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Special issue: Graphs and Groups, Spectra and Symmetries — G2S2 2016

ON GRAPHS AND GROUPS, SPECTRA AND SYMMETRIES HELD ON AUGUST 15–28, 2016, NOVOSIBIRSK, RUSSIA

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Abstract. The International Conference and PhD–Master Summer School on Graphs and Groups, Spectra and Symmetries (G2S2) were held on August 15–28, 2016, in Novosibirsk, Akademgorodok, Russia. The main goal of these events was to bring together young researchers and famous mathematicians in the field of graph theory and group theory, especially those involving group actions on combinatorial objects.
Keywords: group theory, finite group action, graph automorphism, algebraic combinatorics, topology, finite geometries

1. General information

The International Conference and PhD-Master Summer School on Graphs and Groups, Spectra and Symmetries (G2S2) were held on August 15–28, 2016, in Novosibirsk, Akademgorodok, Russia. The G2S2 events were organized by Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia, with cooperation of the Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg, Russia.

The organizing committee includes René van Bevern, Sergey Goryainov, Ekaterina Khomyakova, Elena Konstantinova (chair), Denis Krotov, Alexey Medvedev,

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Kristina Rogalskaya, Anna Simonova, Ev Sotnikova, and Ivan Takhonov. The program committee includes Alexander Gavrilyuk, Elena Konstantinova (co-chair), Denis Krotov, Alexander Makhnev, Natalia Maslova, Alexander Mednykh (co-chair), and Andrey Vasil'ev.

The conference was supported by the Russian Foundation for Basic Research, grant 16-31-10290, and the summer school Novosibirsk State University, Project 5-100.

The official site of G2S2 is http://math.nsc.ru/conference/g2/g2s2/.

2. Scientific program

More than 110 experts on finite group theory, graph theory, algebraic combinatorics, low-dimensional geometry and topology from 19 countries (Brazil, Canada, China, Czech Republic, Finland, Germany, Hungary, India, Iran, Israel, Italy, Japan, Slovakia, Slovenia, South Korea, Taiwan, United Kingdom, USA and Russia) participated in the G2S2 events. Young scientists, PhD students, graduate and undergraduate students were presented by 75 participants.

The scientific program of G2S2 events consisted of minicourses, plenary and contributed talks, open problems session. Video lectures of all courses and plenary talks are published online by Youtube (see http://math.nsc.ru/conference/g2/g2s2/video.html).

2.1. **PhD–Master Summer School.**

Four courses, each containing eight 50-minutes lectures, were given by scientists from USA, Canada, UK, and Taiwan.

G2S2 Summer School started with the course on "Another viewpoint of Euler graphs and Hamiltonian graphs" given by Distinguished Professor Lih-Hsing Hsu from Providence University, Taichung, Taiwan, the author of the book "Graph Theory and Interconnection Networks" [18]. He says in the syllabus of the course:

> *Mathematics is the study of topics such as quantity, structure, space, and change. Mathematicians seek out patterns and use them to formulate new conjectures. Mathematicians resolve the truth or falsity of conjectures by mathematical proof. When mathematical structures are good models of real phenomena, then mathematical reasoning can provide insight or predictions about nature. Through the use of abstraction and logic, mathematics developed from counting, calculation, measurement, and the systematic study of the shapes and motions of physical objects. Now, mathematics is used throughout the world as an essential tool in many fields, including natural science, engineering, medicine, finance, and the social sciences.*

It was pointed out by Prof. Hsu that it may appear that there is little left to do in regards to the study of the Hamiltonian property of vertex-transitive graphs unless there is a major breakthrough on the famous Lovász conjecture. However, if we extend the concept of the traditional Hamiltonian property to other Hamiltonicity properties, then there is still much left to explore. In this series of lectures, Prof. Hsu introduced some of these Hamiltonicity properties, namely fault tolerant Hamiltonian, spanning connectivity, and mutually independent Hamiltonicity.

The course on "Graphs and their eigenvalues" was given by Professor Bojan Mohar, who is a professor of mathematics at the University of Ljubljana and the holder of a Canada Research Chair in graph theory at Simon Fraser University in Vancouver, Canada. He has published well over 200 research papers. His main research interests include graph minors, graph coloring, graph algorithms, topological and algebraic graph theory. Together with Carsten Thomassen he coauthored the monograph "Graphs on Surfaces" [31]. In 1990 he was awarded the Slovenian national award for exceptional achievements in science, and in 2009 he was awarded the recognition of an Ambassador of Science in Slovenia. In his course spectral graph theory was presented with some applications to chemistry.

There were two further courses on group theory. The classification of finite simple groups was probably one of the major achievements of the 20th century mathematics. Peter Cameron writes in his Blog on "The discrete mathematics and Big Data" [10]:

> *One of the groups is the Monster, the largest "sporadic" simple group of order about* 10^{54} *whose smallest permutation representation is on about* 10^{20} *points. A single generator for the group would take about* 800 *exabytes of storage! Matrices are better, since there is a representation by matrices of order* 196882 *over the field of two elements; one of these only takes about* 5 *gigabytes.*

The Monster group is linked to the j-function via string theory, the idea that the universe is made of tiny strings vibrating in high dimensions. "The ultimate goal is to unify quantum mechanics and Einstein's theory of gravity. That's a very, very big goal for physics, one of the biggest goals in science," as it was said by John Duncan (cited from [33]).

Professor Alexander A. Ivanov from Imperial College London, UK, the author of the book "The Monster Group and Majorana Involutions" [20], presented Majorana Theory which is a new developing axiomatic approach to the Monster group and its non-associative 196884-dimensional algebra. The Y-conjecture for the Monster group was formulated by J. H. Conway in 1980 and was motivated by an earlier observation by B. Fischer. The conjecture was proved by the lecturer and S. P. Norton in 1990 [19, 32]. In his course, Prof. Alexander A. Ivanov revisited Y-groups via Majorana Theory.

Group theory also led to recent progress on the Graph Isomorphism Problem, which was announced by L. Babai at the end of 2015. Group theory is the foundation of the course "The Cayley Isomorphism Problem" by Professor Ted Dobson from Mississippi State University, USA, and University of Primorska, Slovenia. In general, the Cayley Isomorphism Problem asks for necessary and sufficient conditions for two Cayley graphs of a group to be isomorphic. This course teached techniques to approach the Cayley Isomorphism Problem. Modifications of these techniques were applied to the isomorphism problem on graphs that are highly symmetric but not Cayley graphs nor even vertex-transitive. Applications of these techniques to isomorphism problems for other classes of combinatorial objects were also shown.

2.2. **International Conference.** The program of G2S2 Conference contained 60 talks. The most important directions of the contemporary mathematics were reflected in the lectures given by 19 plenary speakers.

A few talks were devoted to problems on Cayley graphs. Isomorphism problem for Cayley combinatorial objects was presented in the lecture given by Professor

Mikhail Muzychuk (Netanya Academic College, Israel). A Cayley object over a finite group H is any relational structure $\mathcal{R}$ with point set H which is invariant under the group of right translations H_R. The well-known examples of Cayley objects include Cayley graphs, Cayley maps, group codes etc. The isomorphism problem for Cayley objects may be formulated as follows: Given two combinatorial objects over the group H, find whether they are isomorphic or not. In the talk, the results solving the above problem for different classes of objects were presented.

Professor Andrey Vasil'ev (Sobolev Institute of Mathematics, Russia) presented new results on testing isomorphism of central Cayley graphs over an almost simple groups in polynomial time. It was shown that the automorphism group of a central Cayley graph over an explicitly given almost simple group G of order n can be found in time poly(n). This result is based on the joint work with Professor Ilia Ponomarenko (Steklov Institute of Mathematics, St. Petersburg, Russia) who, in turn, gave the lecture on characterizations of association schemes by intersection numbers. He considered a coherent configuration which is characterized by the intersection numbers if every algebraic isomorphism of this configuration to another one is induced by a combinatorial isomorphism; in this case, a coherent configuration is said to be separable. The importance of this notion was explained by the fact that if the coherent configuration of a graph is separable, then the isomorphism of this graph to any other graph can be tested by the Weisfeiler–Leman algorithm [35]. Besides, the separability of a distance-regular graph (or, more general, of an association scheme) means in terms of [5], that the graph is uniquely determined by its parameters. Let us note, that a family of regular coherent non-Schurian graphs, related to extremal graph theory, was presented in the talk given by Professor Matan Ziv-Av (Ben-Gurion University of the Negev, Israel). In his talk, some natural links between algebraic graph theory and extremal graph theory were established.

Colour-preserving automorphisms of Cayley graphs were discussed in the lecture of Professor Klavdija Kutnar (University of Primorska, Slovenia). In her talk, the automorphisms of a Cayley graph that preserve its natural edge colouring were studied and recent results about colour-preserving automorphisms of Cayley graphs were presented. More precisely, she were interested in groups G, such that every such automorphism of every connected Cayley graph on G has a very simple form: the composition of a left translation and a group automorphism. Classes of groups that have the property were found and the orders of all groups that do not have the property were determined. The analogous results were obtained for automorphisms that permute the colours, rather than preserving them.

Cayley graphs with integral spectrum were considered in the lecture of Professor Anton Betten (Colorado State University, USA), where the spectrum of a graph is the set of eigenvalues of the adjacency matrix of the graph, together with their multiplicities. In 1974, F. Harary and A. J. Schwenk [16] initiated the study of graphs with integral spectra, that is, graphs whose eigenvalues are all integral. In the talk, some open problems on integral Cayley graphs were discussed. In particular, there are integral Cayley graphs over the symmetric group. A connection to the representation theory of the symmetric group was also explored.

Special classes of graphs were presented in lectures given by Professor Alexander Gavrilyuk (Institute of Mathematics and Mechanics UB RAS, University of Science and Technology of China) on the characterization of the Grassmann graphs $J_2(2d+2, d)$ and Professor Jack Koolen (University of Science and Technology of

China) on applications of Hoffman graphs. Among these applications are studying graphs with the smallest eigenvalue -3, constructing (regular) graphs with a fixed smallest eigenvalue, and trees with spectral radius three. The great progress was demonstrated in characterizing the Grassmann graphs. It was proved by Alexander Gavrilyuk and Jack Koolen that the Grassmann graph $J_2(2d+2,d)$, $d>2$, is characterized by its intersection array, if at least one of the following holds: (1) the diameter d is odd; (2) the diameter d is large enough. The Terwilliger algebra theory and the Hoffman graphs theory are used to show this result.

When dealing with symmetry properties of combinatorial objects, such as graphs admitting a transitive group action, that is, vertex-transitive graphs, one of the fundamental questions is to determine their full automorphism group. While some symmetries of such objects are obvious, certain additional symmetries remain hidden or difficult to grasp. When this is the case, the goal is to find a reason for their existence and a method for describing them. Along these lines, the above question reads as follows: given a transitive group H acting on a set V of vertices of a graph, determine whether H is its full automorphism group or not. When the answer is no, find a method to describe the additional automorphisms. In the lecture by Professor Dragan Marušič (University of Primorska, Slovenia), it is proposed to study such group "extensions" by considering the existence of odd automorphisms (as opposed to even automorphisms), that is, automorphisms that act as odd permutations on the vertex set of the graph. Recent results in regards to the above problem were considered. A special emphasis was given to the class of cubic symmetric graphs where a complete solution was presented. These results suggest that the even/odd question is likely to uncover certain much more complex structural properties of graphs that go beyond simple arithmetic conditions.

Professor Tatsuro Ito (Anhui University, Hefei, China) presented a survey talk on the classification of (P and Q)-polynomial association schemes. Eiichi Bannai proposed the classification of (P and Q)-polynomial association schemes in his lectures at Ohio State University in the late 70s. He regarded them as finite, combinatorial analogue of compact symmetric spaces of rank 1. In the present lecture, the speaker traced the history back to the late 60s and explained how the concepts of P/Q-polynomial association schemes arose in relation to finite permutation groups, coding/design theory. The progress of the classification in the 80s, 90s and thereafter was also overviewed. The personal view by Professor Tatsuro Ito about the scope for the classification problem was given at the end of his lecture.

Triply even codes obtained from some graphs and finite geometries were considered in the talk given by Professor Akihiro Munemasa (Tohoku University, Sendai, Japan). A triply even code is a binary linear code in which the weight of every codeword is divisible by 8. By C. H. Lam and H. Yamauchi [28], every triply even code of length a multiple of 16 containing the all-ones vector is the structure code of some holomorphic framed vertex operator algebra. Motivated by this fact, Akihiro Munemasa and Koichi Betsumiya [6] classified maximal triply even codes of length 48, and discovered an infinite family which can be obtained from the triangular graphs. In this talk, the speaker presented another infinite family of triply even codes, derived from the odd-orthogonal graphs.

The answer on the question "Is there a $(4,27,2)$ partial geometry?" was given by Professor Patric Östergård (Aalto University, Finland) in his lecture. A partial geometry with parameters (s,t,α) consists of lines and points with the properties that (i) each line has $s+1$ points and two distinct lines intersect in at most one point; (ii) each point is on $t+1$ lines and two distinct point occur on at most one line; and (iii) for each point p that does not lie on a line l, there are exactly α lines through p that intersect l. The question whether there exists a $(4,27,2)$ partial geometry has tantalized researchers during the last couple of decades. Such a partial geometry would have 275 points and 1540 lines and its point graph would be a $(275,112,30,56)$ strongly regular graph. There is a unique strongly regular graph with the aforementioned parameters called the McLaughlin graph. In this talk, a computer search for a $(4,27,2)$ partial geometry starting from the McLaughlin graph was described. After 270 core-years and more than one physical year, the computers claimed that there is no such partial geometry.

A few talks of G2S2 Conference were devoted to topological graph theory problems. Professor Roman Nedela (University of West Bohemia, Czech Republic) in his lecture on hamiltonian cycles in graphs embedded into surfaces payed attention to the hamiltonicity problem. He indicated that although hundreds of papers deals with the problem of existence of a Hamiltonian cycle in a graph, there is a luck of results on the hamiltonicity of cubic graphs. Among others, it is well known that to decide whether a cubic graph is Hamiltonian is an NP-complete problem. The main idea of his talk was to present a new approach to investigate hamiltonicity of graphs. Instead of graphs, he considered graphs embedded into closed surfaces such that each face is bounded by a circuit (no repetitions of vertices are allowed). Such an embedding is called polytopal or circular. By the cycle-double-cover conjecture, every 2-connected graph admits a polytopal embedding. In an embedded graph, the set of hamiltonian cycles split into three classes: contractible, bounding but non-contractible cycles and non-separating cycles. Bounding and contractible hamiltonian cycles were investigated in the talk and the main results were presented. This is joint work with M. Kotrbčík and M. Škoviera.

2-arc-transitive regular covers were considered in the lecture of Professor Shaofei Du (Capital Normal University, China). A cover X of a given graph Y is an homomorphism ϕ from X to Y, locally it is a bijection. This is one of fundamental and important concepts in topological graph theory. Another motivation to study covers might be from classifications of finite arc-transitive graphs, mainly 2-arc-transitive graphs. In his talk, Shaofei Du collected some recent results on 2-arc-transitive regular covers. In particular, by exhibiting some examples he showed the relationships between construction of covers and group extension theory, group representation theory and topological graph theory.

Characterization of finite metric spaces by their isometric sequences was given by Professor Mitsugu Hirasaka (Pusan National University, South Korea). He considered a metric space (X,d) where $d : X \times X \to \mathbb{R}_{\geq 0}$ is a metric function. For $A, B \subseteq X$, it is said that A is isometric to B if there exists a bijection $f : A \to B$ such that $d(x,y) = d(f(x), f(y))$ for all $x, y \in A$. If A is isometric to B, then we write $A \simeq B$. For a positive integer k, the quotient set of $\binom{X}{k}$ by $\simeq$ is denoted by $A_k(X)$, i.e.

$$A_k(X) = \left\{ [A] \mid A \in \binom{X}{k} \right\},$$

where $[A]$ is the isometry class containing A. For a finite metric space (X,d), $(|A_i(X)| : i = 1, 2, \ldots, |X|)$ is called the isometric sequence of X. In his talk, Mitsugu Hirasaka characterized metric spaces X by their isometric sequences, and classified them with the property $|A_2(X)| = |A_3(X)| \leq 3$.

Group factorizations, graphs and characters of groups were considered in the lecture of Professor Lev Kazarin (Yaroslavl P.Demidov State University, Russia). The great survey on the old and new results on the topic was presented in this talk. It was also shown that many problems concerning factorizations remain open.

In the talk given by Professor Yaokun Wu (Shanghai Jiao Tong University, China), the lit-only sigma game and some mathematics around were discussed. Let G be a graph. The vertex space, which is the power set of the vertex set of G, is regarded as a vector space over the binary field $\mathbb{F}_2$. For each vertex v in G, let T_v be the endomorphism of the vertex space mapping the vertex v to $v + \mathrm{N}_G(v)$, where $\mathrm{N}_G(v)$ is the neighbourhood of v in G, and mapping the vertex w to w itself for all other vertices w in G. In other words, T_v can be written as $\mathrm{id} + \mathrm{N}_G(v)v^*$, where v^* is the Kronecker function for v. T_v is a transvection if v is not a loop vertex, namely if $v \notin \mathrm{N}_G(v)$, while T_v is a projection if v is a loop vertex, namely if $v \in \mathrm{N}_G(v)$. In the case that G is loopless, the set of all T_v's, where v runs over the vertex set of G, generates a group, called the lit-only group of the graph G. It is proved in the talk that the lit-only group is a semidirect product of a classical group over $\mathbb{F}_2$ and an elementary abelian 2-group, and explicit description of the orbits of the corresponding group action is given. In the case that G contains loops, the set of all T_v's, which consists of possible transvections and some projections, generates a monoid. The orbits of this monoid action were also described.

Professor Yuriy Tarannikov (Moscow State University, Russia) gave a talk on plateaued Boolean functions with the same spectrum support. He started with a brief survey on plateaued functions, regular functions and related topics, including connections with problems on subgraphs of the Hamming graph. A Boolean function f is defined as a map $\mathbb{F}_2^n \to \mathbb{F}_2$. A plateaued Boolean function is a Boolean function whose Walsh coefficients take values $\{0, \pm 2^c\}$ for some integer c, where the Walsh coefficient $W_f(u)$ (also known as a spectral coefficient), $u \in \mathbb{F}_2^n$, is defined as the real-valued sum $W_f(u) = \sum\limits_{x \in \mathbb{F}_2^n} (-1)^{f(x) + <x,u>}$. The given collection of Walsh coefficients defines the Boolean function uniquely. If the spectrum support of a plateaued Boolean function f is known (i.e., the set of all vectors $u \in \mathbb{F}_2^n$ such that $W_f(u) \neq 0$), then only the signs of all Walsh coefficients are known; so, the plateaued Boolean function is not defined uniquely. For the majority of spectrum supports S including the full space $\mathbb{F}_2^n$, n even, $n > 8$, the number of plateaued functions with this spectrum support S is unknown whereas for some specific families the number of functions with such spectrum support was found. Some of such spectra constructions were demonstrated and their symmetries were analyzed in the talk. These results are based on a joint work with A. V. Khalyavin and M. S. Lobanov, which is published in the G2S2 Special Issue [23].

In his lecture "Enumeration of maps and coverings", Professor Alexander Mednykh (Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia) gave an analytical survey of the results and methods used in enumeration theory of branched coverings of Riemann surfaces. In particular, the main counting principle for finding the number of conjugacy classes in a finitely generated group obtained by the author in 2006 was mentioned. Some old and new applications of this method to Riemann surfaces and maps were given. It was also noted that the method successfully works for counting coverings of three dimensional Euclidean manifolds. The most results of this lecture were obtained in collaboration with Jin Ho Kwak, Roman Nedela and Valery Liskovets.

As it was noticed by Alexander Ivanov, the source of many ideas discussed during the conference was Com^{2}Mac, The Combinatorial and Computational Mathematics Center, POSTECH, Korea (1999–2008), supervised by Professor Jin Ho Kwak (Beijing Jiaotong University, China), who participated at the G2S2 events. 15 participants of G2S2 were the members of this institution for a while.

2.3. **Open problems session.** The last day of the G2S2 conference was marked by the open problems session, caused the huge enthusiasm among participants. Shortly, we represent the result of the discussion as follows.

2.3.1. **Alexander Mednykh, Novosibirsk, Russia.** Consider a finite connected graph G, allowed to have multiple edges but without loops. We endow each edge of G with the two possible orientations. Prescribe to each oriented edge of G an element of some Abelian group (a *flow*). Then we define *the Jacobian group* $\mathrm{Jac}(G)$ of the graph G as the (maximal) Abelian group generated by flows obeying the following two Kirchhoff laws: K_1 — the flow through each vertex of G vanishes; K_2 — the flow along each closed directed path W in G vanishes.

The notion of the Jacobian group of a graph, which is also known as the Picard group, the critical group, and the dollar or sandpile group, was independently introduced by many authors [14, 4, 7, 3]. This notion arises as a discrete version of the Jacobian in the classical theory of Riemann surfaces. It also admits a natural interpretation in various areas of physics, coding theory, and financial mathematics. The Jacobian group is an important algebraic invariant of a finite graph. In particular, its order coincides with the number of spanning trees of the graph. Equivalent definitions of the group $\mathrm{Jac}(G)$ can be found in papers [29, 8, 24].

Denote the vertex set of G by $V(G)$. For given $u, v \in V(G)$, we set a_{uv} equal to the number of edges between the vertices u and v. The matrix $A = A(G) = \{a_{uv}\}_{u,v\in V(G)}$, is called the *incidence matrix* of the graph G. The degree $d(v)$ of a vertex $v \in V(G)$ is defined as $d(v) = \sum_u a_{uv}$. Let $D = D(G)$ be the diagonal matrix indexed by the elements of $V(G)$ with $d_{vv} = d(v)$. The matrix $L = L(G) = D(G) - A(G)$ is called *the Laplacian matrix* of the graph G.

There is the following useful relation between the structure of the Laplacian matrix and the Jacobian of a graph G. Consider the Laplacian $L(G)$ as a homomorphism $\mathbb{Z}^{|V|} \to \mathbb{Z}^{|V|}$, where $|V| = |V(G)|$ is the number of vertices in G. The cokernel $\mathrm{coker}\,(L(G)) = \mathbb{Z}^{|V|}/\mathrm{im}\,(L(G))$ is an Abelian group. Let

$$\mathrm{coker}\,(L(G)) \cong \mathbb{Z}_{d_1} \oplus \mathbb{Z}_{d_2} \oplus \cdots \oplus \mathbb{Z}_{d_{|V|}}$$

be its Smith normal form satisfying the conditions $d_i \big| d_{i+1}$, $(1 \le i \le |V|)$. If the graph is connected, then the groups $\mathbb{Z}_{d_1}, \mathbb{Z}_{d_2}, \ldots, \mathbb{Z}_{d_{|V|-1}}$ are finite and $\mathbb{Z}_{d_{|V|}} = \mathbb{Z}$. In this case, $\mathrm{Jac}(G) \cong \mathbb{Z}_{d_1} \oplus \mathbb{Z}_{d_2} \oplus \cdots \oplus \mathbb{Z}_{d_{|V|-1}}$ is the Jacobian of the graph G. In other words, $\mathrm{Jac}(G)$ is isomorphic to the torsion subgroup of $\mathrm{coker}\,(L(G))$.

Problem 1. *Find a canonical one-to-one correspondence between the set of spanning trees of a finite connected graph G and the elements of the Abelian group $\mathrm{Jac}(G)$.*

Problem 2. *Find a natural geometric interpretation for generators of cyclic groups in the canonical decomposition*

$$\mathrm{Jac}(G) \cong \mathbb{Z}_{d_1} \oplus \mathbb{Z}_{d_2} \oplus \cdots \oplus \mathbb{Z}_{d_{|V|-1}}.$$

2.3.2. **Anton Betten, Colorado, USA.** Let $\mathrm{Cay}(\mathrm{Sym}_n, S)$ be a Cayley graph on the symmetric group with respect to the following generating sets: $S = \{(12), (23), \dots (n-1, n)\}$ or $S = \{(12), (13), \dots (1, n)\}$. One can replace S by another set of half-inversions.

Problem 3. *The question is whether the Cayley graph obtained in such a way is integral. That is, is it a graph whose eigenvalues are all integral?*

2.3.3. **Ŝtefan Gyürki, Banká Bystrica, Slovakia.** A *directed strongly regular graph* with parameters (v, k, t, λ, μ) is a regular directed graph on n vertices with valency k, such that every vertex is incident with t undirected edges, and the number of paths of length 2 directed from a vertex x to another vertex y is λ, if there is a directed edge from x to y, and μ otherwise.

Let L be a loop of order n (quasigroup with the multiplication $\{\cdot\}$ and an identity element e). Let us define a digraph Γ with the set of vertices

$$V(\Gamma) = \{1, 2, \dots, n\} \times \{1, 2, \dots, n\} \times \mathbb{Z}_3$$

and the set of directed edges

$$\begin{aligned} D(\Gamma) : (x, y, i) &\rightarrow (z, y, i), \quad \forall z \neq x \\ (x, y, i) &\rightarrow (x, z, i), \quad \forall z \neq x \\ (x, y, i) &\rightarrow (x \cdot y, z, i+1) \quad \forall z, \\ (x, y, i) &\rightarrow (z, y \cdot x, i-1) \quad \forall z. \end{aligned}$$

(The operations on the third coordinate are taken modulo 3.) We proved that the defined digraph Γ is a directed strongly regular graph with parameter set

$$(v, k, t, \lambda, \mu) = (3n^2, 4n-2, 2n, n, 4).$$

Problem 4. *(Gyürki, Klin) Is it true that non-isotopic loops L produce non-isomorphic graphs* Γ?

Problem 5. *Is it true that* $\mathrm{Aut}(\Gamma) \cong \mathrm{Par}(L)$ *where* Par(L) *is the autoparatopy group of the loop L?*

With the aid of a computer we checked that the answer for both questions is positive when $n \leq 6$.

2.3.4. **Sven Reichard, Dresden, Germany.** Let $q = p^s$ be a prime power. Consider the field $F = \mathrm{GF}(q)$. Let $V = F^2$ be a linear space endowed by the map $d : (u, v) \rightarrow \det\left(\begin{smallmatrix} u_1 & v_1 \\ u_2 & v_2 \end{smallmatrix}\right)$. Consider a subgroup $K \leq F^\star$ of the multiplicative group $F^\star$ of the field F and set $\Omega = (V \setminus \{0\})/K$. If $K = F^\star$, then $\Omega = PG(1, q)$ is the projective plane. Now we suppose that $K \neq F^\star$ and $K \neq 1$. This gives the map $\bar{d} : \Omega \rightarrow F^\star/K$. Also, we have $\mathrm{Aut}(V, d) = \mathrm{SL}(2, q)$. Define graph Γ with the set of vertices $V = \Omega$ and the adjacency relation $uK \sim vK$ iff $D(u, v) \in K$. Then $\mathrm{SL}(2, q)$ does not act faithfully. But $\mathrm{PSL}(2, q) \leq \mathrm{Aut}(\Gamma)$.

Problem 6. *Show that for q sufficiently large, there are no other automorphisms.*

Let us remark that Γ is an antipodal distance regular graph of diameter 3.

2.3.5. **Bojan Mohar, Burnaby, Canada.** Let G be a Cayley graph. Denote by $Z(G)$ the cycle space of G over $GF(2)$ formed by its Eulerian subgraphs. Let $\mathrm{Ham}(G)$ be subspace of $Z(G)$ generated by Hamiltonian cycles. Also, denote by $Z_t(G)$ the subspace generated by the cycle of length $\geq t$.

Problem 7. $\mathrm{Ham}(G) \leq Z(G)$ *is a subspace of codimension* ≤ 2.

The case of codimension 2 is realized for the Prism graph $\mathrm{Pr}(5) = C_5 \times K_2$.

Problem 8. *Is it true that* $\exists c > 0$ *such that for every Cayley graph of order* n *we have*

$$Z(G) = Z_{c\sqrt{n}}(G).$$

2.3.6. **Natalia Maslova, Ekaterinburg, Russia.** Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e., the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel graph* (or the *prime graph*) $\Gamma(G)$ of G; in this graph, the vertex set is $\pi(G)$ and different vertices p and q are adjacent if and only if $pq \in \omega(G)$.

It is known that if $\Gamma(G)$ contains a 3-coclique then G is non-solvable. It was proved in [15] that a graph Γ is isomorphic to the Gruenberg–Kegel graph of a finite solvable group if and only if the complement to Γ is 3-colorable and triangle-free. The following problem arises.

Problem 9. *Is there a graph whose complement is triangle-free, is not* 3*-colorable, and the graph is isomorphic to the Gruenberg–Kegel graph of an appropriate finite non-solvable group?*

I. Gorshkov and N. Maslova [Algebra and Logic, 2016, to appear] proved that the Gruenberg–Kegel graph $\Gamma(G)$ of a finite almost simple group G is isomorphic to the Gruenberg–Kegel graph of an appropriate finite solvable group if and only if the complement to $\Gamma(G)$ is triangle-free. Thus, if $\Gamma(G)$ gives a solution of Problem 9 in the positive then G is not almost simple.

2.4. **G2S2 Special Issue.** The G2S2 Special Issue contains the papers based on the plenary and contributed talks. Totally, 12 manuscripts were submitted, 7 papers are accepted to publish, and 4 manuscripts are still under review.

S. V. Avgustinovich, E. N. Khomyakova, and E. V. Konstantinova presented a manuscript on multiplicities of eigenvalues of the Star graph [2]. The Star graph S_n, $n \geqslant 2$, is the Cayley graph on the symmetric group Sym_n generated by the set of transpositions $\{(1\ 2), (1\ 3), \ldots, (1\ n)\}$. The authors consider the spectrum of the Star graph as the spectrum of its adjacency matrix. In 2012, R. Krakovski and B. Mohar [25] proved that the spectrum of S_n is integral. More precisely, they showed that for $n \geqslant 2$ and for each integer $1 \leqslant k \leqslant n-1$ the values $\pm(n-k)$ are eigenvalues of the Star graph S_n with multiplicity at least $\binom{n-2}{k-1}$. In this paper, analytic formulas for multiplicities of eigenvalues $\pm(n-k)$ for $k = 2, 3, 4, 5$ in the Star graph are obtained. It is also proved that any fixed integer has multiplicity at least $2^{\frac{1}{2}n\log n(1-o(1))}$ as an eigenvalue of S_n.

Spectral properties of Cayley graphs are also considered in the paper by M. Ghorbani and F. N. Larki [11], where the spectra of Cayley graphs of order p^2q (p and q are prime numbers and $2 < p < q$) are determined via their character table. The main results of this paper are related to the formula obtained in [9] for computing the spectrum of a Cayley graph $\Gamma = \mathrm{Cay}(G, S)$ with respect to the character table of G, where S is a symmetric normal subset of G.

Some simple groups which are determined by their character degree graphs are investigated in the manuscript of S. Heydari and N. Ahanjideh [17]. Throughout this paper, it is supposed that all groups are finite, G is a group, and $\rho(G)$ is the set of prime divisors of the irreducible character degrees of G. The character degree graph $\Delta(G)$ of G is a graph with the vertex set $\rho(G)$, and two vertices a and b are adjacent in $\Delta(G)$ if ab divides some irreducible character degree of G. In this paper, it is shown that some simple groups are uniquely determined by their orders and character degree graphs. In particular, it is concluded that M_{12} is not determined uniquely by its order and its character degree graph.

In the paper by M. Jalali-Rad and A. R. Ashrafi entitled "Erdös-Ko-Rado properties of some finite groups" [21], a subgroup G of the symmetric group Sym_n is considered. A subset A of G is said to be intersecting if for any pair of permutations $\sigma, \tau \in A$ there exists i, $1 \leq i \leq n$, such that $\sigma(i) = \tau(i)$. The group G has the Erdös-Ko-Rado property if the size of any intersecting subset of G is bounded above by the size of a point stabilizer in G. The group G has the strict Erdös-Ko-Rado property if every intersecting set of maximum size is the coset of the stabilizer of a point. In the paper, the Erdös-Ko-Rado and strict Erdös-Ko-Rado properties of the groups V_{8n}, U_{6n}, T_{4n} and SD_{8n} are investigated.

Automorphism groups of cyclotomic schemes over finite near-fields are considered in the paper by D. V. Churikov and A. V. Vasil'ev [12]. They proved that apart from a finite number of known exceptions, the automorphism group of a nontrivial cyclotomic scheme over a finite near-field $\mathbb{K}$ is isomorphic to a subgroup of the group $\mathrm{A\Gamma L}(1, \mathbb{F})$, where $\mathbb{F}$ is a field with $|\mathbb{F}| = |\mathbb{K}|$. Moreover, it was shown that the automorphism group of such a scheme is solvable if the base group of the scheme is solvable.

The paper by A. A. Makhnev and L. Yu. Tsiovkina concerns the classification of arc-transitive antipodal distance-regular graphs of diameter 3, which emerged after the classification of distance-transitive covers of complete graphs [13]. The authors investigated antipodal distance-regular graphs of diameter three and valency $q(q^{d-1}-1)/(q-1)$ with arc-transitive automorphism group which induces an almost simple permutation group on the antipodal classes with the socle isomorphic to $\mathrm{PSL}_d(q)$, where $d \geq 3$. It is proved that such a graph is necessarily bipartite [30].

The classification of S-rings over the elementary group $\mathbb{Z}_2^6$ of order 64 up to scheme isomorphism is presented in the manuscript submitted by S. Reichard [34]. There have been several previous efforts to enumerate all S-rings over small groups. In 2014, M. Ziv-Av stated in [36] that "For the groups of order 64 (especially for E_{64}), an innovative approach is necessary, as the current algorithms cannot finish the calculations in a reasonable time". The results by S. Reichard are obtained using the Weisfeiler–Lehman algorithm [35] which reduces the number of sets that need to be considered, as well as the need to look at all possible partitions. It is shown that there are 2082 schemes in total.

Strongly regular graphs with the same parameters as the symplectic graph are investigated by S. Kubota [27]. Orbit partitions of the automorphism group of the symplectic graph is considered and Godsil–McKay switching is applied to get the following results. Four families of strongly regular graphs with the same parameters as the symplectic graphs are found including the one discovered by A. Abiad and

W. H. Haemers [1]. It is proved that the switched graphs are non-isomorphic to each other by considering the number of common neighbors of three vertices.

Relations between semifield projective plane and its coordinatizing semifield using a linear space and a spread set are considered in the manuscript submitted by O. V. Kravtsova [26]. The results describing the relationship between autotopisms and automorphisms of finite semifield and collineations of the corresponding semifield plane are stated. In particular, the geometrical sense of an involution automorphism and its stabilizer is discussed. The results are illustrated by the examples of semifields of orders 64 and 81.

In their manuscript [22], V. V. Kabanov and A. V. Mityanina gave a description of all strictly Deza line graphs and strictly Deza graphs which are a union of closed neighborhoods of two non-adjacent vertices. A class of claw-free strictly Deza graphs which contains a 3-coclique is considered.

3. Social program

The organizers of G2S2 invested all possible efforts to guarantee a warm atmosphere and took care of any technical problems. They as well as the young students emphasized how glad they were that mathematicians from so many countries had come all the way to Siberia to meet their Russian colleagues.

The participants enjoyed the excursions taking a place in the Academic town and Novosibirsk city, and a few very friendly parties. Social program of G2S2 events also included sport activities (football, volleyball, beach volleyball).

It is worth to mention some comments left by a participant of the conference:

> *It was a great pleasure to come to Novosibirsk and give a talk on the conference G2S2. The conference was well-organized and very enjoyable. I hope that in the future we can expand our collaboration and friendship between scientists in our very different countries. We can learn a lot from each other. Hopefully in the future our students can travel freely between our countries and experience the wonderful opportunities in science.*

4. Acknowledgment

G2S2 organizers thank all participants for their interesting talks and very warm scientific atmosphere during the events. Our special thanks for all chairmen and chairwomen of the different sessions (minicourses, plenary talks, contributed talks). All of them showed their international qualification by giving substantial input to a big variety of mathematical aspects. We thank referees of the papers submitted to the G2S2 Special Issue whose suggestions significantly improved the quality and presentation of the most of the papers.

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