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# NON-UNIFIABILITY IN LINEAR TEMPORAL LOGIC OF KNOWLEDGE WITH MULTI-AGENT RELATIONS

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Abstract. The paper is devoted to the study of the unification problem in the linear temporal logic of knowledge with multi-agent relations (denoted in the sequel as $LFPK$). This logic is based on frames (models) with time points represented by integer numbers from $Z$ and the information clusters $C^i$ for $i \in Z$ with multi-agent accessibility relations $R_i$.

The first main result is a theorem describing a criterion for formulas to be not unifiable in $LFPK$. The second one is a construction of a basis for all inference rules passive in $LFPK$.

**Keywords:** unification, modal temporal logic, passive inference rules.

## 1. Introduction

The investigation of unification in various logical systems is one of highly developing areas of modern mathematical logic and computer science. Arisen in the field of Computer Science, mainly in the form of the possibility of transforming two different terms into syntactically equivalent ones (by the replacing its variables, cf. [1, 2]), the problem eventually changed the course to the study of semantic equivalence ([3, 4]).

For the majority of non-classical logics (modal, intuitionistic, temporal, etc.) there are special dual equational theories of algebraic systems, so their unification problems are interpreted into the corresponding logic-unificational counterparts ([5, 6, 7]). The basic unification problem can be generalized to a more difficult question: whether the formula can be converted into a theorem after replacing only a part of the variables (keeping the rest, as a set of parameters, intact). This problem has been studied and solved for some modal and intuitionistic logics (cf. e.g. V.

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Rybakov [8, 9, 10] for the case of intuitionistic logic itself and modal logics S4 and Gr$z$).

The unification in intuitionistic logic and in propositional modal logics over $K4$ was investigated by S. Ghilardi by offering technique of projective formulas, see [11, 12, 13, 14, 15] (with the application of ideas on projective algebras, using techniques of projective formulas). In these papers the problem of constructing finite complete sets of unifiers was solved for the considered logic and efficient algorithms were found. Such an approach proved to be useful and effective in dealing with the admissibility and the basis of admissible rules (cf. Jerabek [16, 17, 18], Iemhoff, Metcalfe [19, 20]). If algorithms for construction of computable finite sets of unifiers are found, it directly gives a solution of the admissibility problem.

Temporal logic is also a very dynamic area of mathematical logic and computer science (cf. Gabbay and Hodkinson [21, 22, 23]). In particular, LTL (linear temporal logic) has a significant applications in the field of Computer Science (cf. Manna, Pnueli [24, 25], Vardi [26, 27]). A solution of the problem of admissibility for rules in $LTL$ was found by Rybakov [28], the basis of admissible rules in $LTL$ has been discovered by Babenyshev and Rybakov in [29] (and for the case without the operator Until it was done in [30]).

Rybakov has solved the unification problem for formulas with coeficients in $LTL$ [31, 32]; its analogs were also solved for the basic modal and intuitionistic logic in [33, 34]. In particular, in [31] it was proved that not all unified in $LTL$ formulas are projective, and in [32] the projectivity of any unified formulas in $LTL_u$ was proved (it is a fragment of $LTL$ with the operator Until only, no NEXT). In the paper of Dzik and Wojtylak [35] the same result was obtained for the modal linear logic $S4.3$.

V.Rybakov in [36] found a description of all non-unifiable formulas in a broad class of modal logics: in the extensions of S4 (Theorem 1 below) and $[K4 + \Box\bot \equiv \bot \in \mathcal{L}]$ (Theorem 2) and also constructed finite bases for rules passive in these logics. Using results from [37], following closely to this technique, in this our paper we find a criterion for non-unifiability of formulas in the linear temporal logic of knowledge with multi-agent relations – $LFPK$ and construct a basis for inference rules passive in this logic.

## 1. Necessary definitions, known facts, and notation

Before describing the main results of the paper, it will be useful to recall some definitions and known results related to the issue of unification and the logic $LFPK$ in general. For more detailed studies of the results presented in this section (as well as their proofs), we refer to [36] and [37]

Firstly, we recall what does it mean that a formula is unifiable in a logic. Let $\mathcal{L}$ be a logic with the formula $\phi(p, q)$ which describes the equivalent formulas (e.g. for propositional calculus PC, $\phi(p, q) := (p \to q)\&(q \to p))$. We say that a formula $\alpha$ is equivalent to a formula $\beta$ in $\mathcal{L}$, and we write $\alpha \equiv_{\mathcal{L}} \beta$, if $\vdash_{\mathcal{L}} \phi(\alpha, \beta)$. For convenience, $\phi(\alpha, \beta)$ may be shortly denoted as $\alpha \equiv \beta$. We consider here a logic $\mathcal{L}$ as a set of formulas.

**Definition 1.** *A formula $\alpha(p_1, \ldots, p_n)$ is unifiable in an algebraic logic $\mathcal{L}$ iff there is a tuple of formulas $\delta_1, \ldots, \delta_n$ such that $\alpha(\delta_1, \ldots, \delta_n) \in \mathcal{L}$*

**Definition 2.** *Formulas $\alpha(p_1,\dots,p_n)$ and $\beta(p_1,\dots,p_n)$ are said to be unifiable in an algebraic logic $\mathcal{L}$ iff there is a tuple of formulas $\delta_1,\dots,\delta_n$: $[\alpha(\delta_1,\dots,\delta_n) \equiv \beta(\delta_1,\dots,\delta_n)] \in \mathcal{L}$. In this case, the tuple $\delta_1,\dots,\delta_n$ is called an unifier for these two formulas.*

**Corollary 1** ([36, 2.7])**.** *For the logics $SIL$, $S4_{ext}$, $K4+\Box\bot\equiv\bot$, the unifiers for unifiable formulas can be effectively found among sequences of formulas $\top$ and $\bot$.*

**Theorem 1** ([36, 2.10])**.** *For any modal logic $\mathcal{L}$ extending $S4$ and any modal formula $\alpha$, $\alpha$ is not unifiable in $\mathcal{L}$ iff the formula $\Box\alpha \to \left[\bigvee_{p\in Var(\alpha)} \Diamond p \wedge \Diamond\neg p\right]$ is provable in $\mathcal{L}$ (that is, this formula belongs to $\mathcal{L}$, as to the set of its theorems).*

**Theorem 2** ([36, 2.11])**.** *For any modal logic $\mathcal{L}$ extending K4 with $\Box\bot\equiv\bot\in\mathcal{L}$ and any modal formula $\alpha$, $\alpha$ is not unifiable in $\mathcal{L}$ iff the formula $\Box\alpha\wedge\alpha \to \left[\bigvee_{p\in Var(\alpha)} \Diamond p \wedge \Diamond\neg p\right]$ is provable in $\mathcal{L}$.*

**Theorem 3** ([37, 3.1])**.** *A modal formula $\alpha$ is non-unifiable in $LTK$ iff the formula $\Box_{\leq}\alpha \to \left[\bigvee_{p\in Var(\alpha)} \Diamond_{\leq} p \wedge \Diamond_{\leq}\neg p\right]$ is a theorem in $LTK$.*

**Definition 3.** *For any given rule $r := A/B$, $r$ is a consequence of a sequence of rules $r_1 := A_1/B_1, \dots, r_n := A_n/B_n$ in a logic $\mathcal{L}$ if there is a derivaton in $\mathcal{L}$ for the conclusion $B$ from the premise $A$, as a hypothesis, by means of rules from $r_1, \dots, r_n$, theorems of $\mathcal{L}$ and postulated rules of $\mathcal{L}$ (e.g. modus ponense for classical propositional logic or the intuitionistic logic).*

**Definition 4.** *A set of rules $BR$ is a basis for a set of rules $RS$ in a logic $\mathcal{L}$ if any rule $r\in RS$ is a consequence of some rules from $BR$ in $\mathcal{L}$.*

## 2. Semantics of $LFPK$

The alphabet of the language $\mathcal{L}^{LFPK}$ includes a countable set of propositional variables $P := \{p_1,\dots,p_n,\dots\}$, brackets (, ) default Boolean logical operations and a variety of unary modal operators $\{\Box_F,\Box_P,\Box_1,\dots,\Box_n\}$. Every propositional variable $p\in P$ is a well-formed formula (wff), and if $A,B$ are any wff, then so are $(A\vee B),(A\wedge B),(A\to B),\neg A,\Box_F A,\Box_P A,\ \Box_i A,\ i\in\{1,\dots,n\}$. We abbreviate $Fma(\mathcal{L}^{LFPK})$ as a set of all wff in the language $\mathcal{L}^{LFPK}$ (hereinafter referring to the *formula* will be understood as formula from the set $Fma(\mathcal{L}^{LFPK})$).

Logical operations $\Diamond_F$, $\Diamond_P$, $\Diamond_i$ are defined by means of logical operations $\Box_F$, $\Box_P$, $\Box_i$ as follows: $\Diamond_F=\neg\Box_F\neg$, $\Diamond_P=\neg\Box_P\neg$, $\Diamond_i=\neg\Box_i\neg$.

The meaning of the described modal operations are defined as follows. $\Box_P A$: $A$ is true at all previous and at the current time point; $\Box_F A$: $A$ is true at the given time point and will be true at all future ones. $\Box_i A$ means that $A$ is true at all informational states which available to the agent $i$.

Semantics for the language $\mathcal{L}^{LFPK}$ models linear and discrete stream of computational processes at which each point in time is associated with an integer number $n\in\mathbb{Z}$.

**Definition 5.** *Temporal k-modal Kripke-frame is a tuple $T=\langle W_T,R_1,R_2,\dots,R_k\rangle$, where $W_T$ is a non-empty set of worlds, $R_1,\dots,R_k$ are some binary relations on $W_T$, where $R_2=R_1^{-1}:=\{(a,b)|\,(b,a)\in R_1\}$ is a converse relation to $R_1$.*

**Definition 6.** *Let $F=\langle W_F,R_1,\dots,R_k\rangle$ be a Kripke-frame. For all $R_i$, an $R_i$-cluster (if exists) is a subset $C^{R_i}\in W_F$ such that*

(i) $\forall v, z \in C^{R_i} : (vR_iz)\&(zR_iv)$ *and*
(ii) $\forall z \in W_F, \forall v \in C^{R_i} : ((vR_iz\&zR_iv) \Rightarrow z \in C^{R_i})$.

*For any relation* $R_i$ *and any* $v \in W_F$, $C^{R_i}(v)$ *is the* $R_i$*-cluster such that* $v \in C^{R_i}(v)$. *We call* $C^{R_i}(v)$ *the cluster generated by* $v$.

**Definition 7.** *An LFPK-frame is a temporal* $(n+2)$*-modal Kripke-frame*

$$T = \langle Z_T, R_F, R_P, R_1, \dots, R_n\rangle$$

*where* $R_P = R_F^{-1}$ *and:*

a. $Z_T$ *is the disjoint union of clusters of agents* $C^t$, $t \in \mathbb{Z}$, *and* $C^{t_1} \bigcap C^{t_2} = \emptyset$ *if* $t_1 \neq t_2$;

b. $\forall t_1, t_2 \in Z$, *if* $t_1 \leq t_2$ *then* $\forall a \in C^{t_1}, \forall b \in C^{t_2}(aR_Fb)$ *and* $(bR_Pa)$. *None other relations via* $R_P$ *and* $R_F$ *are allowed.*

c. $R_1, \dots, R_n$ *are some equivalence relations in each separate cluster* $C^t$.

*We call any such frame LFPK-frame.*

Frames of this class model situations in which each agent has some information in the current temporary state $C^t$. Any temporary state $C^t$ consists of a set of information points available at $t$. The relations $R_F$ and $R_P$ are time connections on a linear stream of information points, wherein for two points $w$ and $z$ the term $wR_Fz$ means that either $w$ and $z$ are available at time $t$, or $z$ will be available in future in subsequent time w.r.t. $w$. Conversely, the term $wR_Pz$ means that either $w$ and $z$ are also available at the same time $t$, or $z$ was available at previous time w.r.t. $w$. Each relation $R_i$, $i = 1, ..., n$ reflects the information available to a particular agent $i$ in the current time point only, but for any time point.

**Definition 8.** *Model* $M_T$ *on a LFPK-frame* $T$ *is a tuple* $M_T = \langle T, V\rangle$, *where* $V$ *is a valuation of a set of propositional letters* $p \in P$ *on* $T$, *i.e.* $\forall p \in P\ [V(p) \subseteq Z_T]$. *Given a model* $M_T = \langle T, V\rangle$, *where* $T$ *is a LFPK-frame* $Z_T$. *Then* $\forall w \in Z_T$:

a. $\langle T, w\rangle \Vdash_V p \Leftrightarrow w \in V(p)$;

b. $\langle T, w\rangle \Vdash_V \Box_F A \Leftrightarrow \forall z \in Z_T(wR_Fz \Rightarrow \langle T, z\rangle \Vdash_V A)$;

c. $\langle T, w\rangle \Vdash_V \Box_P A \Leftrightarrow \forall z \in Z_T(wR_Pz \Rightarrow \langle T, z\rangle \Vdash_V A)$;

d. $\forall i \in I, \langle T, w\rangle \Vdash_V \Box_i A \Leftrightarrow \forall z \in Z_T(wR_iz \Rightarrow \langle T, z\rangle \Vdash_V A)$.

e. $\langle T, w\rangle \Vdash_V A \vee B \Leftrightarrow [(\langle T, w\rangle \Vdash_V A)$ *or* $(\langle T, w\rangle \Vdash_V B)]$;

f. $\langle T, w\rangle \Vdash_V A \wedge B \Leftrightarrow [(\langle T, w\rangle \Vdash_V A)$ *and* $(\langle T, w\rangle \Vdash_V B)]$;

l. $\langle T, w\rangle \Vdash_V A \to B \Leftrightarrow [(\langle T, w\rangle \Vdash_V B)$*or not*$(\langle T, w\rangle \Vdash_V A)]$;

i. $\langle T, w\rangle \Vdash_V \neg A \ \Leftrightarrow\ [not(\langle T, w\rangle \Vdash_V A)]$;

*The relation* $\Vdash_V$ *here is the truth relation on the element* $w$ *of the model* $M$. *Namely,* $\langle T, w\rangle \Vdash_V A$ *means that* $A$ *is true on the element* $w$ *in the model* $\langle T, V\rangle$. *If the formula* $A$ *is true on any element of a frame* $T$ *w.r.t. any valuation* $V$, *we say* $A$ *is true on the frame* $T$ *and write* $T \Vdash A$.

**Definition 9.** *Temporal Linear Future/Past logic LFPK (of agents knowledge) is the set of all LFPK formulas valid (true) on all LFPK-frames:*

$$LFPK := \{A \in Fma(\mathcal{L}^{LFPK}) \mid \forall T, \text{ where } T \text{ is an } LFPK\text{-frame, } (T \Vdash A)\}.$$

*If a formula* $A$ *belongs to* $LFPK$, *then we say that* $A$ *is a theorem of* $LFPK$.

It is clear that the logic $LFPK$ is closed w.r.t. the rules – modus ponens rule $(A, A \to B/B)$ and the generalization rules: $A/\Box_j A(j \in \{F, P, 1, \dots, n\})$. We consider these rules as postulated for $LFPK$.

## 3. A criterion of non-unifiability

**Theorem 4.** *A formula $A$ is non-unifiable in $LFPK$ iff the formula*

$$\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right]$$

*is a theorem in $LFPK$.*

*Proof.* 1. We prove this theorem by reduction to contradiction. Assume that

$$\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right] \in LFPK,$$

but at the same time the formula $A$ is unifiable in $LFPK$.

Then by the definition of unifiers, there is a substitution $g$ s.t. $g(A) \in LFPK$. By the fact that $LFPK$ is closed under the substitution, we obtain

$$g(\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right]) \in LFPK.$$

Let us consider an arbitrary $LFPK$-frame $T$.

Consider the valuation $V$ for all variables $q$ of formulas $g(p)$, where $p \in Var(A)$, on $T$, where $V(q) = \oslash$. Then it is easy to check by the induction on the length of any formula $B$ constructed on variables $q$ that

$$\forall b \in T, \forall c \in T : b \Vdash_V B \Leftrightarrow c \Vdash_V B.$$

Consequently

$$\forall b \in T : b \nVdash_V \bigvee_{p\in Var(A)} \neg\Box_F\Box_P g(p) \wedge \neg\Box_F\Box_P\neg g(p).$$

At the same time

$$\forall b \in T : b \Vdash_V \Box_F\Box_P g(A).$$

Thereby

$$\forall b \in T : b \nVdash_V g(\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right]),$$

which contradicts the hypothesis

$$g(\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right]) \in LFPK.$$

2. On the contrary, asume that the formula $A$ is non-unifiable in $LFPK$ but at the same time $\Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right] \notin LFPK$.

Then there is a certain frame $T$, that disproves this formula

$$\exists a \in T : \langle T, a\rangle \nVdash_V \Box_F\Box_P A \to \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right].$$

That is, $\langle T, a\rangle \Vdash_V \Box_F\Box_P A$ and $\langle T, a\rangle \nVdash_V \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right]$.

Since $\langle T, a\rangle \nVdash_V \left[\bigvee_{p\in Var(A)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right], \forall p \in Var(A)$ : either

$$(1)\, \forall b \in T : (aR_F b \Rightarrow b \Vdash_V p)\&(aR_P b \Rightarrow b \Vdash_V p),$$

or

$$(2)\, \forall b \in T : (aR_F b \Rightarrow b \Vdash_V \neg p)\&(aR_P b \Rightarrow b \Vdash_V \neg p).$$

Choose a substitution $g$ for all of the variables $p$ from the formula $A$ so that to satisfy the following conditions: $\forall p \in Var(A) : g(p) = \top$ if (1) holds and $g(p) = \bot$ in the case if (2) is true. Then $g$ is a unifier of the formula $A$. Therefore, the formula $A$ is unifiable in $LFPK$.

□

## 4. Passive Inference Rules

**Definition 10.** *Let $r := A_1, \ldots, A_k/\beta$ be an inference rule. The rule $r$ is said to be passive for $LFPK$ if for any substitution $g$ of formulas instead of variables in $r$ the condition $g(A_1) \in LFPK \,\&\ldots\&\, g(A_k) \in LFPK$ does not hold. In other words, $r$ is passive rule if the formulas from its premise have no common unifiers.*

**Theorem 5.** *The rules $r_m := \frac{\bigvee_{1\le i\le m} \neg\Box_F\Box_P p_i \wedge \neg\Box_F\Box_P\neg p_i}{\bot}$ form a basis for all passive inference rules in the logic $LFPK$.*

*Proof.* It is the case that

$$\Box_F\Box_P\left(\bigvee_{1\le i\le m} \neg\Box_F\Box_P p_i \wedge \neg\Box_F\Box_P\neg p_i\right) \to$$
$$\left(\bigvee_{1\le i\le m} \neg\Box_F\Box_P p_i \wedge \neg\Box_F\Box_P\neg p_i\right) \in LFPK,$$

and hence by Theorem 4 the formula $A = \bigvee_{1\le i\le m} \neg\Box_F\Box_P p_i \wedge \neg\Box_F\Box_P\neg p_i$ is not unifiable in the logic $LFPK$, i.e any rule $r_m$ is passive.

Let us assume that a rule $t_1 := A_1, \ldots, A_k/B$ is passive for $LFPK$. Then the rule $t_2 := A_1 \wedge \cdots \wedge A_k/B$ is also passive and the formula $A_1 \wedge \cdots \wedge A_k$ is not unifiable in $LFPK$. Applying Theorem 4, we conclude that

$$\Box_F\Box_P(A_1 \wedge \cdots \wedge A_k) \to \left[\bigvee_{p\in Var(A_1\wedge\cdots\wedge A_k)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p\right] \in LFPK. \quad (1)$$

After applying Gödel's rule to the premise of $t_2$ we derive $\Box_F\Box_P(A_1 \wedge \cdots \wedge A_k)$ (2). From (1) and (2) with modus ponens we derive the formula

$$\bigvee_{p\in Var(A_1\wedge\cdots\wedge A_k)} \neg\Box_F\Box_P p \wedge \neg\Box_F\Box_P\neg p.$$

From this formula, applying the rule $r_m$, where $m$ is the number of variables in the conjunction of $A_1 \wedge \cdots \wedge A_k$, we can derive the formula $\bot$. And from the implementation $\bot \to B \in LFPK$, by modus ponens we obtain $B$. Thus, all rules $r_n$ form a basis for the rules passive in $LFPK$. □

## References


[1] A. Robinson, *A machine oriented logic based on the resolution principle*, J. of the ACM, **12**:1 (1965), 23–41. Zbl 0139.12303

[2] D. Knuth, P. Bendix, J. Leech, ed., *Simple Word Problems in Universal Algebras*, Pergamon Press, (1970), 263–297. Zbl 0188.04902

[3] F. Baader, W. Snyder, *Unification theory. In: Robinson J. A., Voronkov A., editors. Handbook of Automated Reasoning. I*, Elsevier Science Publishers, (2001), 447–533. Zbl 1011.68126

[4] F. Baader, S. Ghilardi, *Unification in modal and description logics*, Logic Jnl IGPL, **19**:6 (2011), 705–730. Zbl 1258.03018

[5] F. Baader, B. Morawska, *Unification in the description logic EL*, Logical Methods in Computer Science, **6** (2010), 1–31. Zbl 1214.68379

[6] F. Baader, P. Narendran, *Unification in a description logic with transitive closure of roles*, Logic for Programming, Artificial Intelligence, and Reasoning, LPAR 2001, **2250**, LNCS, Springer, (2001), 217–232.

[7] F. Baader, B. Morawska, *Unification of concept terms in description logics*, Journal of Symbolic Computation, **31** (2001), 277–305.

[8] V.V. Rybakov, *Problems of Substitution and Admissibility in the Modal System Grz and in Intuitionistic Propositional Calculus*, Annals of Pure and Applied Logic, **50**:1 (1990), 71–106. Zbl 0709.03009

[9] V.V. Rybakov, *Rules of Inference with Parameters for Intuitionistic logic*, J. of Symbolic Logic, **57**:3 (1992), 912–923. Zbl 0788.03007

[10] V.V. Rybakov, *Admissible Logical Inference Rules. Series: Studies in Logic and the Foundations of Mathematics*, Elsevier Sci. Publ., North-Holland, **136**, (1997). Zbl 0872.03002

[11] S. Ghilardi, *Unification Through Projectivity*, J. of Logic and Computation, **7**:6 (1997), 733–752. Zbl 0894.08004

[12] S. Ghilardi, *Unification, finite duality and projectivity in varieties of Heyting algebras*, Annals of Pure and Applied Logic, **127**1–3 (2004), 99–115. Zbl 1058.03020

[13] S. Ghilardi, *Unification in Intuitionistic logic*, Journal of Symbolic Logic, **64**:2 (1999), 859–880. Zbl 0930.03009

[14] S. Ghilardi, *Best solving modal equations*, Annals of Pure and Applied Logic, **102**:3 (2000), 183–198. Zbl 0949.03010

[15] S. Ghilardi, L. Sacchetti, *Filtering Unification and Most General Unifiers in Modal Logic*, Journal of Symbolic Logic, **69**:3 (2004), 879–906. Zbl 1069.03011

[16] E. Jerabek, *Admissible rules of modal logics*, Journal of Logic and Computation, **15** (2005), 411–431. Zbl 1077.03011

[17] E. Jerabek, *Independent bases of admissible rules*, Logic Journal of the IGPL, **16** (2008), 249–267. Zbl 1146.03008

[18] E. Jerabek, *Rules with parameters in modal logic I*, CoRR abs/1305.4912, (2013).

[19] R. Iemhoff, *On the admissible rules of intuitionistic propositional logic*, Journal of Symbolic Logic, **66** (2001), 281–294. Zbl 0986.03013

[20] R. Iemhoff, G. Metcalfe, *Proof theory for admissible rules*, Annals of Pure and Applied Logic, **159** (2009), 171–186. Zbl 1174.03024

[21] D.M. Gabbay, I.M. Hodkinson, M.A. Reynolds, *Temporal Logic: - Mathematical Foundations and Computational Aspects*, Clarendon Press, Oxford, **1** (1994). Zbl 0921.03023

[22] D.M. Gabbay, I.M. Hodkinson, *An axiomatisation of the temporal logic with Until and Since over the real numbers*, Journal of Logic and Computation, **1** (1990), 229–260. Zbl 0744.03018

[23] D.M. Gabbay, I.M. Hodkinson, *Temporal Logic in Context of Databases. In J. Copeland, editor, Logic and Reality, Essays on the legacy of Arthur Prior*, Oxford University Press, (1995).

[24] Z. Manna, A. Pnueli, *The Temporal Logic of Reactive and Concurrent Systems: Specification*, Springer, **1** (1992).

[25] Z. Manna, A. Pnueli, *Temporal Verification of Reactive Systems: Safety*, Springer, **1** (1995).

[26] M. Vardi, *An automata-theoretic approach to linear temporal logic*, Y.Banff Higher Order Workshop, Available at http://citeseer.ist.psu.edu/vardi96automatatheoretic.html, (1995), 238–266.

[27] M. Y. Vardi, *Reasoning about the past with two-way automata, Larsen K.G., Skyum S., Winskel G., editors.* ICALP, LNCS, Springer, 1443 (1998), 628–641. Zbl 0909.03019

[28] V.V. Rybakov, *Linear temporal logic with until and next, logical consecutions*, Annals of Pure and Applied Logic, **155** (2008), 32–45. Zbl 1147.03008
[29] S. Babenyshev, V. Rybakov, *Linear temporal logic LTL: basis for admissible rules*, Journal of Logic and Computation, **21** (2011), 157–177. Zbl 1241.03014
[30] V.V. Rybakov, *Logical Consecutions in Discrete Linear Temporal Logic*, J. of Symbolic Logic, **70**:4 (2005), 1137–1149. Zbl 1110.03010
[31] V.V. Rybakov, *Writing out Unifiers in Linear Temporal Logic*, J. Logic Computation, **22(5)**, (2012), 1199–1206.
[32] V.V. Rybakov, *Projective formulas and unification in linear temporal logic $LTLU$*, Logic Journal of the IGPL, **22**:4 (2014), 665–672. Zbl 06403093
[33] V.V. Rybakov, *Unifiers in transitive modal logics for formulas with coefficients (meta-variables)*, Logic Jnl IGPL, **21**:2 (2013), 205–215. Zbl 1278.03050
[34] V.V. Rybakov, *Writing out unifiers for formulas with coefficients in intuitionistic logic*, Logic Jnl IGPL, **21**:2 (2013), 187–198. Zbl 1277.03004
[35] W. Dzik, P. Wojtylak, *Projective unification in modal logic*, Logic Journal of IGPL, **20**:1 (2012), 121–153. Zbl 1260.03041
[36] V.V. Rybakov, M. Terziler, C. Gencer, *An essay on unification and inference rules for modal logics*, Bulletin of the Section of Logic, **28**:3 (1999), 145–157. Zbl 0952.03019
[37] S.I. Bashmakov, *Unification and inference rules in the multi-modal logic of knowledge and linear time LTK*, Journal of Siberian Federal University. Mathematics and Physics, **9**:2 (2016), 149–157.

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