,q + 1)_m} = m f(q_0)_m.\right.$$  

Thus in both cases $m f(q_0) \in \omega(U)$.

Let

$$f(q_0) = \frac{[q^{(n_1)}_0 - \epsilon^{(n_1)}]}{(n,q + 1)_m},$$

where $l > 0, p^{l} - 1 + 1 + n_1 = n$. Suppose that $n_1$ and $m$ have common prime divisors. Denote the smallest of those by $r$. Consider the number $p^{l}(q^{(n_1)/r} - 1)$ lying in $\omega(U)$. This number is divisible by $f(q_0)$. By (1) we have

$$(q^{(n_1)/r} - \epsilon^{(n_1)/r})_r = \left(\frac{q^{(n_1)}_0 - \epsilon^{(n_1)}}{r}\right)_r \geq \frac{m_r(q^{(n_1)}_0 - \epsilon^{(n_1)})}{r},$$

and

$$(q^{(n_1)/r} - \epsilon^{(n_1)/r})_u = (q^{(n_1)}_0 - \epsilon^{(n_1)})_u \geq m_u(q^{(n_1)}_0 - \epsilon^{(n_1)})_u \geq m_u f(q_0)_u$$

for every $u \in \pi(m), u \neq r$. Observe that $(n_1, m, r - 1) = 1$ by choice of $r$. Hence if $r$ divides $q^{(n_1)}_0 - \epsilon^{(n_1)}$ then $r$ divides $q_0 + 1 = (q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_1)}_0 + 1 - q^{(n_1)/r} - 1)$ as well, and so $m_r(q^{(n_1)}_0 - \epsilon^{(n_1)})_r / r \geq m_r f(q_0)_r$. And if $r$ does not divide $q^{(n_1)}_0 - \epsilon^{(n_1)}$, then $m_r f(q_0)_r = m_r \leq (q^{(n_1)/r} - \epsilon^{(n_1)/r})_r$, since $r$ divides $q_0 + 1$. Thus $m f(q_0)$ divides $p^{l}(q^{(n_1)/r} - \epsilon^{(n_1)/r})$.

Suppose that $(n_1, m) = 1$. Then Lemma 4 asserts that $f(q_0)$ divides $f(q)$. Applying (2), we have that

$$f(q)_m \equiv \left[\frac{[q^{(n_1)}_0 - \epsilon^{(n_1)}]}{(n,q + 1)_m} \geq \frac{m[q^{(n_1)}_0 - \epsilon^{(n_1)}]}{(n,q + 1)_m} = m f(q_0)_m.\right.$$  

Thus $m f(q_0)$ divides $f(q)$.

Finally, let $f(q_0) = p^{l}[q^{(n_1)}_0 - \epsilon^{(n_1)}, q^{(n_2)}_0 - \epsilon^{(n_2)}, \ldots, q^{(n_l)}_0 - \epsilon^{(n_l)}]$, where $l \geq 0$. A direct application of (1) shows that $m f(q_0)$ divides $f(q)$.

The proposition and the theorem are proved.
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