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DOI 10.17377/semi.2016.13.002УДК 519.724
MSC 60K25;90B15**STABILITY AND INSTABILITY OF A RANDOM MULTIPLE
ACCESS MODEL WITH ADAPTIVE ENERGY HARVESTING**

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ABSTRACT. We introduce a model for the classical synchronised multiple access system with a single transmission channel and a randomised transmission protocol (ALOHA). We assume in addition that there is an energy harvesting mechanism, and any message transmission requires a unit of energy. Units of energy arrive randomly and independently of anything else. We analyze stability and instability conditions for this model.

Keywords: random multiple access, stochastic energy harvesting, (in)stability, ALOHA algorithm, generalized Foster criterion.

1. INTRODUCTION

Nowadays the idea of usage of energy harvesting in communication systems is of a common interest, due to many applications. For example, sensor networks with rechargeable batteries that are harvesting energy from the environment can significantly extend the lifetime of the system. Another example of using energy harvesting was introduced in [1], where the authors considered a multi-user system with the base station that employs technology OFDM and a wireless channel to transmit both data and energy to its users. Usage of an energy harvesting mechanism in systems with the random multiple access presents new challenges and, particularly, in determining their stability regions. In [2], the authors considered a model with a decentralized energy harvesting mechanism, with assuming individual power supplies for a finite number of transmitting nodes, and studied their stability properties. In this paper, we consider another model (see fig. 1) with infinite

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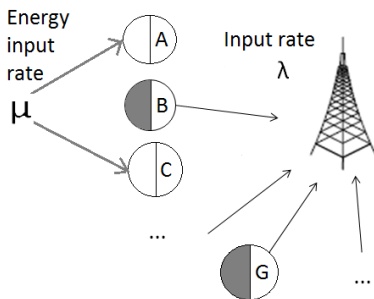


FIGURE 1. Random access of infinite number of users, where each of them is equipped with a battery for storing energy.

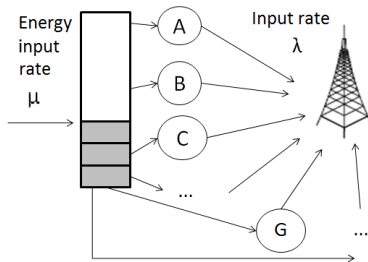


FIGURE 2. Random access of infinite number of users with common energy supply.

number of transmitters and separate power supplies per each transmitting node (with certain limitations on the intensity of supply) and examine its stability and instability. Earlier (see [3] and fig. 2), we considered random multiple access models with a common (centralized) renewable power supply and infinite number of transmitters. We proved that an additional energy limitation on a common energy supply may stabilize the system. At the same time, the model from [3] is simplistic and far from the practice (in comparison with that from [2]). On the other hand, if one considers infinite number of users in the model of [2], the system may lose stability.

Recall that the classical ALOHA (see, e.g., [4]) algorithm is always unstable in the system with infinitely many users, and has a non-empty stability region if the number of users is finite.

In this paper, we study a model with infinite number of transmitters/users having individual power supplies, and with energy harvesting mechanism with intensity that depends on the number of active transmitters. We are interested in conditions on the energy harvesting algorithm for (in)stability of the model.

2. THE MODEL

This is a system with a single transmission channel and messages that arrive and then depart after a successful transmission. Time is slotted, with ξ_n being the number of messages that arrive within time slot $[n-1, n)$ (the n -th time slot). We assume $\{\xi_n\}$ to form an i.i.d. sequence with a general distribution on non-negative

integers with finite mean $\lambda \in (0, 1)$. Every message is equipped with a battery of a unit volume for storing energy. Each message arrives into the system with empty battery and waits for energy harvesting (another model where messages arrive into the system with units of energy was analyzed in [5]). The harvesting mechanism within any time slot is as follows: if a message does not have energy for transmission, its battery receives a unit of energy with probability $\mu > 0$ or nothing otherwise, independently of everything else. If the battery is full, no new units of energy are accepted further (equivalently, an energy unit may arrive, but then is rejected by the user since the battery is already full).

Within each time slot, any message with energy is transmitted with probability p (where $p \in (0, 1]$ is fixed) and stays silent with probability $1 - p$. If there is a transmission of only one message, it is successful, and the message leaves the system. If two or more messages are transmitted simultaneously, a collision occurs, all messages stay in the system, but lose their units of energy (in other words, each unit of energy is used for a single transmission attempt only).

The dynamics of the model may be described as follows. Assume that at the start of, say, the $(n + 1)$ st time slot, there are ξ_n “new” messages that just arrived and q_n “old” messages that arrived earlier. Let $v_n \leq q_n$ be the number of old messages that have energy units for transmission. Let $\{U_{n,i}, -\infty < n < \infty, i \geq 1\}$ and $\{\widehat{U}_{n,i}, -\infty < n < \infty, i \geq 1\}$ be two independent families of i.i.d. random variables that are uniformly distributed in the interval $(0, 1)$. Let $\mathbf{I}(A)$ be the indicator function of event A : it takes value 1 if the event occurs, and value 0 otherwise. Then $B_n(k, p) = \sum_{i=1}^k \mathbf{I}(U_{n,i} < p)$, $-\infty < n < \infty, k \geq 0$ and $\widehat{B}_n(k, \mu) = \sum_{i=1}^k \mathbf{I}(\widehat{U}_{n,i} < \mu)$, $-\infty < n < \infty, k \geq 0$ are two mutually independent families of random variables with binomial distributions that do not depend on anything else. We will also use random variables $D_n(k, 1 - p) := k - B_n(k, p) = \sum_{i=1}^k \mathbf{I}(U_{n,i} > p)$ that have a binomial distribution with parameters k and $1 - p$.

The pairs (q_n, v_n) form a two-dimensional time-homogeneous Markov chain with the following dynamics:

$$\begin{aligned} q_{n+1} &= q_n - \mathbf{I}(B_n(v_n, p) = 1) + \xi_n, \\ v_{n+1} &= v_n - B_n(v_n, p) + \widehat{B}_n(q_n - v_n + \xi_n, \mu). \end{aligned}$$

Note that we can replace $v_n - B_n(v_n, p)$ by $D_n(v_n, p)$ in the last line.

In practice, the intensity μ of energy harvesting is *adaptive*: it depends on the number q of the messages in the system, it decreases when q increases, see e.g. [7], [8]. In this paper, we assume that the dependence is inverse proportional: for $n = 0, 1, \dots$, the intensity $\mu = \mu_n$ in the n -th time slot is given by

$$(1) \quad \mu_n = \mu(q_n) = \frac{c}{q_n},$$

for some $c > 0$.

3. MAIN RESULTS

We say that the system is *stable* if the underlying Markov chain is positive recurrent, and *unstable* if $q_n + v_n \rightarrow \infty$ in probability, as $n \rightarrow \infty$.

To analyse the (in)stability of (q_n, v_n) , we introduce an auxiliary one-dimensional Markov chain \widetilde{V}_n by the following recursion:

$$(2) \quad \widetilde{V}_{n+1} = \widetilde{V}_n - B_n(\widetilde{V}_n, p) + \eta_n,$$

where $\{\eta_n\}_{n=0}^{\infty}$ is an i.i.d. sequence of Poisson random variables with parameter c . One can easily apply the classical Foster criterion and condition $\mathbf{P}(\eta_1 = 0) > 0$ to conclude that this Markov chain is *ergodic*: it admits a unique stationary distribution, say π (see, e.g., [6]) and converges to it in the total variation norm, for any initial value $\tilde{V}_0 \in \mathbb{Z}_+$. Further, note that π is a Poisson distribution with parameter c/p and that the Markov chain is geometrically ergodic, i.e. its distribution converges to the stationary one geometrically fast – see Lemma 1 below.

Let a random variable \tilde{V} have the distribution π and do not depend on anything else. Then random variable $B_1(\tilde{V}, p)$ has a Poisson distribution with parameter c and $\mathbf{P}(B_1(\tilde{V}, p) = 1) = ce^{-c}$.

Theorem 1. *If, for some $c > 0$,*

$$\mu(q) = \min \left\{ \frac{c}{q}, 1 \right\}$$

then the stochastic system described by the 2-dimensional Markov chain

$$\begin{cases} q_{n+1} = q_n - \mathbf{I}(B_n(v_n, p) = 1) + \xi_n, \\ v_{n+1} = v_n - B_n(v_n, p) + \hat{B}_n(q_n - v_n + \xi_n, \mu(q_n)) \end{cases}$$

is stable if $\lambda < ce^{-c}$ and unstable if $\lambda > ce^{-c}$.

Remark 1. *The origin $(0, 0)$ is achievable from all other states $(x, y) \in \mathbb{Z}_+^2$ of the 2-dimensional Markov chain (v_n, q_n) . Since $\mathbf{P}(\xi_1 = 0) > 0$, the Markov chain is aperiodic. Therefore, if the Markov chain is positive recurrent, it is also ergodic, i. e. there is a stationary distribution, say ϕ , on \mathbb{Z}_+^2 such that, for any initial value (q_0, v_0) , the distribution of (q_n, v_n) converges to ϕ in the total variation norm*

$$\sup_{A \subset \mathbb{Z}_+^2} |\mathbf{P}((q_n, v_n) \in A) - \phi(A)| \rightarrow 0, \quad n \rightarrow \infty.$$

Remark 2. *The stability condition attains its maximum value e^{-1} for $c = 1$ and doesn't depend on parameter p .*

Remark 3. *More or less straightforward modifications of the proof of Theorem 1 lead to the following results. If, instead of (1), we assume that*

$$q \cdot \mu(q) \rightarrow \infty, \quad \text{as } q \rightarrow \infty,$$

then the system is similar to the classical ALOHA, which is unstable. On the other hand, if we assume that

$$q \cdot \mu(q) \rightarrow 0, \quad \text{as } q \rightarrow \infty,$$

then the system is unstable again as it does not have enough energy to transmit the «old» messages.

4. AUXILIARY LEMMA

We summarize a number of simple, but important properties in the following lemma.

Lemma 1. *Let $\{Z_n\}$ be an i.i.d. sequence of non-negative integer-valued random variables with finite mean $\mathbf{E}Z_0$. Assume it does not depend on i.i.d. random variables $\{U_{n,i}, -\infty < n < \infty, i \geq 1\}$ having the uniform- $(0, 1)$ distribution.*

(I) For any initial value W_0 and for any $p \in (0, 1)$, the sequence

$$(3) \quad W_{n+1} = W_n - B_n(W_n, p) + Z_n \equiv D_n(W_n, 1 - p) + Z_n$$

is ergodic.

(II) For $m \leq n$ define operators

$$D_{m:n}(k, 1-p) = D_n(D_{n-1}(\dots(D_{m+1}(D_m(k, 1-p), 1-p)\dots), 1-p)$$

and note that random variable $D_{m:n}(k, 1-p)$ has a binomial distribution with parameters k and $(1-p)^{n-m+1}$. Then a stationary sequence $\{W^{(n)}\}$ given by

$$(4) \quad W^{(n)} = Z_{n-1} + \sum_{j=1}^{\infty} D_{(n-j-1):(n-2)}(Z_{n-j-1}, 1-p)$$

has finite mean $\mathbf{E}W^{(n)} = \mathbf{E}Z_0/p$ and forms a stationary solution to recursive equation (3),

$$W^{(n+1)} = D_n(W^{(n)}, 1-p) + Z_n.$$

(III) Let Q be the distribution of $W^{(0)}$. Then, for any initial value W_0 ,

$$(5) \quad \mathbf{P}(W_n = W^{(n)}) = \mathbf{P}(W_l = W^{(l)}), \quad \text{for all } l \geq n) \rightarrow 1$$

and, in particular,

$$\sup_{A \subset \mathbb{Z}_+^2} |\mathbf{P}(W_n \in A) - Q(A)| \leq \mathbf{P}(W_n \neq W^{(n)}) \rightarrow 0, \quad \text{as } n \rightarrow \infty.$$

In particular, if W_0 and Z_0 have finite exponential moments, then

$$\mathbf{P}(W_n \neq W^{(n)}) \leq K_1 e^{-K_2 n},$$

for some $K_1, K_2 > 0$ and for all $N \geq 0$.

(IV) For any $n \geq 0$,

$$W_n \leq W_0 + W^{(n)} \quad \text{a.s.}$$

and

$$\mathbf{E}W_n \leq \mathbf{E}W_0 + \mathbf{E}Z_0/p.$$

(V) Let $\{\tilde{Z}_n\}$ be any other sequence of non-negative and integer-valued random variables, such that $0 \leq \tilde{Z}_n \leq Z_n$ a.s., for all n . Consider a recursion

$$\tilde{W}_{n+1} = D_n(\tilde{W}_n, 1-p) + \tilde{Z}_n$$

with integer initial value $0 \leq \tilde{W}_0 \leq W_0$. Then

$$\tilde{W}_n \leq W_n \quad \text{a.s., for all } n \geq 0.$$

(VI) In particular, when Z_0 is a Poisson random variable with parameter c , formula (4) and the splitting and composition theorems imply that every $W^{(n)}$ has Poisson distribution with parameter c/p .

The PROOF of all statements of the lemma is straightforward. First, we verify that the right-hand side of (4) is finite. This follows from finiteness of $\mathbf{E}Z_0$ since expectation of the sum of non-negative random variables always equals to the sum of expectations, and

$$\begin{aligned} \mathbf{E}Z_{n-1} + \sum_{j=1}^{\infty} \mathbf{E}(D_{(n-j-1):(n-1)}(Z_{n-j-1}, 1-p)) &= \sum_{j=0}^{\infty} \mathbf{E}Z_0 \cdot (1-p)^j \\ &= \mathbf{E}Z_0 \cdot \frac{1}{p} < \infty. \end{aligned}$$

Therefore, the sum in the right-hand side of (4) must contain only finite number of non-zero elements. Further, if $\mathbf{E}e^{K_0 Z_0}$ is finite for some $K_0 > 0$, then, for $K \in (0, K_0)$,

$$\mathbf{E}e^{KW^{(0)}} = \prod_{n=0}^{\infty} \mathbf{E} (1 + (e^K - 1)(1 - p)^n)^{Z_0}$$

where all terms in the right product are finite and, as $K \downarrow 0$ and uniformly in n ,

$$\mathbf{E} (1 + (e^K - 1)(1 - p)^n)^{Z_0} = 1 + (1 + o(1))K(1 - p)^n \mathbf{E}Z_0$$

so the product is finite too. Indeed, for any $h > 0$,

$$1 + hZ_0 \leq (1 + h)^{Z_0} \leq e^{hZ_0} \leq 1 + hZ_0 + \frac{h^2 Z_0^2}{2} e^{hZ_0} \leq 1 + hZ_0 + \frac{h^2 Z_0^4}{4} + \frac{h^2 e^{2hZ_0}}{4}$$

and then

$$1 + h\mathbf{E}Z_0 \leq \mathbf{E}(1 + h)^{Z_0} \leq 1 + h\mathbf{E}Z_0 + O(h^2), \quad \text{as } h \downarrow 0.$$

Then one can take $h = (e^K - 1)(1 - p)^n$ and note that $e^K - 1 = K(1 + o(1))$, as $K \downarrow 0$.

Next, at any time n , we have a set of W_n elements, each of which disappears at time $(n + 1)$ with probability p , independently of everything else, while Z_n new elements arrive. Then the original W_0 elements disappear in a (random) finite time, say T , and after that all W_n include only new items that arrive after time 0. Further, if $\mathbf{E}e^{K_0 Z_0}$ is finite for some $K_0 > 0$, then, for k sufficiently small and uniformly in n ,

$$\begin{aligned} \mathbf{P}(T > n) &= \mathbf{P}(D_{0:(n-1)}(Z_0, 1 - p) \geq 1) \\ &= \mathbf{E} (1 - (1 - (1 - p)^n)^{Z_0}) \\ &\leq \mathbf{P}(Z_0 > Kn) + 1 - (1 - (1 - p)^n)^{Kn} \\ &\leq \tilde{K}e^{-Kn} + (1 - p)^n Kn(1 + o(1)) \\ &= o(e^{-rn}), \end{aligned}$$

for some \tilde{K} and for any $r < \min\left(K, \log \frac{1}{1-p}\right)$. Similar observations hold for $W^{(n)}$: all elements of $W^{(0)}$ leave by finite time, say \hat{T} , which has a finite exponential moment if Z_0 does. Therefore W_n and $W^{(n)}$ coincide after time $\max(T, \hat{T}) \leq T + \hat{T}$ which also has a finite exponential moment.

5. PROOF OF THEOREM 1

We start with the proof of stability. Let $X_n = (q_n, v_n)$.

By the generalized Foster criterion (see, e.g., [6]), it is enough to find a test («Lyapunov») function $L(x) \geq 0$ and a (sufficiently large) positive integer N_0 such that the set

$$D = \{x \in \mathbb{Z}_+^2 : L(x) \leq N_0\},$$

is compact and, for a properly chosen positive and bounded from above integer-valued function $g(x)$, the Markov chain has a bounded mean drift on D ,

$$(6) \quad \sup_{x \in D} \mathbf{E} (L(X_{g(X_0)}) - L(X_0) \mid X_0 = x) \equiv K < \infty,$$

and the drift is uniformly negative on the complement D^c of D : for any $x \in D^c$,

$$(7) \quad \mathbf{E} (L(X_{g(X_0)}) - L(X_0) \mid X_0 = x) \leq -\varepsilon$$

where ε is a fixed positive constant. We choose the test («Lyapunov») function L by

$$L(x) = q + v, \quad \text{for } x = (q, v) \in \mathbb{Z}_+^2, \quad v \leq q,$$

and then a proper choice of function $g(x)$ leads to the stability result.

Note that since

$$\mathbf{E}(q_1 - q_0 \mid (q_0, v_0)) = \lambda - \mathbf{P}(B_0(v_0, p) = 1 \mid (q_0, v_0)) \leq 1$$

and

$$\begin{aligned} \mathbf{E}(v_1 - v_0 \mid (q_0, v_0)) &= -pv_0 + (q_0 - v_0 + \lambda) \mathbf{I}(q_0 \leq c) \\ &+ c \frac{q_0 - v_0 + \lambda}{q_0} \mathbf{I}(q_0 > c) \leq 2(q_0 - v_0 + \lambda) \end{aligned}$$

the drift (6) is bounded from above on the set D , for any choice of constant N_0 .

Recall that $v_0 \leq q_0$, so if we choose N_0 such that $\frac{N_0}{2} > c$, then $q_0 > c$ for $X_0 \in D^c$. Therefore, the drift (7) is uniformly negative for $g(x) = 1$ for all $(q_0, v_0) \in D^c$, such that $v_0 > \frac{c+\lambda+1}{p}$.

Now we complete the stability proof, with choosing sufficiently large values of N_0 and k such that, with $g(x) \equiv k$, inequality (7) uniformly holds for all $x = (q_0, v_0) \in D^c$ such that $v_0 \leq \frac{c+\lambda+1}{p}$.

Let $\{Y_n\}$ be i.i.d. random variables that do not depend on $\{\xi_n\}$ and have distribution

$$\mathbf{P}(Y_n > x) = \sup_{q \geq c} \mathbf{P}(B_1(q, \mu(q)) > x).$$

By Chernoff's inequality, for $x \geq c$ and any $\alpha > 0$, the right-hand side of the latter equality does not exceed

$$\sup_{q \geq c} \mathbf{E} e^{\alpha B_1(q, c/q)} e^{-\alpha x} = \sup_{q \geq c} \left(1 + (e^\alpha - 1) \frac{c}{q} \right)^q e^{-\alpha x} \equiv C_1 e^{-\alpha x}$$

where C_1 is finite since $\left(1 + (e^\alpha - 1) \frac{c}{q} \right)^q \rightarrow \exp(c(e^\alpha - 1)) < \infty$ as $q \rightarrow \infty$. Therefore the distribution of Y_1 is proper and, moreover, has a finite exponential moment.

Let now

$$(8) \quad Z_n = Y_n + \xi_n \quad \text{and} \quad W_{n+1} = D_n(W_n, 1-p) + Z_n, \quad \text{with} \quad W_0 = v_0.$$

Then, by property (V) of Lemma 1,

$$\mathbf{E}(v_n \mid v_0) \leq \mathbf{E}(W_n \mid W_0) \leq v_0 + C_2$$

where

$$C_2 = \mathbf{E}(Y_1 + \xi_1)/p \equiv (\mathbf{E}Y_1 + \lambda)/p.$$

Next, choose $\delta > 0$ such that $\lambda + \delta < ce^{-c}$ and consider the auxiliary Markov chain \tilde{V}_n . Since it is ergodic, for given $\delta > 0$ and $R := (c + \lambda + 1)/p$ (we assume R to be an integer), there exists $l \in \mathbb{Z}_+$ such that, for any $n \geq l$,

$$(9) \quad \sup_{\tilde{v}_0 \leq R} \left| \mathbf{P}(B_n(\tilde{V}_n, p) = 1) - ce^{-c} \right| < \frac{\delta}{3}.$$

Then we choose $k > l$ such that

$$(10) \quad -\varepsilon := k\lambda + C_2 - (k-l)(ce^{-c} - \delta) < 0.$$

Now choose C_3 so large that, for the sequence $\{W_n\}$ defined in (8) and with initial value $W_0 = R$, the probability of event

$$A(C_3) = \{W_i \leq C_3 \text{ for all } 0 \leq i \leq k\}$$

is not smaller than $1 - \delta/3$.

The famous Poisson theorem allows the following simple extension: as $q \rightarrow \infty$, the distribution of random variable $B_1(q - v + \xi_1, c/q)$ converges in the total variation norm to the Poisson distribution with parameter c , uniformly in all $v \in \{0, 1, \dots, C_3\}$. Therefore, one can choose \hat{q} such that the total probability distance between the distribution of $B_1(q - v + \xi_1, c/q)$ and Poisson distribution with parameter c does not exceed $\delta/3$, for all $q \geq \hat{q}$.

Finally, let $N_0 = \hat{q} + k + R$. For any (q_0, v_0) with $q_0 + v_0 \geq N_0$ and $v_0 \leq R$, we have $q_i \geq \hat{q}$ for all $0 \leq i \leq k$, since $q_{i+1} \geq q_i - 1$ a.s. Then we may put all bounds together to conclude that, under the assumptions from above, for any $i = l + 1, \dots, k$,

$$|\mathbf{P}(B_i(v_i, p) = 1 \mid (q_0, v_0)) - ce^{-c}| \leq \delta$$

and, therefore, (7) holds. Indeed, with $x = (q_0, v_0)$ where $q_0 + v_0 \geq N_0$ and $v_0 \leq R$, we have

$$\begin{aligned} \mathbf{E}(L(X_k) - L(X_0) \mid X_0 = x) &= \mathbf{E}(q_k - q_0 \mid X_0 = x) + \mathbf{E}(v_k - v_0 \mid X_0 = x) \\ &= C_2 + k\lambda - \sum_{i=1}^k \mathbf{P}(B_i(v_i, p) = 1 \mid X_0 = x) \\ &\leq C_2 + k\lambda - \sum_{i=l+1}^k \mathbf{P}(B_i(v_i, p) = 1 \mid X_0 = x) \leq -\varepsilon \end{aligned}$$

where ε is from (10). This completes the proof of the stability part of the theorem.

The proof of the second part of the theorem is based on Theorem 2.1 from [9]. Assume one may define a nonnegative function $\tilde{L}(x)$ in such a way that, for a properly chosen bounded positive integer-valued function $\tilde{g}(x)$ and for any initial value $X_0 = x$ with $\tilde{L}(x) \geq \tilde{N}_0$, for some fixed $\tilde{N}_0 > 0$, the Markov chain has a positive drift

$$(11) \quad \mathbf{E}(\Delta_{\tilde{g}(x)} \mathbf{I}(\Delta_{\tilde{g}(x)} \leq M) \mid X_0 = x) \geq \tilde{\varepsilon}$$

where $\Delta_{\tilde{g}(x)} = \tilde{L}(X_{\tilde{g}(x)}) - \tilde{L}(x)$ given $X_0 = x$, and the sequence

$$(12) \quad \left(\Delta_{\tilde{g}(x)}^-\right)^2 = (\min\{0, \Delta_{\tilde{g}(x)}\})^2$$

is uniformly integrable in all x such that $\tilde{L}(x) \geq \tilde{N}_0$. Here $\tilde{\varepsilon}$ and M are fixed positive constants. By Theorem 2.1 from [9], under conditions (11)-(12) imply that, for any $x \in \mathbb{Z}_+^2$,

$$P\left(\tilde{L}(X_n) \rightarrow \infty \mid X_0 = x\right) = 1.$$

We will apply this Theorem for the following choice of functions \tilde{L} and \tilde{g} : for $x = (q, v)$, we let $\tilde{L}(x) = q$, and $\tilde{g}(x) = 1$ if $v > \tilde{R}$ and $\tilde{g}(x) = \tilde{k}$ if $v \leq \tilde{R}$. We choose \tilde{R} and \tilde{k} below. Before that, we like to mention that our choice of functions leads to redundancy of condition (12) since $q_{i-1} \geq q_i - 1$ a.s.. Also, since $\lambda = \mathbf{E}\xi_1$ is

finite, the increments $\Delta_{\tilde{g}(x)}$ are uniformly integrable in x and, therefore, condition (11) is equivalent to

$$(13) \quad \mathbf{E}(\Delta_{\tilde{g}(x)}) \geq \tilde{\varepsilon},$$

for some positive $\tilde{\varepsilon}$.

Let \tilde{R} be any positive integer such that

$$\gamma := \sup_{n \geq \tilde{R}} \mathbf{P}(B_1(n, p) = 1) < \lambda.$$

Since $\mathbf{P}(B_1(n, p) = 1) \rightarrow 0$ as $n \rightarrow \infty$, one can always find such \tilde{R} . Then, indeed, for $v_0 > \tilde{R}$

$$\mathbf{E}(q_1 - q_0 \mid X_0 = (q_0, v_0)) \geq \lambda - \gamma > 0.$$

Assume now that $\tilde{R} > 1$ and consider the case where q_0 is large and $v_0 \leq \tilde{R}$. On the essence, we repeat the scheme of the last part of the proof of stability, but with considering the opposite inequalities.

First, we choose $0 < \tilde{\delta} < \lambda - ce^{-c}$ and find then \tilde{l} such that, for all $n \geq \tilde{l}$,

$$(14) \quad \sup_{\tilde{v}_0 \leq \tilde{R}} \left| \mathbf{P}(B_n(\tilde{V}_n, p) = 1) - ce^{-c} \right| < \frac{\tilde{\delta}}{3}.$$

Then we choose $\tilde{k} > \tilde{l}$ such that

$$\tilde{\varepsilon} := \tilde{k}\lambda - \tilde{l} - (\tilde{k} - \tilde{l})(ce^{-c} + \tilde{\delta}) > 0.$$

Then we choose \tilde{C} so large that, for the sequence $\{W_n\}$ defined in (8) and with initial value $W_0 = \tilde{R}$, the probability of event

$$A(\tilde{C}) = \{W_i \leq \tilde{C} \text{ for all } 0 \leq i \leq n\}$$

is not smaller than $1 - \tilde{\delta}/3$.

Finally we choose \hat{q} such that the total probability distance between the distribution of $B_1(q - v + \xi_1, c/q)$ and Poisson distribution with parameter c does not exceed $\tilde{\delta}/3$, for all $q \geq \hat{q}$ and $v \leq \tilde{C}$.

Then, putting altogether, we obtain that if $q_0 > \hat{q} + \tilde{k}$, then

$$\mathbf{E}(q_{\tilde{k}} - q_0 \mid X_0 = (q_0, v_0)) \geq \tilde{\varepsilon}$$

if $v_0 \leq \tilde{R}$. This completes the proof of the second part of the theorem.

6. CONCLUSION

We studied stability of the random access system under stochastic energy harvesting for a model with infinite number of transmitting nodes. We assumed that the harvesting intensity of individual power batteries depends on the number of active users. General stability and instability conditions have been obtained.

The model introduced in this paper assumes an individual energy supply mechanism for each users. One may comment that, in existing wireless systems, the energy harvesting mechanism is such that each user has an individual storage element (battery or capacitor) which is charged by the common energy mechanism (see, for instance, [1]). If the number of users is relatively low, then the amount of energy arriving at a single user does not depend on the total number of users that are storing the energy. However, if the number of users becomes sufficiently large, the energy arriving at an individual user starts to decrease. This effect was discussed

in [7], [8]. Thus, the models introduced in this paper reflects the features of wireless systems with an energy harvesting mechanism that has a single base station and a large number of user devices.

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