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AMENABILITY OF CLOSED SUBGROUPS AND ORLICZ SPACES

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Abstract. We prove that a closed subgroup H of a second countable locally compact group G is amenable if and only if its left regular representation on an Orlicz space $L^\Phi(G)$ for some Δ_2-regular N-function Φ almost has invariant vectors. We also show that a noncompact second countable locally compact group G is amenable if and ony if the first cohomology space $H^1(G, L^\Phi(G))$ is non-Hausdorff for some Δ_2-regular N-function Φ.

Keywords: locally compact group, amenable group, second countable group, closed subgroup, N-function, Orlicz space, 1-cohomology.

1. Introduction

Throughout, we assume all topological groups separated.

A locally compact topological group is called *amenable* [7] if there exists a G-invariant mean on $L^\infty(G)$ or, equivalently, G possesses the *fixed point property*: for every action of G by continuous affine transformations on a nonempty convex compact subset Q of a locally convex space W, there is a fixed point for G in Q.

Let V be a Banach G-module, i.e., a real or complex Banach space endowed with a continuous linear representation $\alpha : G \to \mathcal{B}(V)$. We say that V *almost has invariant vectors* if, for every compact subset $F \subset G$ and every $\varepsilon > 0$, there exists a unit vector $v \in V$ such that $\|\alpha(g)v - v\| \le \varepsilon$ for all $g \in F$. Here $\mathcal{B}(V)$ stands for the space of all bounded linear endomorphisms of a Banach space V.

Let V be a normed space of functions $f : G \to \mathbb{R}$ ($f : G \to \mathbb{C}$) such that if $f \in V$ then, for every $g \in G$, the function

$$\lambda_G(g)f(x) = f(g^{-1}x), \quad x \in G,$$

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lies in V and $\|\lambda_G(g)f\|_V = \|f\|_V$. Then $\lambda_G : G \to B(V)$ is called the *left regular representation* of G in V.

Examples of such function spaces V are given by the space $L^p(G)$ of all real-valued functions on G integrable to the power p over G with respect to a left-invariant Haar measure μ_G. Instead of the L^p spaces, one can consider more general Orlicz spaces $L^\Phi(G)$ of real-valued functions on G with the finite "gauge" norm

$$\|f\|_{(\Phi)} = \inf\left\{k > 0 \, : \, \int\limits_G \Phi\left(\frac{f(x)}{k}\right) d\mu_G(x) \leq 1\right\}.$$

for an N-function Φ (N-functions are defined in Section 2). Orlicz spaces on locally compact groups were considered in [5, 10] and more recently in [1, 2, 14, 15].

In [20] (see also [19, Theorem 8.3.2]), Stegeman proved that, for a locally compact group, the following conditions (P_p) (called Reiter's conditions) are equivalent for all $p \geq 1$:

(P_p) for every compact set $F \subset G$ and every $\varepsilon > 0$, there exists a function $f \in L^p(G)$ with $f \geq 0$ and $\|f\|_{L^p(G)} = 1$ such that $\|\lambda_G(z)f - f\|_{L^p(G)} < \varepsilon$ for all $z \in F$.

In [7] Eymard extended this equivalence to quotients G/H of locally compact groups by closed subgroups and proved that conditions (P_p) are equivalent to the amenability of G/H.

In [15, Proposition 2, pp. 387–389], Rao proved that a locally compact group G is amenable if and only if, given a Δ_2-regular N-function Φ, G satisfies the property

(P_Φ) for every compact set $F \subset G$ and every $\varepsilon > 0$, there exists a function $f \in L^\Phi(G)$ with $f \geq 0$ and $\|f\|_{L^{(\Phi)}(G)} = 1$ such that $\|\lambda_G(z)f - f\|_{L^{(\Phi)}(G)} < \varepsilon$ for all $z \in F$.

Here $\|\cdot\|_{L^{(\Phi)}(G)}$ stands for the gauge norm in the space $L^\Phi(G)$.

In 2005, Bourdon, Martin, and Valette established the following [4, Lemma 2]:

Theorem A. *Suppose that $p \geq 1$. Let X be a countable set on which a countable group H acts freely. The following are equivalent:*

(i) *The natural "permutation" representation λ_X of H on $L^p(X)$ almost has invariant vectors;*

(ii) *H is amenable.*

In [11], in an attempt to generalize this assertion, we proved:

Theorem B. Assume that $p \geq 1$. Let G be a second countable locally compact group and let H be a closed subgroup in G. The following are equivalent:

(i) The left regular representation of H on $L^p(G)$ almost has invariant vectors;

(ii) H is amenable.

The paper is organized as follows: In Section 2, we recall some basic notions concerning N-functions and Orlicz spaces. Section 3 contains some necessary information on integration on locally compact groups and homogeneous spaces. In Section 4, we prove a generalization of Theorem B, where $L^p(G)$ is replaced by the Orlicz space $L^\Phi(G)$ for any Δ_2-regular N-function Φ (Theorem 1). In Section 5, using the equivalence of amenability and the fulfillment of the above condition (P_Φ) and a general result by Guichardet, we deduce that the nonreduced and reduced first cohomology of a noncompact second countable locally compact group coincide if and only if it is not amenable.

2. N-Functions and Orlicz Function Spaces

Definition 1. *A function $\Phi : \mathbb{R} \to \mathbb{R}$ is called an N-function if*

(i) *Φ is even and convex;*

(ii) *$\Phi(x) = 0 \Longleftrightarrow x = 0$;*

(iii) $\lim_{x\to 0} \frac{\Phi(x)}{x} = 0; \quad \lim_{x\to\infty} \frac{\Phi(x)}{x} = \infty.$

An N-function Φ has left and right derivatives (which can differ only on an at most countable set, see, for instance, [17, Theorem 1, p. 7]). The left derivative φ of Φ (we write $\varphi = \Phi'$ below) is left continuous, nondecreasing on $(0, \infty)$, and such that $0 < \varphi(t) < \infty$ for $t > 0$, $\varphi(0) = 0$, $\lim_{t\to\infty} \varphi(t) = \infty$. The function

$$\psi(s) = \inf\{t > 0 : \varphi(t) > s\}, \quad s > 0,$$

is called the *left inverse* of φ.

The functions Φ, Ψ given by

$$\Phi(x) = \int_0^{|x|} \varphi(t)dt, \quad \Psi(x) = \int_0^{|x|} \psi(t)dt$$

are called *complementary N-functions.*

The N-function Ψ complementary to an N-function Φ can also be expressed as

$$\Psi(y) = \sup\{x|y| - \Phi(x) : x \geq 0\}, \quad y \in \mathbb{R}.$$

N-functions are classified in accordance with their growth rates as follows:

Definition 2. *An N-function Φ is said to satisfy the Δ_2-condition for large x (for small x, for all x), which is written as $\Phi \in \Delta_2(\infty)$ ($\Phi \in \Delta_2(0)$, or $\Phi \in \Delta_2$), if there exist constants $x_0 > 0$, $K > 2$ such that $\Phi(2x) \leq K\varphi(x)$ for $x \geq x_0$ (for $0 \leq x \leq x_0$, or for all $x \geq 0$); and it satisfies the ∇_2-condition for large x (for small x, or for all x), denoted symbolically as $\Phi \in \nabla_2(\infty)$ ($\Phi \in \nabla_2(0)$, or $\Phi \in \nabla_2$) if there are constants $x_0 > 0$ and $c > 1$ such that $\Phi(x) \leq \frac{1}{2c}\Phi(cx)$ for $x \geq x_0$ (for $0 \leq x \leq x_0$, or for all $x \geq 0$).*

Henceforth, let Φ be an N-function and let (Ω, Σ, μ) be a measure space.

Definition 3. *The set $\tilde{L}^\Phi = \tilde{L}^\Phi(\Omega) = L^\Phi(\Omega, \Sigma, \mu)$ is defined to be the set of measurable functions $f : \Omega \to \mathbb{R}$ such that*

$$\rho_\Phi(f) := \int_\Omega \Phi(f)d\mu < \infty.$$

Proposition 1. [18] *The set $\tilde{L}^\Phi$ is a vector space in the following cases:*
(i) $\mu(\Omega) < \infty, \quad \Phi \in \Delta_2(\infty)$;
(ii) $\mu(\Omega) = \infty, \quad \Phi \in \Delta_2$.
(iii) Ω *is countable,* μ *is the counting measure on* $\Omega, \quad \Phi \in \Delta_2(0)$.

Definition 4. *The linear space*

$$L^\Phi = L^\Phi(\Omega) = L^\Phi(\Omega, \Sigma, \mu) = \{f : \Omega \to \mathbb{R} \textit{ measurable } : \rho_\Phi(af) < \infty \textit{ for some } a > 0\}$$

is called an Orlicz space on (Ω, Σ, μ).

For an Orlicz space $L^\Phi = L^\Phi(\Omega, \Sigma, \mu)$, the N-function Φ is called Δ_2-*regular* if $\Phi \in \Delta_2(\infty)$ when $\mu(\Omega) < \infty$ or $\Phi \in \Delta_2$ when $\mu(\Omega) = \infty$ or $\Phi \in \Delta_2(0)$ for μ a counting measure.

Let Ψ be the complementary N-function to Φ.

Below we as usual identify two functions equal on a set of measure zero.

If $f \in L^\Phi$ then the functional $\|\cdot\|_\Phi$ (called *the Orlicz norm*) defined by

$$\|f\|_\Phi = \|f\|_{L^\Phi(\Omega)} = \sup\left\{\left|\int_\Omega fg\, d\mu\right| : \rho_\Psi(g) \leq 1\right\}$$

is a seminorm. It becomes a norm if μ satisfies the *finite subset property* (see [17, p. 59]): if $A \in \Sigma$ and $\mu(A) > 0$ then there exists $B \in \Sigma$, $B \subset A$, such that $0 < \mu(B) < \infty$.

The *gauge* (or *Luxemburg*) *norm* of a function $f \in L^\Phi$ is defined by the formula

$$\|f\|_{(\Phi)} = \|f\|_{L^{(\Phi)}(\Omega)} = \inf\left\{k > 0 \,:\, \rho_\Phi\left(\frac{f}{k}\right) \le 1\right\}.$$

This is a norm without any constraint on the measure μ (see [17, p. 54, Theorem 3]).

Suppose that the measure μ satisfies the finite subset property. As is proved in [16, Chapter 10], a left-invariant Haar measure on a locally compact group has this property.

It is well known that the Orlicz and gauge norms are equivalent, namely (see, for example, [17, pp. 61–62]):

$$\|f\|_{(\Phi)} \le \|f\|_\Phi \le 2\|f\|_{(\Phi)}.$$

We will need the following version of Hölder's inequality for Orlicz spaces [17, p. 62]:

Hölder's Inequality. *If Φ and Ψ are two complementary N-functions then $fg \in L^1$ and*

$$\|fg\|_1 \le \|f\|_{(\Phi)}\|g\|_\Psi \quad (\ \|fg\|_1 \le \|f\|_\Phi\|g\|_{(\Psi)}\ ). \tag{1}$$

3. Integration on Locally Compact Groups and Borel Sections

Recall some basic facts and definitions from the theory of integration on locally compact groups.

Let G be a locally compact group and let H be a closed subgroup in G. Denote by μ_G and μ_H left-invariant Haar measures on G and H respectively and denote by π the projection $G \to G/H$.

Denote by Δ_K the modulus of a locally compact group K.

Given a function f and a class $u \in G/H$, take an arbitrary representative x in u and consider the function $\alpha : y \to f(xy)$ on H. If α is integrable over H, the left invariance of μ_H implies that $\int_H f(xy)\, d\mu_H(y)$ is independent of the choice of x with $\pi(x) = u$.

It is well known that the homogeneous space G/H admits a *quasi-G-invariant* measure $\mu_{G/H}$ on H which is unique up to equivalence. Here the "quasi-G-invariance" means that all left translates of $\mu_{G/H}$ by the elements of G are equivalent to $\mu_{G/H}$. The measure $\mu_{G/H}$ can be described as follows (see [3, Chapter VII, 2.5] or [7]).

(a) There exists a positive continuous function ρ on G such that $\rho(xy) = \frac{\Delta_H(y)}{\Delta_G(y)}\rho(x)$ for all $x \in G$ and $y \in H$.

Put $\mu_{G/H} = (\rho\mu_G)/\mu_H$ (see [3, Definition 1 in Chapter VII, 2.2]).

(b) If $f \in L^1(G, \rho\mu_G)$ then the set of $\overline{x} = \pi(x) \in G/H$ for which $y \mapsto f(xy)$ is not μ_H-integrable is $\mu_{G/H}$-negligible, the function $\overline{x} = \pi(x) \mapsto \int_H f(xy)d\mu_H(y)$ is $\mu_{G/H}$-integrable, and

$$\int_G f(x)\rho(x)\, d\mu_G(x) = \int_{G/H} d\mu_{G/H}(\overline{x}) \int_H f(xy)\, d\mu_H(y).$$

(c) There exists a nonnegative continuous function h on G with $\int_H h(xy)dy = 1$ for all $x \in G$ such that a function w on G/H is $\mu_{G/H}$-measurable ($\mu_{G/H}$-integrable) if and only if $h(w \circ \pi)$ is $\rho\mu_G$-measurable ($\rho\mu_G$-integrable). If $w \in L^1(G/H, \mu_{G/H})$ then

$$\int\limits_{G/H} w(\bar{x})\, d\mu_{G/H}(\bar{x}) = \int\limits_{G} h(x)w(\pi(x))\rho(x)\, d\mu_G(x).$$

Note that a second countable locally compact space is Polish (*polonais*) (see [3]). As follows from Dixmier's lemma (see [6]), if G is a Polish group and H is a closed subgroup in G then there exists a Borel section $\sigma : G/H \to G$ (in particular, $\pi \circ \sigma = \mathrm{id}_{G/H}$). We will need the following technical assertion (see [11] for a proof):

Lemma 1. *Suppose that G is a second countable locally compact group, H is a closed subgroup in G, $\sigma : G/H \to G$ is a Borel section, and $f \in L^1(G, \rho\mu_G)$. Then, in the above notations,*

$$\int\limits_{G} f(x)\rho(x)\, d\mu_G(x) = \int\limits_{H} d\mu_H(y) \int\limits_{G/H} f(\sigma(\overline{x})y)\, d\mu_{G/H}(\overline{x}).$$

4. Amenability of Closed Subgroups

The main result of this section is as follows:

Theorem 1. *Assume that Φ is a Δ_2-regular N-function. Let G be a second countable locally compact group and let H be a closed subgroup in G. The following are equivalent:*

(i) *The left regular representation of H on $L^\Phi(G)$ almost has invariant vectors;*

(ii) *H is amenable.*

Proof. Put $\Phi' = \varphi$ and let Ψ be the complementary N-function to Φ.

Observe first that (ii) implies (i) by the equivalence of amenability and the fulfillment of the Rao–Reiter condition (P_Φ), established by Rao in [15, Proposition 2, pp. 387–389]:

(P_Φ) For every compact set F and every $\varepsilon > 0$, there exists a function $f \in L^\Phi(H)$ with $f \geq 0$ and $\|f\|_{L^{(\Phi)}(H)} = 1$ such that $\|\lambda_H(z)f - f\|_{L^{(\Phi)}(H)} < \varepsilon$ for all $z \in F$.

Now, prove (i)$\Rightarrow$(ii). Suppose that $L^\Phi(G)$ almost has invariant vectors for H and deduce from this that H meets Reiter's condition (P_1).

By the Δ_2-regularity of Φ, we conclude from [17, Proposition 8, p. 79] that

$$S := \sup\bigl\{\rho_\Psi(\varphi \circ |v|) \,:\, v \in L^\Phi(G),\, \|v\|_{L^{(\phi)}(G)} \leq 1\bigr\} < \infty.$$

Take $\varepsilon > 0$ and a compact set $F \subset H$; choose $f \in L^\Phi(G)$, $\|f\|_{L^{(\Phi)}(G)} = 1$, such that

$$\|\lambda_G(z)f - f\|_{L^{(\Phi)}(G)} \leq \frac{\varepsilon}{2(S+1)} \tag{2}$$

for all $z \in F$. Assume without loss of generality that $f \geq 0$ (taking $|f|$ instead of f if necessary).

Note that

$$|\Phi(a) - \Phi(b)| \leq |a-b|\,(\varphi(a) + \varphi(b)), \quad a, b \geq 0, \tag{3}$$

because φ is monotone and nonnegative (cf. [15, p. 388]).

Put $u = \Phi \circ f$. Since Φ is Δ_2-regular, we have ([12, p. 78]):

$$\int\limits_{G} u(x)\, d\mu_G(x) = \int\limits_{G} \Phi(f(x))\, d\mu_G(x) = \|f\|_{L^{(\Phi)}(G)} = 1.$$

For $z \in F$, using (3), Hölder's inequality (1), (2), and the inequality

$$\|v\|_{L^\Psi(G)} \leq \rho_\Psi(v) + 1,$$

we infer

$$\|\lambda_G(z)u-u\|_{L^1(G)}=\int_G |\Phi(f(z^{-1}x))-\Phi(f(x))|\,d\mu_G(x)$$
$$\le \int_G |f(z^{-1}x)-f(x)|\;|\varphi(f(z^{-1}x))+\varphi(f(x))|\;d\mu_G(x)$$
$$\le \|\lambda_G(z)f-f\|_{L^{(\Phi)}(G)}\|\varphi\circ\lambda_G(z)f+\varphi\circ f\|_{L^{\Psi}(G)}$$
$$\le \|\lambda_G(z)f-f\|_{L^{(\Phi)}(G)}\big(\|\varphi\circ\lambda_G(z)f\|_{L^{\Psi}(G)}+\|\varphi\circ f\|_{L^{\Psi}(G)}\big)$$
$$=2\|\varphi\circ f\|_{L^{\Psi}(G)}\|\lambda_G(z)f-f\|_{L^{(\Phi)}(G)}\le 2(\rho_\psi(\varphi\circ f)+1)\,\|\lambda_G(z)f-f\|_{L^{(\Phi)}(G)}$$
$$\le 2(S+1)\|\lambda_G(z)f-f\|_{L^{(\Phi)}(G)}<\varepsilon.$$

Now, let $\sigma : G/H \to G$ be a Borel section. Consider the function

$$U(y)=\int_{G/H}\frac{u(y\sigma(\overline{x}))}{\rho(\sigma(\overline{x}))}\,d\mu_{G/H}(\overline{x}),\quad y\in H,$$

where ρ is the function described in Section 3. By Lemma 1, since u is nonnegative, we have

$$\|U\|_{L^1(H)}=\int_G u(x)d\mu_G(x)=1.$$

Involving Lemma 1 again, we obtain the following estimates:

$$\|\lambda_H(z)U-U\|_{L^1(H)}=\int_H\left|\int_{G/H}\frac{u(z^{-1}y\sigma(\overline{x}))-u(y\sigma(\overline{x}))}{\rho(\sigma(\overline{x}))}\,d\mu_{G/H}(\overline{x})\right|d\mu_H(y)$$
$$\le\int_H d\mu_H(y)\int_{G/H}\left|\frac{u(z^{-1}y\sigma(\overline{x}))-u(y\sigma(\overline{x}))}{\rho(\sigma(\overline{x}))}\right|d\mu_{G/H}(\overline{x})$$
$$=\int_G\frac{|u(z^{-1}x)-u(x)|}{\rho(x)}\rho(x)\,d\mu_G(x)=\int_G|u(z^{-1}x)-u(x)|\,d\mu_G(x)=\|\lambda_H(z)u-u\|_{L^1(G)}.$$

So, if $z\in F$ then $\|\lambda_H(z)U-U\|_{L^1(H)}<\varepsilon$. Thus, H has property (P_1) and hence is amenable. Theorem 1 is proved. □

Remark 1. Theorem 1 is informative only if H is noncompact since (i) and (ii) are both fulfilled when H is compact.

5. First Cohomology and Amenability

Let G be a topological group and let V be a topological G-module, i.e., a real or complex topological vector space endowed with a linear representation $\pi : G\times V\to V$, $(g,v)\mapsto\pi(g)v$. The space V is called a *Banach G-module* if V is a Banach space and π is a representation of G by isometries of V. Introduce the notation:

$$Z^1(G,V):=\{b:B\to V \text{ continuous } \mid b(gh)=b(g)+\pi(g)b(h)\}\quad (\textit{1-cocycles});$$
$$B^1(G,V)=\{b\in Z^1(G,V)\mid(\exists v\in V)\,(\forall g\in G)\;b(g)=\pi(g)v\}\quad(\textit{1-coboundaries});$$
$$H^1(G,V)=Z^1(G,V)/B^1(G,V)\quad(\textit{1-cohomology with coefficients in } V).$$

Endow $Z^1(G,V)$ with the topology of uniform convergence on compact subsets of G and denote by $\overline{B}^1(G,V)$ the closure of $B^1(G,V)$ in this topology. The quotient $\overline{H}^1(G,V)=Z^1(G,V)/\overline{B}^1(G,V)$ is called the *reduced 1-cohomology* of G with coefficients in the G-module V.

The following assertion was established by Guichardet (see [8, Théorème 1]):

Lemma 2. *Let G be a locally compact second countable group and let V be a Banach module such that*

$$V^G := \{v \in V \mid \pi(g)v = v \text{ for all } g \in G\} = 0.$$

Then the following are equivalent:

(i) $H^1(G,V) = \overline{H}^1(G,V)$;

(ii) *V does not almost have invariant vectors, that is, there exists a compact subset $F \subset G$ and $\varepsilon > 0$ such that* $\sup_{g \in F} \|\pi(g)v - v\| \geq \varepsilon\|v\|$ *for all $v \in V$.*

As is well known, if a locally compact group G is noncompact then the Haar measure of the whole group is infinite [9, Theorem 15.9]. Hence, constant functions are not integrable over G. Therefore, $L^\Phi(G)^G = 0$. Thus, combining Lemma 2 with the Rao–Reiter condition (P_Φ), we obtain the following generalization of Corollary 2.4 in [13, p. 86] to coefficients in an Orlicz space:

Proposition 2. *Suppose that Φ is a Δ_2-regular N-function. If G is a noncompact second countable locally compact group then the following are equivalent:*

(i) $H^1(G, L^\Phi(G)) = \overline{H}^1(G, L^\Phi(G))$;

(ii) *G is not amenable.*

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