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# THE CONFERENCE "DYNAMICS IN SIBERIA", NOVOSIBIRSK, FEBRUARY 26 – MARCH 4, 2017

I.A. DYNNIKOV, A.A. GLUTSYUK, A.E. MIRONOV, I.A. TAIMANOV, A.YU. VESNIN

Abstract. In this article abstracts of talks of the Conference "Dynamics in Siberia" held in Sobolev Institute of Mathematics, February 26 – March 4, 2017 are presented.

**Keywords:** dynamical systems, geometry, integrable systems, mathematical physics.

The second international conference "Dynamics in Siberia" was held at the Sobolev Institute of Mathematics SB RAS (Novosibirsk) from February 26 to March 4, 2017 (for information on the previous conference see [1]). Members of the program committee were as follows: I.A. Dynnikov, A.A. Glutsyuk, A.E. Mironov, I.A. Taimanov and A.Yu. Vesnin.

More than 50 experts on dynamical systems, mathematical physics, geometry and topology participated in the conference. The conference program consisted of plenary talks and short talks. The talks were made by well-known experts from Moscow, St. Petersburg, Novosibirsk, Chelyabinsk, Gorno-Altaisk, Nizhny Novgorod, Grozny, Kemerovo, Krasnoyarsk, Tomsk, Ufa, Magadan, Yakutsk and also by well-known mathematicians from China, France, Germany, Italy, Japan, Taiwan, Belarus. More than 20 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

The conference was supported by the Russian Foundation for Basic Research (grant 17-01-20042) and by International Mathematical Center of Novosibirsk State University.

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Dynnikov, I.A., Glutsyuk, A.A., Mironov, A.E., Taimanov, I.A., Vesnin, A.Yu. Conference "Dynamics in Siberia", Novosibirsk, February 26 – March 4, 2017.

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## Program (Plenary talks)

**February 29**

| | |
|---|---|
| 10:00-10:50 | D. Treschev (*Moscow*) Arnold diffusion in a priori unstable case. |
| 10:50-11:40 | J. Zhou (*Beijing, China*) Poisson structures in theories of modular forms, elliptic functions, and invariant theory. |
| 12:00-12:50 | Y. Zhang (*Beijing, China*) Bihamiltonian integrable hierarchies and their tau structures. |
| 12:50-13:40 | O. Pochinka (*Nizhny Novgorod*) On Morse-Smale systems on manifolds. |

**February 28**

| | |
|---|---|
| 9:00-09:50 | T. Shiota (*Kyoto, Japan*) Finite dimensional orbits of soliton equations. |
| 9:50-10:40 | S. Dobrokhotov (*Moscow*) Punctured Lagrangian Manifolds and the Propagation of the Leading Edge Front for Linear Equations of the Water Waves and Crystal Lattice. |
| 11:00-11:50 | S. Liu (*Beijing, China*) Matrix Integral, Hodge Integral, and Integrable system. |
| 11:50-12:40 | R. Vitolo (*Lecce, Italy*) Geometry of third-order homogeneous Hamiltonian operators and integrable systems. |
| 12:40-13:30 | V. Roubtsov (*Angers, France*) Painleve monodromy varieties: geometry and quantisation. |

**March 1**

| | |
|---|---|
| 9:00-9:50 | V. Nazaikinskii (*Moscow*) Lagrangian manifolds associated with some problems in abstract analytic number theory. |
| 9:50-10:40 | H. Ma (*Beijing, China*) Uniqueness of closed self-similar solutions to $\sigma_a^k$-curvature flow. |
| 11:00-11:50 | D. Zuo (*Hefei, China*) Extended affine Weyl groups of BCD type, Frobenius manifolds and their Landau-Ginzburg superpotentials. |
| 11:50-12:40 | A. Gaifullin (*Moscow*) On the analytic continuation of the volume of a non-Euclidean simplex. |

**March 2**

| | |
|---|---|
| 10:00-10:50 | S. Bolotin (*Moscow*) Singularities of the potential and integrability in Hamiltonian systems with two degrees of freedom. |
| 10:50-11:40 | A. Tsiganov (*Saint Petersburg*) Backlund transformations and new bi-Hamiltonian systems on the plane, sphere and ellipsoid. |
| 12:00-12:50 | A. Chupakhin (*Novosibirsk*) The abnormalities of a human brain vasculature as the singularities of a haemodynamics mathematical model. |

**March 3**

| | |
|---|---|
| 10:00-10:50 | A. Shafarevich (*Moscow*) Laplace operator and a wave equation on polyhedrons. |
| 10:50-11:40 | U. Semmelmann (*Stuttgart, Germany*) Killing tensors on Riemannian manifolds. |

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## Plenary talks

### Сингулярности потенциала и интегрируемость в гамильтоновых системах с двумя степенями свободы

*С. Болотин (Москва)*

Рассматривается задача о полиномиальных по импульсу интегралах гамильтоновых систем с двумя степенями свободы при фиксированном значении полной энергии (условных интегралах по Биркгофу). Предполагается, что потенциал имеет несколько сингулярных точек. Показано, что при наличии условных полиномиальных интегралов сумма степеней сингулярностей не превосходит удвоенной эйлеровой характеристики конфигурационного пространства. Доказательство основано на введении комплексной структуры на конфигурационном пространстве и оценке степени дивизора, отвечающего старшей по импульсу степени в интеграле. При дополнительных условиях доказана также положительность топологической энтропии. Доклад основан на совместной работе с В.В.Козловым.

### Punctured Lagrangian Manifolds and the Propagation of the Leading Edge Front for Linear Equations of the Water Waves and Crystal Lattice

*S. Dobrokhotov (Moscow)*[1]

We consider the Cauchy problem with localized initial data for linear two-dimensional equations of the water wave theory and two-dimensional equations of the crystal lattice. We show that the asymptotics of the solutions of such problems is described by Lagrangian manifolds with punctured point and that the leading edge of the waves is a special caustic. We also show that the asymptotic solution in a neighborhood of the leading edge is described by the Hamiltonian system corresponding to the limit wave equation and by coefficients determining the dispersion in the linearized Boussinesq equation approximating the equations in question.

### On the analytic continuation of the volume of a non-Euclidean simplex

*A. Gaifullin (Moscow)*

We shall construct a special unipotent filtration on the space of multi-valued analytic functions on a complex analytic manifold $X$. We shall study an important partial case of $X$ being the complement of a union of affine hypersurfaces in $\mathbb{C}^m$ defined over reals. For such $X$, we shall prove a non-trivial sufficient condition for a multi-valued analytic function $F$ to be totally real in a domain $U \subset \mathbb{R}^m \subset \mathbb{C}^m$. Here a multivalued function is said to be totally real in $U$ if all branches of it are real in $U$.

Further, we shall apply these results to the function expressing the volume of an $n$-dimensional simplex in Lobachevsky space (or in sphere) from the hyperbolic cosines (respectively, cosines) of its edge lengths. We shall prove that this function is totally real in 'geometric' domain in the spherical case and in even-dimensional Lobachevsky spaces. Also, we shall prove a natural analogue of this results in the case of odd-dimensional Lobachevsky spaces.

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[1]This work was done together with V.E.Nazaikinskii and was supported by the Russian Science Foundation (project N 16-11-10282).

Originally, these results came from the problem on volumes of flexible polyhedra in non-Euclidean spaces. However, I think that these results can be interesting by themselves. So, in this talk I would like to discuss this particular topic rather than its application to flexible polyhedra.

### Matrix Integral, Hodge Integral, and Integrable system

*S.-Q. Liu (Beijing, China)*

Matrix integral is a classical topic in mathematics. It is introduced by physicist E. Wigner, and has many interesting applications in physics, probability theory, mathematical statistics, numerical analysis, and number theory. It is revealed by the celebrated Witten conjecture that matrix integral is also the bridge among two-dimensional quantum gravity, the moduli space of stable curves, and the Korteweg-de Vries (KdV) hierarchy. Hodge integrals are the integrals of certain natural cohomological classes on the moduli space of stable curves, which are very important in modern mathematical physics. In our previous work, we showed that Hodge integral is also related to a certain generalization of the KdV hierarchy. We also conjectured a mysterious relation between matrix Integral and Hodge Integral. Recently, we proved this conjecture. This is a joint work with Boris Dubrovin, Di Yang, and Youjin Zhang.

### Lagrangian manifolds associated with some problems in abstract analytic number theory

*V. Nazaikinskii (Moscow)*

We define the entropy and study typical shapes of elements of an arithmetic semigroup with power-law or exponential asymptotics of the counting function of abstract primes and show how these problems naturally give rise to Lagrangian manifolds and quasi-thermodynamic models. Relations to other fields and possible applications are discussed. The talk is based on results obtained jointly with V.P. Maslov.

### Painleve monodromy varieties: geometry and quantisation

*V. Roubtsov (Angers, France)*

We introduce the concept of decorated character variety for the Riemann surfaces arising in the theory of the Painleve differenti-al equations. Since all Painleve differential equations (apart from the PVI) exhibit Stokes phenomenon, we show that it is natural to consider Riemann spheres with holes and bordered cusps on such holes. The decorated character variety is considered here as complexification of the bor-dered cusped Teichmu?ller space. We also show how to obtain the confluence procedure of the Painleve differential equations in geometric terms. A quantisation and a relation to geometric type cluster algebras.

### Killing tensors on Riemannian manifolds

*U. Semmelmann (Stuttgart, Germany)*

In my talk I will give a survey on results for Killing tensors on Riemannian manifolds. I will explain the relation to first integrals and present a few new proofs, together with several new examples.

**Laplace operator and a wave equation on polyhedra**
*A. Shafarevich (Moscow)*

We study properties of Laplace operator and wave equation on a $2D$ polyhedron. Laplacians are defined in terms of extension theory and a number of their properties can be described using the natural complex structure on the polyhedron. In particular, we study kernels of Laplacians and give simple examples of harmonic functions. We also study certain properties if the wave equation; in the simplest situation we obtain the analog of the Poisson formula. For the short-wave approximation we study corresponding singular Lagrangian manifolds.

**Finite dimensional orbits of soliton equations**
*T. Shiota (Kyoto, Japan)*

A solution of hierarchy of soliton equations having a finite dimensional orbit is often described by a line bundle or a torsion-free rank one sheaf on an algebraic curve. I would like to discuss some aspects of such solutions, including
- study of periodic initial value problem for the K-dV in 1970s
- Burchnall-Chaundy-Krichever theory
- Novikov's conjecture and generalizations
- Calogero-Moser-type system for motion of poles of a solution
- Abelian solutions to soliton equations

**Arnold diffusion in a priori unstable case**
*D. Treschev (Moscow)*

Let $F : D_1 \to D_2$ be a diffeomorphism of two domains on a plane. Consider a parallel bundle of light rays through $D_1$ perpendicular to the plane. We consider the following problem.

Find a system of mirrors which transforms this bumdle to a parallel bundle through $D_2$ perpendicular to the plane such that any ray through $x \in D_1$ is transformed to the ray through $F(x) \in D_2$.

**Backlund transformations and new bi-Hamiltonian systems on the plane, sphere and ellipsoid**
*A. Tsiganov (Saint Petersburg)*

Using Jacobian arithmetic for hyperelliptic curves, we can identify well-known cryptographic algorithms and protocols with various schemes of the discretization of continuous Hamiltonian flows in classical mechanics. Most interesting to us fact is that the corresponding auto Backlund transformations also yield new canonical variables on the original phase space, which are useful to construction of new integrable systems in the framework of the Jacobi method. As an example, we show how to construct new bi-Hamiltonian systems with integrals of motion of third, fourth and sixth order in momenta on the plane, sphere and ellipsoid.

### The geometry of homogeneous third-order Hamiltonian operators and applications to integrable systems

*R. Vitolo (Lecce, Italy)*

In this talk we will consider third-order homogeneous Hamiltonian operators, introduced by B.A. Dubrovin and S.P. Novikov in 1984. It was recently found that they are in correspondence with quadratic line complexes, which are algebraic varieties in the space of all lines of a complex projective space. This enables us to classify them for a number of components lower than or equal to 4. Third-order homogeneous Hamiltonian operators determine families of hydrodynamic-type systems with interesting geometric properties; many of them are integrable. Moreover, third-order homogeneous operators can be arranged, together with first-order homogeneous operators, in trios of mutually compatible operators. This mechanism was observed for the KdV hierarchy and the Camassa-Holm hierarchy. We generalize it to two-component systems using the above classification; we are able to find multi-parametric families of Miura-inequivalent integrable bi-Hamiltonian systems.

Joint work with E.V. Ferapontov, P. Lorenzoni, M.V. Pavlov, A. Savoldi.

### Bihamiltonian integrable hierarchies and their tau structures

*Y. Zhang (Beijing, China)*

Starting from a so-called flat exact semisimple bihamiltonian structures of hydrodynamic type, we construct a Frobenius manifold structure and a tau structure for the associated principal hierarchy. We then classify the deformations of the principal hierarchy which possess tau structures.

### Poisson structures in theories of modular forms, elliptic functions, and invariant theory

*J. Zhou (Beijing, China)*

The Omega process of Cayley in the classical invariant theory of binary forms is a Poisson structure on the complex 2-plane. We will show that it induces some natural Poisson structures in the theories of modular forms, elliptic functions, and invariant theory of binary polyhedral groups. As a corollary, these induced Poisson structures induce simple deformation quantizations which we conjecture to coincide with the deformation quantizations constructed by Kontsevich.

### Extended affine Weyl groups of BCD type, Frobenius manifolds and their Landau-Ginzburg superpotentials

*D. Zuo (Hefei, China)*

For the root systems of type $B_l, C_l$ and $D_l$, we show the existence of Frobenius manifold structures on the orbit spaces of the extended affine Weyl groups that correspond to any vertex of the Dynkin diagram. It also depends on certain additional data. We also construct LG superpotentials for these Frobenius manifold structures.

This talk based on the joint work with Boris Dubrovin, Ian Strachan and Youjin Zhang (arXiv:1510.08690).

## Short talks

### Semi-classical asymptotics for operator pencil with applications to plasma physics

*A. Anikin (Moscow)*

We propose a useful trick for calculating asymptotic solutions of spectral problems for pseudo-differential operators with a matrix-valued symbol. Within the standard approach, one needs to study a classical system, where the Hamiltonian is one of the eigenvalues of the matrix symbol (called effective Hamiltonians or terms). However, in complicated physical problems effective Hamiltonians cannot be calculated analy-tically. We show that the asymptotic solutions actually can be written out by means of objects connected with a classical system whose Hamiltonian is the determinant of the symbol. As an example, we discuss stationary solutions of a system of equations describing the motion of cold plasma in TOKAMAK.

### Some remarks on quantum logic and a Hilbert space axiom in the orthodox quantum mechanics

*Yu. Brezhnev (Tomsk)*

This is (some discussion) topic on foundations for nonrelativistic quantum mechanics (QM) and its mathlogical maintanance. There is known Birkhoff's problem of derivation of the Hilbert space lattices (so called quantum logic by Birkhoff-von Neumann (1936)) independently of the Hilbert space axiom in orthodoxal QM. Root of the problem lies in a linearity property of the state space (superposition principle) followed by support with a scalar product structure. We proposed several natural principles (postulats) entailing this superposition principle and scalar product and answering the question why numeric domain in QM should be a complex number set C.

### Differentiation of Hyperelliptic Functions: Genus 3 Case

*E. Bunkova (Moscow)*[2]

In [1] the problem of differentiation of Abelian functions was described. It has deep relations [2] with KdV equations theory. Our approach to the problem uses the results of [3].

Let us consider hyperelliptic curves of genus $g$ in the model

$$\mathcal{V}_\lambda = \{(x, y) \in \mathbb{C}^2 \colon y^2 = x^{2g+1} + \lambda_4 x^{2g-1} + \lambda_6 x^{2g-2} + \ldots + \lambda_{4g} x + \lambda_{4g+2}\}.$$

It depends on the parameters $\lambda = (\lambda_4, \lambda_6, \ldots, \lambda_{4g}, \lambda_{4g+2}) \in \mathbb{C}^{2g}$.

Denote by $\mathcal{B} \subset \mathbb{C}^{2g}$ the subspace of parameters such that $\mathcal{V}_\lambda$ is non-singular for $\lambda \in \mathcal{B}$. Let $\mathcal{U}$ be the space of the fiber bundle $\pi : \mathcal{U} \to \mathcal{B}$ with fiber over $\lambda \in \mathcal{B}$ the Jacobian $\mathcal{J}_\lambda$ of the curve $\mathcal{V}_\lambda$.

An *Abelian function* is a meromorphic function on $\mathbb{C}^g$ with a lattice of periods $\Gamma \subset \mathbb{C}^g$ of rank $2g$. We say that an Abelian function is a meromorphic function on the complex torus $T^g = \mathbb{C}^g/\Gamma$.

[2]This work was supported in part by Young Russian Mathematics award, Royal Society International Exchange grant and the RFBR project 17-01-00366 A.

A *hyperelliptic function of genus $g$* is a smooth function defined on an open dense subset of $\mathbb{C}^g \times \mathcal{B}$, such that for each $\lambda \in \mathcal{B}$ it's restriction on $\mathbb{C}^g \times \lambda$ is Abelian with $T^g$ the Jacobian $\mathcal{J}_\lambda$ of $\mathcal{V}_\lambda$. Denote the field of hyperelliptic functions on $\mathcal{U}$ by $\mathcal{F}$.

The general statement of the Problem of Differentiation of Abelian Functions is given in [1]. We consider it's special case, namely

(1) find the generators of the $\mathcal{F}$-module $\mathcal{F}$ of derivations of the field $\mathcal{F}$ and their action on $\mathcal{F}$,
(2) describe the structure of Lie algebra $\mathcal{F}$.

In this talk we consider the genus 3 case of this Problem and give explicit formulas for genuses 1, 2 and 3.

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## Compatibility of different classes of almost Hermitian 6-manifolds

*N. Daurtseva (Kemerovo)*

Let $(M, g, J, \omega)$ is almost Hermitian manifold. Any structure of the triple $(g, J, \omega)$ defines some special sets of almost Hermitian structures (a.H.s.) Really, Riemannian metric $g$ defines the space $\mathcal{AO}_g^+$ of all positively oriented $g$-orthogonal almost complex structures. This space is in one-to-one correspondence with the space $\mathcal{H}_g = \{(g, J, \omega_J) : J \in \mathcal{AO}_g^+, \omega_J(X, Y) = g(JX, Y)\}$ of all a.H.s. with the same metric $g$. Almost complex structure $J$ defines the set $\mathcal{M}_J$ of all $J$-orthogonal metrics on $M$ and gives rise to space $\mathcal{H}_J = \{(g, J, \omega_g) : g \in \mathcal{M}_J, \omega_g(X, Y) = g(JX, Y)\}$ of a.H.s. with the same almost complex structure $J$. Analogically, 2-form $\omega$ defines the space $\mathcal{A}_\omega^+$ of $\omega$-tamed almost complex structures and gives rise to space $\mathcal{H}_\omega = \{(g_J, J, \omega) : J \in \mathcal{A}_\omega^+, g_J(X, Y) = \omega(X, JY)\}$ of a.H.s. with the same 2-form.

In Gray-Herwella classification [1] the class $\mathcal{W}$ of almost Hermitian manifolds $(M, g, J, \omega)$ is decomposed into sum of classes $\mathcal{W} = \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$. Each class is characterized by some additional symmetry properties for 3-form $\nabla\omega$. The question about compatibility between different classes of almost Hermitian 6-manifolds $(M, g, J)$ through the a.H.s. in $\mathcal{H}_J$ and $\mathcal{H}_\omega$ is reaseached.

The list of incompatible a.H.s. $(g, J, \omega) \in \mathcal{H}_J$ for 6-manifolds is shown in paper [2]. For $\mathcal{H}_\omega$ at [3] was shown that if 6-manifold $(M, g, J, \omega) \in \mathcal{W}_1$, then for any other a.H.s. $(g_I, I, \omega) \in \mathcal{H}_\omega$ 6-manifold $(M, g_I, I, \omega) \notin \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$.

For another classes of almost Hermitian manifold $(M^6, g, J, \omega)$ we have the following theorem.

**Theorem 1.** *Let $(M, g, J, \omega)$ is almost Hermitian 6-manifold.*

*1). If $(M, g, J, \omega) \in \mathcal{W}_1$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_2 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$ for any $I \in \mathcal{A}_\omega^+$;*

*2). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_2$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_3 \oplus \mathcal{W}_4$ for any $I \in \mathcal{A}_\omega^+$;*

*3). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_3$ strictly, then $(M, g_I, I, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$, but not in $\mathcal{W}_2$ for any $I \in \mathcal{A}_\omega^+$;*

*4). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_4, \mathcal{W}_2 \oplus \mathcal{W}_4$ or $\mathcal{W}_4$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$ for any $I \in \mathcal{A}_\omega^+$;*

*5). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$, then $(M, g_I, I, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3$ for any $I \in \mathcal{A}_\omega^+$;*

*6). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_2$ for any $I \in \mathcal{A}_\omega^+$;*

*7). If $(M, g, J, \omega) \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_4$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_3$ for any $I \in \mathcal{A}_\omega^+$;*

*8). If $(M, g, J, \omega) \in \mathcal{W}_2$, then $(M, g_I, I, \omega) \in \mathcal{W}_2$ for any $I \in \mathcal{A}_\omega^+$;*

*9). If $(M, g, J, \omega) \in \mathcal{W}_2 \oplus \mathcal{W}_3$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_1 \oplus \mathcal{W}_4$ for any $I \in \mathcal{A}_\omega^+$;*

*10). If $(M, g, J, \omega) \in \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_1$ for any $I \in \mathcal{A}_\omega^+$;*

*11). If $(M, g, J, \omega) \in \mathcal{W}_3$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_4$ for any $I \in \mathcal{A}_\omega^+$;*

*12). If $(M, g, J, \omega) \in \mathcal{W}_3 \oplus \mathcal{W}_4$ strictly, then $(M, g_I, I, \omega) \notin \mathcal{W}_1 \oplus \mathcal{W}_2$ for any $I \in \mathcal{A}_\omega^+$.*

Remark that we say that manifold belongs to any class *strictly*, if it is in the class, but not in less.

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## The Riemann-Roch theorem for the Dynnikov-Novikov discrete complex analysis

*D. Egorov (Yakutsk)*

We prove an analog of the Riemann–Roch Theorem for the Dynnikov–Novikov discrete complex analysis

## Platonic complexities of hyperbolic 3-manifolds

*E. Fominykh (Chelyabinsk)*

A triangulation of a 3-manifold M into tetrahedra is minimal if there is no triangulation of M into fewer tetrahedra. The tetrahedral complexity of M is the number of tetrahedra in a minimal triangulation. Similarly we can define cubical, octahedral and dodecahedral Platonic complexities of M. In this talk we calculate Platonic complexities for infinite families of hyperbolic 3-manifolds. The talk is based on a joint work with Colin Adams and Vladimir Tarkaev.

## Uncountable families of pairwise disjoint inequivalent wild $k$-disks in $R^n$

*O. Frolkina (Moscow)*

**Definition 1.** *A subset $P \subset R^n$ homeomorphic to a polyhedron is called tame if there exists a homeomorphism $h$ of $R^n$ onto itself such that $h(P)$ is a polyhedron in $R^n$; and called wild otherwise.*

R.H. Bing proved [1] that it is impossible to place an uncountable collection of pairwise disjoint wild closed surfaces in $R^3$.

J. Stallings constructed [2] a family of $\mathfrak{c}$ (continuum cardinality) pairwise disjoint wild 2-disks in $R^3$.

In [3], R.B. Sher modified Stallings' construction so that no two disks of his family are ambiently homeomorphic in the following sense:

**Definition 2.** *Two subsets $A, B \subset R^n$ are ambiently homeomorphic if there exists a homeomorphism of $R^n$ onto itself such that $h(A) = B$.*

We prove the following.

**Theorem 1.** *For each pair of integers $n \geqslant 3$ and $1 \leqslant k \leqslant n-1$ there exists a family of $\mathfrak{c}$ pairwise disjoint wild $k$-disks in $R^n$ such that each of the $k$-disks has simply connected complement in $R^n$.*
*For $n = 3$ we obtain additionally: no two elements of the family are ambiently homeomorphic.*

**Definition 3.** *Let $A, B \subset R^n$. We say that $A$ can be ambiently embedded in $B$ if there exists a homeomorphism $h$ of $R^n$ onto itself such that $h(A) \subset B$. Two subsets $A, B \subset R^n$ are called ambiently comparable if at least one of them can be ambiently embedded into another.*

**Theorem 2.** *For each pair of integers $n \geqslant 3$ and $1 \leqslant k \leqslant n-1$ there exists a family of $\mathfrak{c}$ pairwise disjoint wild $k$-disks in $R^n$ such that no two elements of the family are ambiently comparable.*

We note that statement of Theorem 2 for particular case of $n = 3$, $k = 1$ follows from [4].

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## Towards global bifurcation theory of polynomial dynamical systems

*V. Gaiko (Minsk, Belarus)*

We carry out the global bifurction analysis of low-dimensional polynomial dynamical systems. To control the global bifurcations of limit cycles in planar systems, it is necessary to know the properties and combine the effects of all their rotation parameters [1]. It can be done by means of the development of new bifurcational geometric methods based on the Wintner–Perko termination principle [1]. Using these methods, we present, e. g., a solution of Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the Kukles cubic system [2] and for the general Liénard polynomial system with an arbitrary number of singular points [3]. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles [4].

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## On Algebro–Geometric Approach to Diagonal Semi–Hamiltonian Systems of Hydrodynamic Type

*E. Glukhov (Moscow)*[3]

An algebro–geometric approach to diagonal Semi–Hamiltonian systems of hydrodynamic type is presented. There is a correspondence between such systems and orthogonal curvilinear coordinate systems in spaces of diagonal curvature. The construction proposed in the report is a generalization of the Krichever construction for orthogonal coordinate systems in flat spaces. Formulae for coefficients of diagonal Semi–Hamil-tonian systems of hydrodynamic type are obtained in terms of Baker–-Akhiezer functions on a spectral curve. Families of hydrodynamic integrals and commuting flows for such systems are constructed.

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## On existence of a stable cycle in one asymmetric dynamical system

*V. Golubyatnikov (Novosibirsk)*[4]

We consider nonlinear 6-dimensional dynamical system which describes a model of functioning of one simple gene network introduced in [1]:

$$\frac{dm_1}{dt} = -k_1 m_1 + f_1(p_3); \quad \frac{dp_1}{dt} = \mu_1(m_1 - p_1);$$
$$\frac{dm_2}{dt} = -k_2 m_2 + f_2(p_1); \quad \frac{dp_2}{dt} = \mu_2(m_2 - p_2);$$
$$\frac{dm_3}{dt} = -k_3 m_3 + f_3(p_2); \quad \frac{dp_3}{dt} = \mu_3(m_3 - p_3),$$

Here $k_j$ and $\mu_j$ are positive coefficients, the variables in this system describe concentrations $m_j$ of mRNAs and corresponding proteins $(p_j)$ in the gene network, thus they are non-negative. The functions $f_j$ are smooth and monotonically decreasing, they describe negative feedbacks in the gene networks.

We find sufficient conditions of existence of a stable cycle in the phase portrait of this system and describe its location in its phase portrait.

In the dimensionless symmetric particular case $k_1 = k_2 = k_3 = 1$, $\mu_1 = \mu_2 = \mu_3$, $f_1(p) = f_2(p) = f_3(p) = f(p) \equiv \alpha(1 + p^{\gamma})^{-1} + \alpha_0$ this sysetm was studied in [1], [2].

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## On Embedding of Morse–Smale diffeomorphisms in topological flows

*E. Gurevich (Nizhny Novgorod)*

The report is devoted to discussing results of papers [2]-[3] obtained in collaboration with V. Grines and O. Pochinka.

One of the important indicators of an adequacy of the numerical solution of an autonomous system of differential equations is the topological conjugacy of the obtained discrete model to the time-one shift map of the initial flow. In this connection the question on necessary and sufficient conditions for the embedding of a cascade in a flow naturally arises.

In [1] there were stated necessary conditions of the embedding of a Morse–Smale diffeomorphism (structurally stable diffeomorphism with finite non-wandering set) $f\colon M^n \to M^n$ in a topological flow. These conditions are: 1) the non-wandering set $\Omega_f$ coincides with the set of fixed points; 2) the restriction of the diffeomorphism $f$

[4]The work was supported by RFBR, grant 15-01-00745.

on every invariant manifold of any fixed point $p \in \Omega_f$ preserves its orientation; 3) if for any different saddle points $p, q \in \Omega_f$ the intersection $W^{\mathrm{s}}_p \cap W^{\mathrm{u}}_q$ is not empty then it does not contain any compact connected components.

Palis has proved, that in the case $n = 2$ this conditions are not only necessary but also sufficient. In [2] the case $n = 3$ was considered. It was shown that there is an additional obstruction to the embedding of such diffeomorphisms in topological flows, with is connected with a possibility of a non-trivial embedding of separatrices of saddle points in the ambient manifold, and the necessary and sufficient conditions of the embedding of a 3-dimensional Morse–Smale diffeomorphism in a topological flow were obtained.

In [3] it was announced that for the class $G(S^n)$ of Morse–Smale diffeomorphisms without heteroclinic intersections, defined on the sphere $S^n$ of the dimension $n \geqslant 4$ and satisfying to Palis conditions there are no other obstructions and the following theorem is true.

**Theorem.** *Any diffeomorphism $f \in G(S^n)$, $n \geqslant 4$, is embedded to a topological flow.*

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## Characteristic invariants of hyperbolic systems

*O. Kaptsov (Krasnoyarsk)*

We develop a method that allows us to derive reductions and solutions to hyperbolic systems of partial differential equations. In this report we consider systems of partial differential equations in two and $n$ independent variables. We introduce an operator of differentiation in the direction of characteristics of the system and corresponding invariants of characteristics of order $k$. We prove that if a function $h$ defined on the $k$-th order jet space $J^{(k)}$ is constant along a vector field on the solutions of the system of partial differential equations, then this function is an invariant of characteristics. These functions generalize well-known Riemann invariants. As applications we consider the gas dynamics system and ideal magnetohydrodynamics equations. In special cases, we find solutions of these equations depending on some arbitrary functions.

## Capture into parametric autoresonance in non-linear oscillator

*O. Kiselev (Ufa)*

We discuss a capture into parametric autoresonance for an equation

$$u'' + (1 + 4\epsilon \cos(\Omega(t,\epsilon)t) \sin(u) = 0, \quad 0 < \epsilon \ll 1. \tag{1}$$

Oscillations of such pendulum with amplitude of order $\epsilon$ can be considered as linear oscillations. Such approach allows us to obtain a resonant frequencies without of any

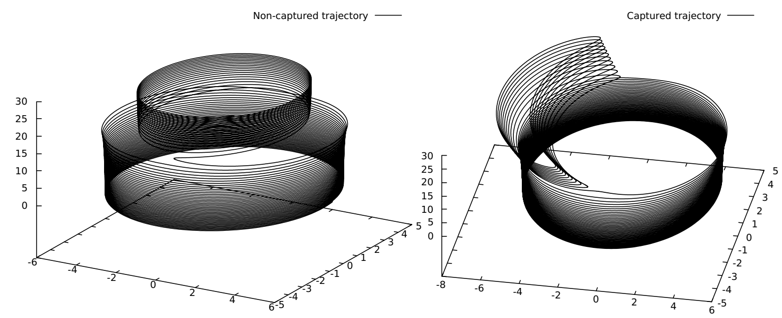


Рис. 1. On the left-hand side one can see a solution of (2) with initial condition $\psi = 5\exp(0.15i)$ at $t = 0$. The trajectory turns at $t \sim 20$. On the right-hand side one can see a solution of (2) with initial condition $\psi = 5\exp(0.19i)$ at $t = 0$. The graph shows how this trajectory is captured at $t \sim 20$. Both trajectories are constructed by Runge-Kutta method of 4-th order with step 0.001.

additional calculations. For primary order such frequencies are defined by Mathieu functions. The primary resonance takes place near $\Omega = 2$.

In the resonant interval the solutions grow up to order $\sqrt{\epsilon}$ and the linear approach becomes invalid. Here it is important to study non-linear effects. The growth of the amplitude of solutions up to order $\sqrt{\epsilon}$ is typical for non-linear parametric resonance.

The parametric autoresonance is more sophisticated phenomenon, see [1]. It arises when a phase of the oscillations is captured by a perturbation with slow changing frequency for a lot applications from Faraday waves [2] and plasmas [3] up to quantum phenomena [4]. In the parametric resonance only part of the trajectories can be captured. Two cases are appropriated to study by asymptotic methods. There are solutions of (2) with small amplitude or large amplitude. The oscillations with intermediate amplitudes can be investigated numerically for example.

The amplitude of parametric autoresonance is defined by

$$i\psi' + (\lambda^2\tau - |\psi|^2)\psi + \overline{\psi} = 0, \quad \tau = \epsilon t. \tag{2}$$

This equation defines an evolution of amplitude of non-linear oscillator like (1). This equation can be rewritten in more elegant form of equation for the parametric autoresonance, which looks like as one equation of second order:

$$\varphi'' + 4\lambda^2\tau\sin(\varphi) + 2\sin(2\varphi) - 2\lambda^2 = 0. \tag{3}$$

Our goal is studying of the capture into parametric resonance of large amplitude solutions of (2) as $\tau \to \infty$. It is convenient to investigate such solutions of (2) using a special depending on an inverse value of small parameter:

$$\psi = \varepsilon^{-1}\Psi(\tau, \varepsilon), \quad 0 < \varepsilon \ll 1.$$

Here $\varepsilon^{-1}$ is a parameter of solution, which defines an amplitude of oscillations of $\psi$. After substitution (2) one gets:

$$i\varepsilon^2\Psi' + (\lambda^2\varepsilon^2\tau - |\Psi|^2)\Psi + \varepsilon^2\overline{\Psi} = 0. \tag{4}$$

Note that $\varepsilon$ and $\epsilon$ are small parameters which correspond to different problems. Parameter $\epsilon$ is perturbation of the non-linear oscillator and parameter $\varepsilon$ is a formal parameter which is useful for studying large solutions of (2) and which is derived from (1) for small amplitude oscillations of non-linear oscillator. A condition $\varepsilon \ll \sqrt{\epsilon}$ should be true for an asymptotic formalism which is considered here. The order of amplitudes of oscillations, which are considered here, is *intermediate small:* $\sqrt{\epsilon} \ll |A(\tau)| \ll 1$.

There exist stable focuses for the equation (3).

The stable focuses of (3) define the captured solutions.

Beginning at some large $\tau = \tau_0$ up to infinity the measure of captured trajectories has a following asymptotics

$$M \sim \frac{16\pi\lambda^2}{\tau_0}, \quad \tau_0 \to \infty.$$

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**Grassmann extensions of Yang-Baxter maps related to NLS type equations**

*S. Konstantinou-Rizos (Grozny)*

In this talk, we present some novel endomorphisms between Grassmann extensions of algebraic varieties which possess the Yang-Baxter property. In particular, we consider the cases of the nonlinear Schrödinger (NLS) equation and the derivative NLS equation, and we make use of their associated Darboux transformations to construct ten-dimensional maps which can be restricted to eight-dimensional Yang-Baxter maps on invariant leaves. These results constitute the first attempt to extend the theory of Yang-Baxter maps in the case of Grassmann algebras.

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**Lefschetz trace formulas for flows on foliated manifolds**

*Yu. Kordyukov (Ufa)*

In my talk, I will discuss Lefschetz trace formulas for foliated flows on compact manifolds equipped with codimension one foliations. First, I will recall such a formula, due to J. Alvarez Lopez and myself, in the case when the orbits of the flow are everywhere tranverse to the leaves of the foliation. I will briefly describe the role of Lefschetz trace formulas for foliated flows in Deninger's approach to the study of arithmetic zeta-functions. Then I will consider a case when the flow may have fixed points and describe an approach to Lefschetz trace formulas for such flows based on the pseudodifferential b-calculus on manifolds with boundary developed by R. Melrose. I will report on the recent progress in this direction. This is joint work with J. Alvarez Lopez and E. Leichtnam.

**Subkähler structures and kähler submanifolds**

*E. Kornev (Kemerovo)*

Let $M$ be a real smooth manifold of dimension $n \geq 3$. A bilinear form $\Omega$ on $M$ is refered as regular if its radical

$$\mathrm{rad}\omega = \bigcup \mathrm{rad}\Omega_x : \mathrm{rad}\Omega_x = \{v \in T_xM : \Omega(v, .) = 0\}$$

is regular distribution on $M$. A subtwistor structure on $M$ is the set $(\Omega, D, \Phi, g)$, where $\omega$ is a regular degenerated linear 2-form on $M$, $D$ is an orthogonal complementary to radical of 2-form $\Omega$, $\Phi$ is a special smooth field of tangent spaces endomorphisms associated with 2-form $\Omega$, and $g$ is a Riemannian metric on $M$. A subtwistor structure $(\Omega, D, \Phi, g)$ along with submanifold $Q \subset M$ so that $d\Omega = 0$ and restriction of $\Phi$ to $Q$ is complex structure on $Q$ is called a Subkähler structure. We provide some concepts and results for subtwistor and subkähler structures. In particular, we define the concepts of torsion and induced torsion for subtwistor structure, and provide that any free-torsion subtwistor structure generates a subkähler structure, and any subtwistor structure having zero torsion and induced torsion on $M$ allows represented $M$ as dirrect product of some kähler submanifold and riemannian submanifold. Also, we show how $G$-invariant subkähler structures on homogeneous space $G/H$ allows to obtain homogeneous kähler subspaces in the case of arbitrary dimension. We show relationship between Lie subalgebras in distribution $D$ for subkähler structure on Lie group $G$ and kähler homogeneous submanifolds in homogeneous space $G/H$.

**Heegaard splittings of branched cyclic coverings of connected sums of lens spaces**

*T. Kozlovskaya (Magadan)*

We study relations between two descriptions of closed orientable 3-manifolds: as branched coverings and as Heegaard splittings. An explicit relation is presented for a class of 3-manifolds which are branched cyclic coverings of connected sums of lens spaces, where the branching set is an axis of a hyperelliptic involution of a Heegaard surface.

**The absolute boundary of discrete Heisenberg group**

*A. Malyutin (St. Petersburg)*

A.M.Vershik introduced the notion of absolute boundary (also called the 'absolute') for finitely generated groups. The absolute boundary of a group is a topological space that can be regarded as the boundary at infinity (Dynkin's exit-boundary) of the so-called dynamical graph over the Cayley graph of the group. The absolute boundary contains, in a sense, the Poisson-Furstenberg boundary of the group and is contained in the Martin boundary of the dynamical graph. A part of the absolute boundary can be identified with the set of all minimal positive eigenfunctions of the Laplacian determined by the simple random walk on the group. The absolute boundary of an abelian group is homeomorphic to a closed ball of certain dimension. (The fact that the absolute boundary of the infinite cyclic group is an interval is a reformulation of de Finetti's theorem.) The absolute boundary of the free non-abelian group is homeomorphic to the direct product of the Cantor set by an interval. The next phase in developing the theory of absolute boundary is the case of nilpotent groups. We show that in the case of discrete Heisenberg group with the standard generating set, the absolute boundary is homeomorphic to the disjoint union of a closed 2-disk and a countable set of isolated points whose limit set is the boundary of the 2-disk. In order to find the absolute we need, in particular, to describe the set of all geodesic rays in the (Cayley graph of) Heisenberg group.

**Naturally graded Lie algebras of slow growth**

*D. Millionshchikov (Moscow)*[5]

The growth of a finitely generated infinite-dimensional Lie algebra $\mathfrak{g}$ can be described by the Gelfand-Kirillov dimension which is defined as

$$GK\dim\mathfrak{g} = \limsup_{n\to\infty}\frac{\log\dim V^n}{\log n},$$

where $V^n$ is the subspace in $\mathfrak{g}$ spanned by all elements of length at most $n$ with arbitrary arrangements of brackets. A finite Gelfand-Kirillov dimension means that there exists a polynomial $P(x)$ such that $\dim V^n{<}P(n)$ for all $n{>}1$. Shalev and Zelmanov obtained [6] important results on Lie algebras of linear growth (obviously they have the GK-dimension equal to 1). Petrograsky constructed examples of Lie algebras $\mathfrak{g}$ with non-linear growth but however such that $GK\dim\mathfrak{g} = 1$ [5]. Shalev, Zelmanov defined Lie algebras of maximal class – a special subclass of the positively graded two-generated Lie algebras with the slowest possible growth ($\dim V^n = n{+}1$).

Kac [2] classified under a certain technical condition infinite-dimensional $\mathbb{Z}$-graded simple Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{Z}}\mathfrak{g}_i$ of finite growth in a following sense: $\dim\mathfrak{g}_n \le P(n)$ for some polynomial $P(x)$. Moreover, Kac conjectured that dropping the condition would add only the Witt algebra. Finally Kac's conjecture was proved in 1990 by Mathieu [3].

Fialowski [1] classified $\mathbb{N}$-graded Lie algebras $\mathfrak{g} = \oplus_{i\in\mathbb{N}}\mathfrak{g}_i$ with one-dimensional homogeneous components $\mathfrak{g}_i$ that are multiplicatively generated by two elements from $\mathfrak{g}_1$ and $\mathfrak{g}_2$ respectively. Besides other algebras one can find in her list the positive part of the Witt (Virasoro) Lie algebra $W^+$ and two positively graded Lie

[5]The researcch was made under the support of the RSF grant № 14-11-00414.

algebras $\mathfrak{n}_1$ and $\mathfrak{n}_2$ that are maximal nilpotent subalgebras of twisted loop algebras $A_1^{(1)}$ and $A_2^{(2)}$ respectively.

A Lie algebra $\mathfrak{g}$ is called naturally graded if it is isomorphic to its associated graded Lie algebra $\mathrm{gr}_C\mathfrak{g}$ with respect to the filtration by ideals $C^i\mathfrak{g}$ of the descending central sequence. For instance $\mathrm{gr}_C W^+ \cong \mathfrak{m}_0$, where $\mathfrak{m}_0$ can be defined by its infinite basis $e_1, e_2, \dots$, and structure relations: $[e_1, e_i] = e_{i+1}, i = 2, 3, \dots$ $[e_k, e_l] = 0, k, l \geq 2$. It was proved by Vergne [7] that up to an isomorphism there is the only one naturally graded Lie algebra of maximal class and it is $\mathfrak{m}_0$.

We classify naturally graded Lie algebras with the linear growth $\dim V^n \leq \frac{3}{2}n + const$.

**Theorem**. Let $\mathfrak{g} = \oplus_{i=1}^{+\infty}\mathfrak{g}_i$ be a real naturally graded Lie algebra such that:

$$\dim \mathfrak{g}_i + \dim \mathfrak{g}_{i+1} \leq 3, \forall i \in \mathbb{N}.$$

Then $\mathfrak{g} = \oplus_{i=1}^{+\infty}\mathfrak{g}_i$ is isomorphic to the only one Lie algebra from the following list:

$$\mathfrak{m}_0, \mathfrak{n}_1^{\pm},\ \mathfrak{n}_2, \left\{\mathfrak{m}_0^S \mid S \subset \{3, 5, 7, 9, \dots\}\right\},$$

where $\mathfrak{n}_1^{\pm}$ are special subalgebras in loop Lie algebras $\mathfrak{so}(3, \mathbb{R})$ and $\mathfrak{so}(1, 2, \mathbb{R})$ respectively and $\left\{\mathfrak{m}_0^S \mid S \subset \{3, 5, 7, 9, \dots\}\right\}$ are central extensions of $\mathfrak{m}_0$.

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## On Hamiltonian geometry of associativity equations

*N. Pavlenko (Moscow)*[6]

In the case of three primary fields, the associativity equations or the Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations of the two-dimensional topological quantum field theory can be represented as integrable nondiagonalizable systems of hydrodynamic type (O.I. Mokhov, [1]). After that the question about the Hamiltonian nature of such hydrodynamic type systems arose. O.I. Mokhov and E.V. Ferapontov [2] have shown that the Hamiltonian geometry of these systems essentially depends on the metric of the associativity equations. Namely there are examples of the WDVV equations which are equivalent to the hydrodynamic type systems with local homogeneous first-order Dubrovin-Novikov type Hamiltonian structures,

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and those which are equivalent to the hydrodynamic type systems without such structures.

The classification problem of existence of a local first-order Hamiltonian structure for the associativity equations in the representation of hydrodynamic type system in the case of three primary fields has been solved by O.I. Mokhov and the author. The results of O.I. Bogoyavlenskij and A.P. Reynolds [3] for the three-component nondiagonalizable hydrodynamic type systems are essentially used for the solution. The classification will be presented.

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## Deformation quantization and the action of Poisson vector fields

*G. Sharygin (Moscow)*[7]

As one knows, for every Poisson manifold M there exists a formal noncommutative deformation of the algebra of functions on it; it is determined in a unique way (up to an equivalence relation) by the given Poisson bivector. Let a Lie algebra $g$ act by derivations on the functions on M. The question, which we shall address in my talk is whether it is possible to lift this action to the derivations on the deformed algebra. It is easy to see, that when dimension of $g$ is 1, the only necessary and sufficient condition for this is that the given action is by Poisson vector fields. However, when dimension of $g$ is greater than 1, this method does not work. In this paper we show how one can obtain a series of homological obstructions for this problem, which vanish if there exists the necessary extension. We hope that these obstructions can be of interest in their own right as invariants of Poisson actions.

## Almost graded current algebras on the symmetric square of a curve

*O. Sheinman (Moscow)*

The Krichever-Novikov function and current algebras are almost graded associative (resp., Lie) algebras on algebraic curves. It is a non-trivial question whether their analogs can be constructed for algebraic surfaces. We present an example of such construction on one of simplest algebraic surfaces: the symmetric square of a curve.

[7] The talk is based on the paper arXiv:1612.02673.

**Birational automorphisms**

*K. Shramov (Moscow)*

I will survey various results on groups of birational selfmaps of algebraic varieties, focusing on their finite subgroups. The most interesting examples are provided by birational selfmaps of rationally connected varieties, including classical Cremona groups. In particular, I will explain some recent results concerning Jordan property for these groups.

**Cohen-Macaulay modules over dihedral quasi-invariants and Calogero-Moser systems**

*A. Zheglov (Moscow)*

It is well known that the Calogero-Moser differential operator is (super) integrable for certain potentials, i.e. it determines an algebraically completely integrable quan-tum system. On the other hand, such systems are essentially determined by their spectral variety and spectral sheaf, both being Cohen-Macaulay. Answering a ques-tion of Feigin and Johnston, we determine the spectral surfaces and spectral sheaves of quantum Calogero-Moser systems with rational potentials quasi-invariant with res-pect to the dihedral group $I_2(N)$. It appears that all Cohen-Macaulay sheaves on such surfaces have a nice description similar to the description of the generalised Jacobian for singular rational curves. Unlike the case of curves, not every such sheaf is spectral. The moduli space of spectral sheaves appears to be much more subtle, but its structure indicates the existence of integrable deformations of such Calogero-Moser systems in the ring of differential-difference operators. We were able to produce some explicit examples of such deformations. The talk is based on the joint work with Igor Burban.

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