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LINEAR PROBLEM OF SHOCK WAVE DISTURBANCE ANALYSIS. PART 1: GENERAL SOLUTION, INCIDENCE, REFRACTION AND REFLECTION IN GENERAL CASE

E. V. SEMENKO

Abstract. This article is devoted to the linear problem of shock wave disturbance, where a number of questions related to this problem are considered. A new representation of problem's solution, having completely algebraic form in spectral variables, is found, which allows us to scrutinize the problem, obtain new results and refine known ones. The analytical results are approved and illustrated by numerical calculations.

A whole article is divided into three parts because of a large volume.

In first part, the basic representation of initial-value problem's solution is established, and the basic techniques of its analysis – singular and regular terms detachment, incident, refracted and reflected waves separation – is described. On this basic, the incidence upon the shock, refraction and reflection of waves in general form is inspected.

The peculiarity of refraction, which haven't been noted before, is found: any incident wave may be decomposed into the sum of waves with physically different interaction with shock, namely, one summand interacts with shock, i.e. generates shock disturbance, but doesn't generate any transmitted waves; other summands don't interact with shock, i.e. don't generate shock disturbance, but generate different kinds of transmitted waves.

A post-shock incidence of different kinds of waves and its reflection is inspected, in particular a four-wave configuration at reflection is stated.

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1. Introduction

The propagation of small perturbations of hydrodynamical quantities and small deformations of the shock wave front against the background of the basic flow is considered in the linear problem of the shock wave disturbance. In the original nonlinear problem, the sought hydrodynamical quantities are involved into the usual functional relations (e.g., equation of state), governing Euler equations, and Rankine-Hugoniot conditions on the shock. The piecewise-constant quantities are considered as the basic flow (basic or unperturbed solution), and the surface of their discontinuity (basic or unperturbed shock) is the plane $x = 0$. The linear problem occurs after linearization of the original nonlinear problem on the basic solution. This problem has a long history and may be treated as a classical one; we refer reader to the widely known monograph [1].

The study of the problem can be conventionally divided into two directions.

The first direction is the study of important classes of partial solutions. The class of solutions in the form of plane waves is usually considered, particularly, in connection with studying wave refraction and reflection. This direction is presented in pioneering works [2, 3] and [4]; a fairly complete description and detailed investigation of the refraction and reflection problem can be found in the paper [5] and in many later publications [6, 7, 8, 9, 10, 11, 12, 13, 14], the list is far from being complete.

The second direction is the solution of the initial-value problem, i.e. the problem with additional assignment of the values of the sought quantities at the time instant $t = 0$. As a main source, the publications [15, 16, 17, 18, 19, 20, 21, 22] can be indicated; again the list is far from being full. In particular, it was clarified that the solution of the initial-value problem is unique, and assignment of the initial values is the simplest and most natural way to parameterize the general solution (i.e. the set of all solutions) of the problem.

The works dealing with the initial-value problem differ in many important details, but there is a great similarity between them, namely, they use the similar methods: Fourier transform with respect to the tangent to the unperturbed shock space variables, Laplace transform with respect to time, and solving the remaining system of differential equations with the only variable normal to the shock. For the shock front disturbance, an algebraic linear equation is derived, which is essentially the same in all works up to notations (in the present work it is Eq. (11) in Part 1, Section 4). The specific features of the solution of this linear equation, namely, the presence or absence of singularities in the analyticity region, separate the so-called cases of stability, instability, and "intermediate" case of the D'yakov-Kontorovich instability or neutral stability in modern terms. However, the solution is written out in a quite "asymmetrical" form with respect to the different variables. Moreover, it is also "asymmetrical" with respect to pre-shock/post-shock regions. In particular, the problem ahead of the shock practically cannot be solved, and the boundary values on the shock of the sought quantities (from the pre-shock zone) are actually treated as given (e.g. [16, 17, 22]) or zero (e.g. [15, 18, 19, 20, 21]). This asymmetry and especially the need of solving of the system of differential equations appreciably impede the wave analysis. In particular, the natural decomposition of the solution into a sum of acoustic and entropy-vorticity waves and, moreover, into incident and refracted and/or reflected waves is not considered; it is a separate and, seemingly, hard problem for applied methods.

On the other hand, if only partial solutions in the form of plane waves are considered, solution's decomposition into acoustic and entropy-vorticity and into incident and refracted and/or reflected waves does not produce any problem, and all calculations are algebraic calculations only. In this way, the cases of stability, instability, and neutral stability were first identified (in [2], further this classification was repeatedly specified, checked, and rechecked), and the problem of wave refraction and reflection was investigated in detail (see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]). However, as only partial solutions in specific forms are considered, then some important solutions or parts of solutions can be missed, which leads to some mistakes or misunderstandings, particularly, to the paradoxes in the linear theory, e.g. so-called abnormal amplification at refraction, or spontaneous emission of waves by shock.

A number of papers are devoted to the original nonlinear problem, e.g., [23, 24, 25]; again the list is far from being complete. As the nonlinear problem is much more complicated, the results are significantly less comprehensive. As a rule, all conclusions about the solution properties are based on the Hugoniot (shock) adiabat analysis. In this way, the stability/instability/neutral stability conditions were checked and rechecked again and the cases of instability and neutral stability were analyzed. The general results can be summarized as follows: in the instability case, there are no temporally exponentiating solutions, as the linear theory predicts, but the shock wave splits into several different waves (the linear theory does not predict any splitting of the shock); in the neutral stability case, the reason for the paradox of spontaneous emission is that some agent of the second order of smallness, which is incident upon the shock, is neglected in the linear theory.

Finally, it is worth noting the publications [26, 27] dealing with experimental or numerical investigations of the problem. In work [26], the predictions of the linear theory about spontaneous emission are confirmed; in work [27], the splitting of the shock in the case of instability is investigated numerically, and the conclusions in general confirm the results of [24, 25].

In the present paper, we propose a method of solving the linear problem by means of combining both above-mentioned directions. In general, it is a rather usual technique for solving partial differential equations. The principal feature is that the Fourier transform is applied uniformly and with respect to all variables at once. This "total" Fourier transform immediately converts the problem to a completely algebraic form. It allows one to write out the solution (shock disturbance, pre-shock and post-shock waves) in an explicit and strictly algebraic form, uniformly, and in not too complicated expressions. This, in turn, offers additional possibilities for the solution analysis.

Thus, the obtained algebraic (in spectral variables) formulas allow immediately separate singular terms of solution (e.g. plane waves) and regular terms, i.e. damped waves. Furthermore, as the solution representation (as the Fourier transform) involves actually plane waves, we obtain an absolutely natural way to decompose any solution into a sum of acoustic, entropy, and vorticity waves and into incident and refracted and/or reflected waves. At that we may analyze the obtained summands separately or in any combination. So we can consider refraction or reflection either for solutions in the general form (which has not been ever done before) or for plane waves only (which has been analyzed in previous works of [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]), e.g. separate plane and damped waves at incidence/refraction/reflection, etc. Moreover, as we have an explicit representation of any solution, then we "do not lose" any important agents when we describe the solution properties. It allows us to investigate the problem deeper and more comprehensively, in particular, refine some known results, correct some mistakes and misunderstandings, explain some paradoxes of the linear theory only in its own framework, and somehow "rehabilitate" the linear theory. In particular, we argue that the abnormal amplification at refraction isn't exist, and the agent (source) of spontaneous emission in fact is not neglected under the linear theory.

The article consists of three parts. In first part, we deduce formulas of solution, describe basic techniques of its analysis, and inspect the incidence, refraction and reflection in general case. The second part is devoted to the plane waves propagation (incidence, refraction and reflection) in stability case. In third part, we consider refraction and reflection in neutral stability case, in particular inspect the spontaneous emission. Also in this last part, we combine and briefly describe all obtained results (in Summary), and adduce all necessary details of using the mathematical technique (in Appendix).

The numeration of formulas, sections, etc., is separate in different parts. In the cross-references, we indicate the part, e.g. "see Part 1, Eq. (2)". For reader's convenience, all parts are supplied with the full reference list.

This (first) part is organized as follows. Section 2 contains the problem formulation. In Section 3, we introduce technical terms and notations, basic for following investigations. In Section 4, we construct the initial-value problem's solution (Subsection 4.1); discuss a method of singular and regular parts of solution detachment (Subsection 4.2); and separate the stability, instability, and neutral stability cases (Subsection 4.3). In Section 5, we consider the refraction and reflection in general case: the decomposition of any solution into the sum of acoustic and entropy-vorticity waves (Subsection 5.1); the decomposition of any post-shock wave into the sum of initial (incident), refracted and reflected waves (Subsection 5.2); the pre-shock incidence and refraction (Subsection 5.3); the post-shock incidence and reflection (Subsection 5.4).

2. Formulation of the problem

In the variables (x, y, t), we consider the sought functions: density ρ, velocity $U = (U_x, U_y)$, pressure p, entropy s, specific (per unit volume) internal energy ϵ, temperature T, enthalpy w, and specific energy e. These functions are involved into the functional relations $\rho = \rho(p, s)$ (equation of state), $\epsilon = \epsilon(\rho, s)$, $w = \epsilon + p/\rho$, $e = \rho\epsilon + \rho|U|^2/2$, and the Euler equations: continuity equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho U) = 0,$$

momentum conservation law

$$\rho\left(\frac{\partial U}{\partial t} + (U, \nabla)U\right) + \nabla p = 0,$$

and energy conservation law

$$\frac{\partial e}{\partial t} + \mathrm{div}(U(e+p)) = 0.$$

We consider discontinuous solutions. Let the surface of the discontinuity (shock wave or shock front or simply shock) has the form $x = f(y,t)$, where f is the sought function. If we treat discontinuous functions as weak (generalized) solutions (see e.g. [28, 29]), then, at $x = f(y,t)$, we obtain the standard Rankine-Hugoniot conditions

$$\begin{gathered}
[\rho U_x] = f'_t[\rho] + f'_y[\rho U_y]\,, \\
[\rho U_x^2] + [p] = f'_t[\rho U_x] + f'_y[\rho U_y U_x]\,, \\
[\rho U_x U_y] = f'_t[\rho U_y] + f'_y[\rho U_y^2] + f'_y[p]\,, \\
[U_x(e+p)] = f'_t[e] + f'_y[U_y(e+p)]\,,
\end{gathered}$$

where, as usually, the square brackets denote the jump in a quantity across the discontinuity.

Now we linearize the problem on a partial (basic) solution. As the basic solution, we consider a piecewise-constant solution having a jump on the plane $x = 0$, i.e. $f^0 = 0$ (the superscript 0 denotes the basic solution),

$$\rho^0 = \begin{cases} \rho^+, & x < 0, \\ \rho^-, & x > 0, \end{cases} \qquad \rho^\pm = \mathrm{const}\,,$$

similarly, $U^0 = (u_x^\pm, u_y)$, p^0, s^0. The Rankine-Hugoniot conditions for this solution have the form (see e.g. [1, 5])

$$[\rho^0 u_x] = 0\,, \quad [p^0 + \rho^0 (u_x)^2] = 0\,, \quad [u_y] = 0\,, \quad [w^0 + (u_x)^2/2] = 0\,,$$

and mean definite relations between the constants $\rho^\pm$, $u_x^\pm$, $p^\pm$, and $w^\pm$. These relations lead to the equation

$$[\epsilon^0] + \frac{1}{2}\left[\frac{1}{\rho^0}\right](p^+ + p^-) = 0\,,$$

which is called the shock adiabat (Hugoniot adiabat).

Additionally, we accept some assumptions having clear physical grounds (see [1, 2] and other publications):

$$u_x^+ > u_x^- > 0\,, \quad s^- > s^+\,, \quad p^- > p^+\,; \quad u_x^+ > c^+\,, \quad u_x^- < c^-\,,$$

where $c^\pm = \sqrt{1/\rho'_p(p^\pm, s^\pm)}$ is the sound velocity for the basic solution. It is these assumptions, that ensures the correctness of linearized Cauchy problem.

After the linearization of the original problem (Euler equations and Rankine-Hugoniot conditions) on the basic solution, we obtain for the variations

$$\delta\rho = \delta\rho^\pm\,, \quad \delta U = \delta U^\pm = (\delta U_x^\pm, \delta U_y^\pm)\,, \quad \delta s = \delta s^\pm\,, \quad \pm x \leqslant 0$$

the following system of differential equations at $x \neq 0$:

$$\begin{gathered}
\frac{\partial \delta\rho}{\partial t} + \rho^0 \cdot \mathrm{div}\,\delta U + (U^0, \nabla\delta\rho) = 0\,, \\
\rho^0\left(\frac{\partial \delta U}{\partial t} + (U^0, \nabla)\delta U\right) + c^2\nabla\delta\rho - c^2 r \nabla\delta s = 0\,, \\
\frac{\partial \delta s}{\partial t} + (U^0, \nabla\delta s) = 0\,,
\end{gathered}$$

and the boundary conditions at $x = 0$:

$$[u_x\delta\rho + \rho^0\delta U_x] = [\rho^0](f'_t + u_y f'_y)\,,$$
$$[\delta p + J^0\delta U_x] + [u_x(\rho^0\delta U_x + u_x\delta\rho)] = 0\,,$$
$$[J^0\delta U_y] = -J^0[u_x]f'_y\,,$$
$$[u_x(\delta p + J^0\delta U_x)] + J^0[T\delta s] = J^0[u_x](f'_t + u_y f'_y)\,,$$

where $r = \rho'_s(p^0, s^0) = r^\pm$ is the isobaric derivative of density, $J^0 = \rho^0 u_x$ is the flux across the shock for the basic solution, and $T = e'_s(\rho^0, s^0, U^0)/\rho^0 = T^\pm$ is the temperature for the basic solution.

Thus, we obtain a standard and almost classical formulation of the linear problem of the shock wave disturbance, see [1] and the references therein.

The region $x < 0$ – pre-shock zone, or upstream or ahead of the shock region – is marked by the plus superscript, and the region $x > 0$ – post-shock zone, or downstream or behind the shock region – is marked by the minus superscripts. Further we usually omit the $\pm$ superscripts meant on default. Sometimes only the plus or minus superscript is tacitly meant; all these cases are specially indicated.

For convenience, we introduce the sought vector in a form

$$G = G^\pm(x,y,t) = \begin{pmatrix} \delta\rho \\ \rho^0\delta U_x/c \\ \rho^0\delta U_y/c \\ r\delta s \end{pmatrix}$$

i.e., we reduce all sought quantities to the dimension of density. In this notation, the problem has the following form:
the system of differential equations at $x \neq 0$ is

$$(1)\qquad BG = 0\,,\; B = B^\pm = \begin{pmatrix} \dfrac{d}{dt} & c\dfrac{\partial}{\partial x} & c\dfrac{\partial}{\partial y} & 0 \\ c\dfrac{\partial}{\partial x} & \dfrac{d}{dt} & 0 & -c\dfrac{\partial}{\partial x} \\ c\dfrac{\partial}{\partial y} & 0 & \dfrac{d}{dt} & -c\dfrac{\partial}{\partial y} \\ 0 & 0 & 0 & \dfrac{d}{dt} \end{pmatrix}\,,\; \frac{d}{dt} = \frac{\partial}{\partial t} + (U^0, \nabla)\,;$$

and the boundary conditions at $x = 0$ are

$$(2)\qquad A^+H^+ = A^-H^- + F_0 f\,;\quad H^\pm = A_0^\pm G_\Gamma^\pm\,,\quad G_\Gamma^\pm(y,t) = G^\pm(0,y,t)\,,$$

where

$$A^\pm = A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ u_x & c & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & u_x c & 0 & \rho^0 T/r \end{pmatrix}\,,\quad A_0^\pm = A_0 = \begin{pmatrix} u_x & c & 0 & 0 \\ c & u_x & 0 & -c \\ 0 & 0 & u_x & 0 \\ 0 & 0 & 0 & u_x \end{pmatrix}\,;$$

$$F_0 = \begin{pmatrix} [\rho^0](\partial/\partial t + u_y\partial/\partial y) \\ 0 \\ -J^0[u_x]\partial/\partial y \\ J^0[u_x](\partial/\partial t + u_y\partial/\partial y) \end{pmatrix}\,.$$

So we have Eqs. (1) and (2) for the sought quantities $G(x,y,t)$ and $f(y,t)$.

We denote the initial values by $G_0(x,y) = G_0^{\pm}(x,y) = G^{\pm}(x,y,0)$.

For solving the problem, we employ the Fourier transform. As we solve the initial-value problem at $t > 0$, and the coefficients of the differential equations are different at $\pm x < 0$, we use the Fourier transform in the form

$$\hat{G} = \hat{G}^{\pm}(\xi,\eta,\omega) = \frac{1}{(2\pi)^3} \iiint\limits_{\substack{t>0\\ \pm x<0}} G^{\pm}(x,y,t) e^{-i(\xi x+\eta y-\omega t)}\, dxdydt\,,$$

accordingly,

$$\hat{f}(\eta,\omega) = \frac{1}{(2\pi)^2} \iint\limits_{t>0} f(y,t) e^{-i(\eta y-\omega t)}\, dydt\,;$$

$$\hat{G}_0^{\pm}(\xi,\eta) = \frac{1}{(2\pi)^2} \iint\limits_{\pm x<0} G_0^{\pm}(x,y) e^{-i(\xi x+\eta y)}\, dxdy\,,$$

$$\hat{G}_\Gamma^{\pm}(\eta,\omega) = \frac{1}{(2\pi)^2} \iint\limits_{t>0} G_\Gamma^{\pm}(y,t) e^{-i(\eta y-\omega t)}\, dydt\,.$$

The properties of the introduced (one-sided) Fourier transform are described in Appendix (Part 3, Subsection 6.1).

In the plane of the Fourier transform, we obtain the problem

$$\hat{B}^{\pm}(\xi,\eta,\omega)\hat{G}^{\pm}(\xi,\eta,\omega) = \frac{\hat{G}_0^{\pm}(\xi,\eta) \mp \hat{H}^{\pm}(\eta,\omega)}{2\pi i}\,, \tag{3}$$

$$A^+\hat{H}^+ = A^-\hat{H}^- + i\hat{F}_0\hat{f}\,, \quad \hat{H}^{\pm} = A_0^{\pm}\hat{G}_\Gamma^{\pm}\,; \tag{4}$$

where

$$\hat{B} = \begin{pmatrix} Q & c\xi & c\eta & 0 \\ c\xi & Q & 0 & -c\xi \\ c\eta & 0 & Q & -c\eta \\ 0 & 0 & 0 & Q \end{pmatrix},$$

$Q = Q^{\pm}(\xi,\eta,\omega) = \xi u_x + \eta u_y - \omega$ is a symbol of the operator $-id/dt$;

$$\hat{F}_0 = \begin{pmatrix} [\rho^0](\eta u_y - \omega) \\ 0 \\ -J^0[u_x]\eta \\ J^0[u_x](\eta u_y - \omega) \end{pmatrix}.$$

The inverse matrix for $\hat{B}$ has a form

$$\hat{B}^{-1} = \frac{F}{PQ}\,,$$

$$F = F^{\pm}(\xi,\eta,\omega) = \begin{pmatrix} Q^2 & -c\xi Q & -c\eta Q & -c^2(\xi^2+\eta^2) \\ -\xi cQ & P + c^2\xi^2 & \xi\eta c^2 & \xi cQ \\ -\eta cQ & \xi\eta c^2 & P + \eta^2 c^2 & \eta cQ \\ 0 & 0 & 0 & P \end{pmatrix}, \tag{5}$$

where $P = P^{\pm}(\xi,\eta,\omega) = Q^2 - c^2(\xi^2+\eta^2)$ is a symbol of the wave operator $-d^2/dt^2 + c^2\Delta$.

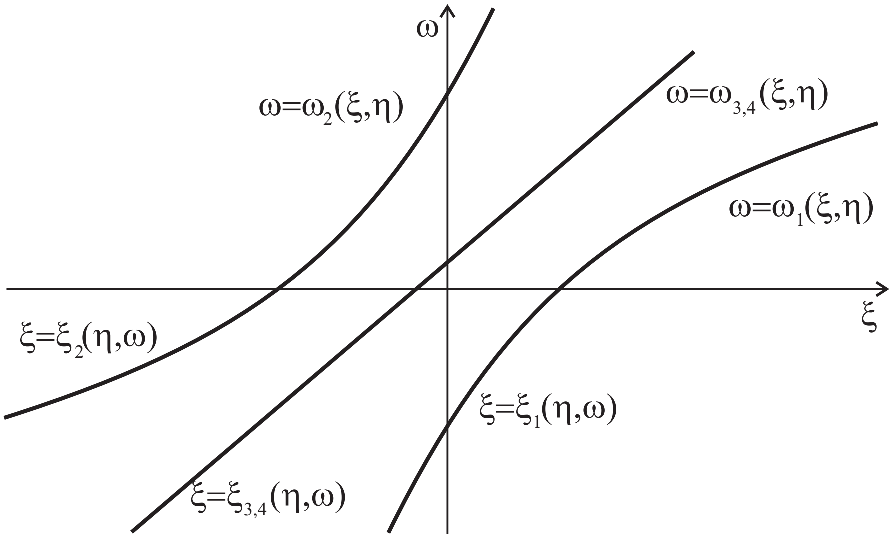

Fig. 1: Roots of P and Q in the pre-shock zone

Obviously, to solve the algebraic problem (3)-(4), the localization of the roots of the functions Q and P is of great importance.

3. Zero localization and some vector bases

We denote the roots of Q by the subscripts 3 and 4:

$$Q(\xi,\eta,\omega)=0 \Longleftrightarrow \omega=\omega_3(\xi,\eta)=\omega_4(\xi,\eta)=\xi u_x+\eta u_y$$

$$\Longleftrightarrow \xi=\xi_3(\eta,\omega)=\xi_4(\eta,\omega)=\frac{\omega-\eta u_y}{u_x};$$

and the roots of P by the subscripts $\pm$:

$$P(\xi,\eta,\omega)=0 \Longleftrightarrow \omega=\omega_\pm(\xi,\eta)=\xi u_x+\eta u_y\pm c\sqrt{\xi^2+\eta^2}$$

$$\Longleftrightarrow \xi=\xi_\pm(\eta,\omega)=\frac{u_x(\omega-\eta u_y)\pm c\sqrt{(\omega-\eta u_y)^2+\eta^2(u_x^2-c^2)}}{u_x^2-c^2}.$$

Ahead of the shock (superscript plus, $u_x>c$), the roots $\xi_\pm(\eta,\omega)$ are real and different at real values of ω, and both are located in the upper half-plane $\operatorname{Im}\xi_\pm>0$ at $\operatorname{Im}\omega>0$. We denote $\omega_1=\omega_-$, $\xi_1=\xi_+$ and $\omega_2=\omega_+$, $\xi_2=\xi_-$.

In the fig.1, we conditionally depict plots of these roots at given η. The plots of roots $\omega_{1,2}$ and $\xi_{1,2}$ respectively are the sections of pre-shock characteristic cone $P^+(\xi,\eta,\omega)=0$ by plane $\eta=\text{const}$.

Behind the shock (superscript minus, $u_x<c$), if ω is real, then the roots $\xi_\pm(\eta,\omega)$ are:

- real and different at $|\omega-\eta u_y|>|\eta|\sqrt{c^2-u_x^2}$,
- complex at $|\omega-\eta u_y|<|\eta|\sqrt{c^2-u_x^2}$,
- coincident $\xi_+=\xi_-$ at $|\omega-\eta u_y|=|\eta|\sqrt{c^2-u_x^2}$ or $\omega=\eta u_y\pm|\eta|\sqrt{c^2-u_x^2}$.

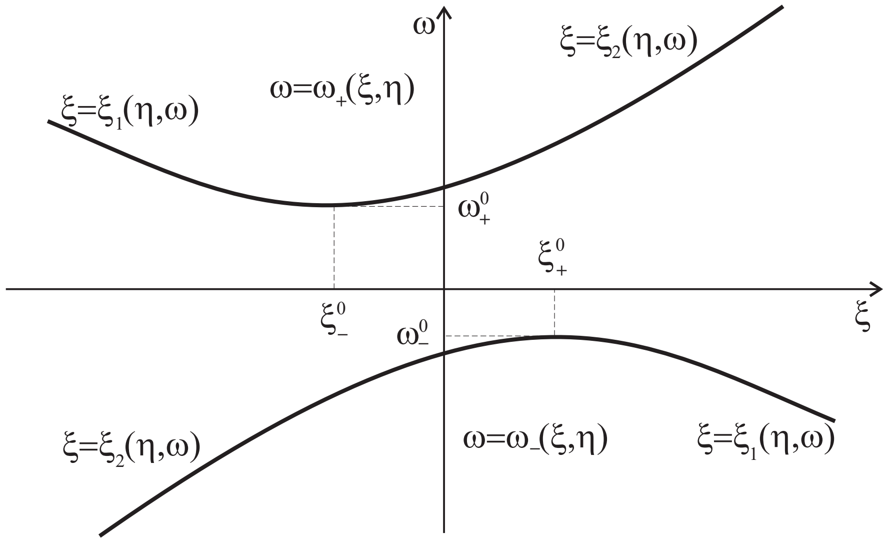

Fig. 2: Roots of P in the post-shock zone

The values

$$\omega^0_\pm = \omega^0_\pm(\eta) = \eta u_y \pm |\eta|\sqrt{c^2 - u_x^2} \text{ or } \xi^0_\pm = \xi^0_\pm(\eta) = \frac{u_x(\omega^0_\mp(\eta) - \eta u_y)}{u_x^2 - c^2},$$

we call critical values. These critical values (or critical angles, up to notations and terms) always appear in studying refraction and/or reflection (see, e.g., [5, 13, 14]).

We define the roots of P for complex ω, $\operatorname{Im}\omega > 0$, as

$$\xi_1(\eta,\omega) = \frac{u_x(\omega - \eta u_y) + c\sqrt{(\omega - \eta u_y)^2 - \eta^2(c^2 - u_x^2)}}{u_x^2 - c^2},$$

$$\xi_2(\eta,\omega) = \frac{u_x(\omega - \eta u_y) - c\sqrt{(\omega - \eta u_y)^2 - \eta^2(c^2 - u_x^2)}}{u_x^2 - c^2},$$

where the branch of the square root is chosen under the following condition: at $\omega = \eta u_y + i\alpha$ and $\alpha > 0$, we have

$$\sqrt{(\omega - \eta u_y)^2 - \eta^2(c^2 - u_x^2)} = \sqrt{-\alpha^2 - \eta^2(c^2 - u_x^2)} = i\sqrt{\alpha^2 + \eta^2(c^2 - u_x^2)}\,.$$

In the fig.2, we depict the roots $\xi_{1,2}$ at fixed η, i.e. the section of post-shock characteristic cone $P^-(\xi,\eta,\omega) = 0$ by plane $\eta = \text{const}$.

The roots $\xi_{1,2}$ are analytic with respect to ω at $\operatorname{Im}\omega > 0$ and $\operatorname{Im}\xi_1(\eta,\omega) < 0$, $\operatorname{Im}\xi_2(\eta,\omega) > 0$. For real ω, we have

$$\xi_1 = \xi_+\,,\quad \xi_2 = \xi_- \quad \text{at} \quad \omega < \omega^0_-\,;$$

$$\xi_1 = \xi_-\,,\quad \xi_2 = \xi_+ \quad \text{at} \quad \omega > \omega^0_+\,;$$

$$\operatorname{Im}\xi_1 < 0\,,\quad \operatorname{Im}\xi_2 > 0 \quad \text{at} \quad \omega \in (\omega^0_-, \omega^0_+)\,;$$

$$\xi_1 = \xi_2 \quad \text{at} \quad \omega = \omega^0_\pm\,;$$

here $\omega^0_\pm = \omega^0_\pm(\eta)$.

Now let us introduce the basis of eigenvectors of the matrix $\hat{B}$ (we write the vectors e as columns and g as rows):

$$e_{01} = e_{01}(\xi,\eta) = \begin{pmatrix} 1 \\ -\dfrac{\xi}{\sqrt{\xi^2+\eta^2}} \\ -\dfrac{\eta}{\sqrt{\xi^2+\eta^2}} \\ 0 \end{pmatrix}, \quad e_{02} = e_{02}(\xi,\eta) = \begin{pmatrix} 1 \\ \dfrac{\xi}{\sqrt{\xi^2+\eta^2}} \\ \dfrac{\eta}{\sqrt{\xi^2+\eta^2}} \\ 0 \end{pmatrix},$$

$$e_{03} = e_{03}(\xi,\eta) = \frac{1}{\sqrt{\xi^2+\eta^2}} \begin{pmatrix} 0 \\ \eta \\ -\xi \\ 0 \end{pmatrix}, \quad e_{04} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

$$g_{01} = g_{01}(\xi,\eta) = \frac{1}{2}\left(1;\ \ -\frac{\xi}{\sqrt{\xi^2+\eta^2}};\ \ -\frac{\eta}{\sqrt{\xi^2+\eta^2}};\ \ -1 \right),$$

$$g_{02} = g_{02}(\xi,\eta) = \frac{1}{2}\left(1;\ \ \frac{\xi}{\sqrt{\xi^2+\eta^2}};\ \ \frac{\eta}{\sqrt{\xi^2+\eta^2}};\ \ -1 \right),$$

$$g_{03} = g_{03}(\xi,\eta) = \frac{1}{\sqrt{\xi^2+\eta^2}} \left(0;\ \ \eta;\ \ -\xi;\ \ 0 \right), \quad g_{04} = \left(0;\ \ 0;\ \ 0;\ \ 1 \right).$$

Actually the eigenvectors e_{0j}, $j = \overline{1,4}$ were used in work by [13], but only in the post-shock zone and in other notations.

Obviously, we have

$$g_{0j} e_{0k} = \delta_{jk} = \begin{cases} 1, & k = j, \\ 0, & k \neq j, \end{cases}$$

so these vectors form biorthogonal bases and

$$e_{01} \otimes g_{01} + e_{02} \otimes g_{02} + e_{03} \otimes g_{03} + e_{04} \otimes g_{04} = E,$$

here the tensor product is simply the product of the matrix $e(4 \times 1)$ and the matrix $g(1 \times 4)$, i.e.

$$\begin{pmatrix} e^1 \\ e^2 \\ e^3 \\ e^4 \end{pmatrix} \otimes \begin{pmatrix} g^1 & g^2 & g^3 & g^4 \end{pmatrix} = \begin{pmatrix} e^1g^1 & e^1g^2 & e^1g^3 & e^1g^4 \\ e^2g^1 & e^2g^2 & e^2g^3 & e^2g^4 \\ e^3g^1 & e^3g^2 & e^3g^3 & e^3g^4 \\ e^4g^1 & e^4g^2 & e^4g^3 & e^4g^4 \end{pmatrix}.$$

We denote $\Omega_{0j} = \Omega_{0j}^{\pm}(\xi,\eta) = e_{0j} \otimes g_{0j}$, $j = \overline{1,4}$.

It is easy to see,

$$\hat{B}e_{01} = (Q - c\sqrt{\xi^2+\eta^2})e_{01}, \quad g_{01}\hat{B} = (Q - c\sqrt{\xi^2+\eta^2})g_{01};$$
$$\hat{B}e_{02} = (Q + c\sqrt{\xi^2+\eta^2})e_{02}, \quad g_{02}\hat{B} = (Q + c\sqrt{\xi^2+\eta^2})g_{02};$$
$$\hat{B}e_{03,04} = Qe_{03,04}, \quad g_{03,04}\hat{B} = Qg_{03,04};$$

i.e. the eigenvalues of the matrix $\hat{B}$ are $\lambda_{1,2} = Q \pm c\sqrt{\xi^2+\eta^2}$ and $\lambda_{3,4} = Q$.

The decomposition of the vector $\hat{G}$ by the basis $\{e_{0j}\}$ has a clear physical meaning. We denote

$$\hat{G}(\xi,\eta,\omega) = \sum_{j=1}^{4} \Omega_{0j}\hat{G} = \sum_{j=1}^{4} \hat{G}_j = \hat{G}_a + \hat{G}_v + \hat{G}_e\,, \tag{6}$$

$$\hat{G}_a = \hat{G}_1 + \hat{G}_2 = \Omega_{01}\hat{G} + \Omega_{02}\hat{G}\,, \quad \hat{G}_v = \hat{G}_3 = \Omega_{03}\hat{G}\,, \quad \hat{G}_e = \hat{G}_4 = \Omega_{04}\hat{G}\,,$$

and, accordingly, obtain decomposition in the physical variables

$$G(x,y,t) = G_a + G_v + G_e \,.$$

Let us decompose the perturbation of the velocity field δU into the sum of solenoidal and potential fields:

$$\delta U = \delta U_s + \delta U_p = \begin{pmatrix} \delta U_{sx} \\ \delta U_{sy} \end{pmatrix} + \begin{pmatrix} \delta U_{px} \\ \delta U_{py} \end{pmatrix}, \quad \operatorname{div} \delta U_s = 0 \,, \quad \operatorname{rot} \delta U_p = 0 \,,$$

then

$$G_e = \begin{pmatrix} \delta\rho \\ 0 \\ 0 \\ r\delta s \end{pmatrix}, \; \delta\rho = r\delta s; \; G_v = \begin{pmatrix} 0 \\ \rho^0 \delta U_{sx}/c \\ \rho^0 \delta U_{sy}/c \\ 0 \end{pmatrix}; \; G_a = \begin{pmatrix} \delta\rho \\ \rho^0 \delta U_{px}/c \\ \rho^0 \delta U_{py}/c \\ 0 \end{pmatrix}, \; \delta\rho = \frac{\delta p}{c^2},$$

i.e. the vector G_e describes the perturbation of entropy and the corresponding perturbation of density at constant pressure $\delta p = 0$, it is called the entropy wave; the vector G_v describes the perturbation of the solenoidal component of the velocity field, it is called the vorticity wave; G_a describes the associated quantities: the perturbation of density at constant entropy $\delta s = 0$ and the potential component of the velocity field, it is called the acoustic or sound wave. This terminology was first proposed in [2, 3] and then became commonly used, see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

For the boundary values $\hat{G}_\Gamma$, we use the following bases:

$$e_1(\eta,\omega) = \begin{pmatrix} 1 \\ -c\xi_1/Q_1 \\ -c\eta/Q_1 \\ 0 \end{pmatrix}, \quad e_2(\eta,\omega) = \begin{pmatrix} 1 \\ -c\xi_2/Q_2 \\ -c\eta/Q_2 \\ 0 \end{pmatrix},$$

$$e_3(\eta,\omega) = e_{03}(\xi_3(\eta,\omega),\eta) = \frac{1}{\sqrt{\xi_3^2+\eta^2}} \begin{pmatrix} 0 \\ \eta \\ -\xi_3 \\ 0 \end{pmatrix}, \quad e_4 = e_{04} \,;$$

$$g_1(\eta,\omega) = \frac{1}{2} \left(\; 1; \quad -c\xi_1/Q_1 \,; \quad -c\eta/Q_1 \,; \quad -1 \; \right),$$

$$g_2(\eta,\omega) = \frac{1}{2} \left(\; 1; \quad -c\xi_2/Q_2 \,; \quad -c\eta/Q_2 \,; \quad -1 \; \right),$$

$$g_3(\eta,\omega) = g_{03}(\xi_3(\eta,\omega),\eta) = \frac{1}{\sqrt{\xi_3^2+\eta^2}} \left(\; 0; \quad \eta; \quad -\xi_3 \,; \quad 0 \; \right), \quad g_4 = g_{04};$$

where

$$Q_1 = Q_1(\eta,\omega) = Q(\xi_1(\eta,\omega),\eta,\omega) = \xi_1 u_x + \eta u_y - \omega,$$

$$Q_2 = Q_2(\eta,\omega) = Q(\xi_2(\eta,\omega),\eta,\omega) = \xi_2 u_x + \eta u_y - \omega.$$

These vectors also form biorthogonal bases with the matrix A_0, i.e. $g_j A_0 e_k = \delta_{jk} s_j$, $j,k = \overline{1,4}$, where

(7)

$$s_j = \frac{1}{(\xi_j)'_\omega}\,, \quad j = \overline{1,4}\,;$$

$$s_1 = \frac{(u_x^2 - c^2)(Q_1 - Q_2)}{2u_x Q_1}\,, \quad s_2 = \frac{(u_x^2 - c^2)(Q_2 - Q_1)}{2u_x Q_2}\,, \quad s_{3,4} = u_x \,.$$

We denote $\Omega_j = e_j \otimes g_j/s_j$, $j = \overline{1,4}$. Hence,

$$\left(\sum_{j=1}^{4} \Omega_j\right) A_0 = \left(\sum_{j=1}^{4} \frac{e_j \otimes g_j}{s_j}\right) A_0 = E\,,$$

and so each boundary vector may be decomposed:

$$\hat{G}_\Gamma = \sum_{j=1}^{4} \Omega_j A_0 \hat{G}_\Gamma = \sum_{j=1}^{4} \Omega_j \hat{H} = \sum_{j=1}^{4} \hat{G}_{j\Gamma} = \hat{G}_{a\Gamma} + \hat{G}_{v\Gamma} + \hat{G}_{e\Gamma}\,, \tag{8}$$

where $\hat{G}_{a\Gamma} = \hat{G}_{1\Gamma} + \hat{G}_{2\Gamma}$ is a boundary value for an acoustic wave, $\hat{G}_{v\Gamma} = \hat{G}_{3\Gamma}$ is a boundary value for vorticity, and $\hat{G}_{e\Gamma} = \hat{G}_{4\Gamma}$ for an entropy wave.

4. General solution

4.1. **Computation.** Now we are ready to find the solution of the initial-value problem (3), (4). From Eq. (3), using the form of the inverse matrix $\hat{B}^{-1}$ (5), we obtain

$$\hat{G} = \frac{F(\hat{G}_0(\xi,\eta) \mp \hat{H}(\eta,\omega))}{2\pi i PQ}$$

with the solvability conditions (Appendix, Part 3, Subsection 6.2, Statement 3).

Ahead of the shock (superscript plus), we have three zeroes of the denominator in the analyticity region $\operatorname{Im}\xi > 0$, $\operatorname{Im}\omega > 0$: $\xi = \xi_j(\eta,\omega)$, $j = \overline{1,3}$; hence, we have three solvability conditions

$$F(\xi_j(\eta,\omega),\eta,\omega)(\hat{G}_0(\xi_j(\eta,\omega),\eta) - \hat{H}(\eta,\omega)) \equiv 0\,, \quad j = \overline{1,3}\,.$$

However,

$$F(\xi_{1,2}) = 2Q_{1,2}^2 e_{1,2} \otimes g_{1,2}\,, \quad F(\xi_3) = -c^2(\xi_3^2 + \eta^2)(e_3 \otimes g_3 + e_4 \otimes g_4)\,,$$

so the solvability conditions have the form $g_j \hat{G}_0(\xi_j) = g_j \hat{H}$, $j = \overline{1,4}$ (we remind $\xi_4 = \xi_3$), and, according to Eq. (8), the boundary vector $\hat{H} = A_0 \hat{G}_\Gamma$ is unambiguously determined by the initial data:

$$\hat{G}_\Gamma = \sum_{j=1}^{4} e_j \frac{g_j \hat{H}}{s_j} = \sum_{j=1}^{4} e_j \frac{g_j \hat{G}_0(\xi_j)}{s_j} = \sum_{j=1}^{4} \Omega_j \hat{G}_0(\xi_j)\,.$$

Consequently, the sought vector $\hat{G}^+$ is unambiguously determined by the initial data $\hat{G}_0^+$.

Behind the shock (superscript minus), the analyticity region is $\operatorname{Im}\xi < 0$, $\operatorname{Im}\omega > 0$; here the polynomial $Q(\xi,\eta,\omega)$ has no roots, and the polynomial $P(\xi,\eta,\omega)$ has one root $\xi = \xi_1(\eta,\omega)$. So the only solvability condition is

$$F(\xi_1,\eta,\omega)(\hat{G}_0(\xi_1,\eta) + \hat{H}(\eta,\omega)) \equiv 0$$

or $g_1 \hat{G}_0(\xi_1) = -g_1 \hat{H}$.

Now let us pass to the boundary condition (4) $A^+\hat{H}^+ = A^-\hat{H}^- + i\hat{F}_0\hat{f}$. We find

$$\hat{H}^- = (A^-)^{-1} A^+ \hat{H}^+ - i(A^-)^{-1}\hat{F}_0\hat{f}\,.$$

Whence, due to the solvability condition behind the shock $g_1^- \hat{G}_0^-(\xi_1^-) = -g_1^- \hat{H}^-$, we obtain

$$-g_1^- \hat{G}_0^-(\xi_1^-) = g_1^- \hat{H}^- = g_1^-(A^-)^{-1} A^+ \hat{H}^+ - i g_1^-(A^-)^{-1}\hat{F}_0\hat{f}$$

or

$$Y_1(\eta,\omega)\hat{f}(\eta,\omega) = Z(\eta,\omega)\,,$$

$$Y_1 = g_1^-(A^-)^{-1}\hat{F}_0\,,\; Z = -ig_1^-(A^-)^{-1}A^+\hat{H}^+ - ig_1^-\hat{G}_0^-(\xi_1^-)\,.$$

This equation for the shock disturbance always appears (in different notations) in all works devoted to the initial-value problem (see e.g. [15, 16, 17, 18, 19, 20, 21, 22]).

Substituting the found value of $\hat{f}$ into the expression for $\hat{H}^-$, we find $\hat{H}^-$ and finally can determine $\hat{G}^-$. Thus, the formulas of the solution of the initial-value problem are:
the pre-shock wave is

$$\hat{G}^+(\xi,\eta,\omega) = \frac{F^+(\xi,\eta,\omega)(\hat{G}_0^+(\xi,\eta) - \hat{H}^+(\eta,\omega))}{2\pi i P^+ Q^+}\,, \tag{9}$$

where

$$\hat{H}^+(\eta,\omega) = A_0^+\sum_{j=1}^{4}\Omega_j^+(\eta,\omega)\hat{G}_0^+(\xi_j^+(\eta,\omega),\eta)\,; \tag{10}$$

the shock disturbance $\hat{f}$ is a solution of the linear equation

$$Y_1(\eta,\omega)\hat{f}(\eta,\omega) = Z(\eta,\omega)\,, \tag{11}$$

where

$$Y_1(\eta,\omega) = g_1^-(\eta,\omega)(A^-)^{-1}\hat{F}_0(\eta,\omega)\,, \tag{12}$$

$$Z(\eta,\omega) = -ig_1^-(\eta,\omega)(A^-)^{-1}A^+\hat{H}^+(\eta,\omega) - ig_1^-(\eta,\omega)\hat{G}_0^-(\xi_1^-(\eta,\omega),\eta))\,; \tag{13}$$

and the post-shock wave is

$$\hat{G}^-(\xi,\eta,\omega) = \frac{F^-(\xi,\eta,\omega)(\hat{G}_0^-(\xi,\eta) + \hat{H}^-(\eta,\omega))}{2\pi i P^- Q^-}\,, \tag{14}$$

where

$$\hat{H}^- = (A^-)^{-1}A^+\hat{H}^+ - i(A^-)^{-1}\hat{F}_0\hat{f}\,. \tag{15}$$

We note that the initial-value problem has an unique solution for any initial data. That means the initial data unambiguously parameterize the general solution.

Certainly, the obtained solution is not new: the initial-value problem has only solution and all solutions obtained before (e.g. [15, 16, 17, 18, 19, 20, 21, 22]), as well as the present solution (9), (11), (14), are the same. But, we obtain a new representation of the solution, strictly algebraic and more or less simple, that provides additional opportunities to analyze it.

Thus, the formulas (9), (11), (14) are usable as for the regular in spectral variables terms (i.e. tending to zero in physical variables) as for the singular terms (i.e. not tending to zero in physical variables, e.g. plane waves). Moreover, these algebraic formulas allow to separate the singular and regular terms of solution and consider them separately or in any combination, both at the analytic and numerical calculations.

4.2. **The singular and regular terms detachment.** The function is regular with respect to the some spectral variable, if it tends to zero when the corresponding physical variable tends to infinity. We are interested in the regulation with respect to the variables ξ and ω, i.e., when the quantity tends to zero at $x \to \infty$ or $t \to \infty$. In particular, the wave tenging to zero at $t \to \infty$, we call damped wave.

The example of singular wave gives the plane wave: $G^{\pm}(x, y, t) = H_0 e^{i(\xi_0 x + \eta_0 y - \omega_0 t)}$, $t > 0$, $\pm x < 0$, where $(\xi_0, \eta_0, \omega_0) = \text{const}$, H_0 is a constant vector. Its Fourier transform has a form (see Appendix, Part 3, Subsection 6.1, Eq. (6.6))

$$\hat{G}^{\pm}(\xi, \eta, \omega) = H_0 \delta(\eta - \eta_0) I_1(\omega - \omega_0) I_1(\pm(\xi - \xi_0)), \quad I_1(\alpha) = \frac{\delta(\alpha)}{2} - \frac{1}{2\pi i \alpha}.$$

Now we describe the general scheme of the singular and regular terms detachment. The formulas of solution (9) – (15) is arranged as follows: eigther we multiply the vector of two variables (initial data $\hat{G}_0(\xi, \eta)$ or boundary value $\hat{G}_\Gamma(\eta, \omega)$) by some matrix $A(\xi, \eta, \omega)$, or substitute the roots $\xi = \xi_j(\eta, \omega)$ into the vector $\hat{G}_0(\xi, \eta)$.

Detachment at the multiplication: let, e.g., $\hat{G}_0$ is singular, $\hat{G}_0 = H_0 \delta(\eta - \eta_0) I_1(\xi - \xi_0)$. Then $A \cdot \hat{G}_0 = A(\xi_0, \eta_0, \omega) H_0 \delta(\eta - \eta_0) I_1(\xi - \xi_0) +$

$$+(A(\xi, \eta, \omega) - A(\xi_0, \eta_0, \omega)) H_0 \delta(\eta - \eta_0) I_1(\xi - \xi_0) = A(\xi_0, \eta_0, \omega) H_0 \delta(\eta - \eta_0) I_1(\xi - \xi_0) -$$

$$- \frac{A(\xi, \eta_0, \omega) - A(\xi_0, \eta_0, \omega)}{2\pi i (\xi - \xi_0)} H_0 \delta(\eta - \eta_0) = \hat{G}_1 + \hat{\varphi},$$

where the term $\hat{G}_1$ is singular and the term $\hat{\varphi}$ is regular with respect to ξ. The same we obtain at the multiplication of matrix A by singular boundary value $\hat{G}_\Gamma = H_0 \delta(\eta - \eta_0) I_1(\omega - \omega_0)$.

Detachment at the change of variable: let $\hat{G}_0 = H_0 \delta(\eta - \eta_0) I_1(\xi - \xi_0)$ again. Due to the change of variable formula (Appendix, Part 3, Subsection 6.3, Eq. (6.12)), we have

$$\hat{G}_0(\xi(\eta, \omega), \eta) = H_0 \delta(\eta - \eta_0) \frac{I_1(\omega - \omega_0)}{\xi'_\omega(\eta_0, \omega_0)} + \hat{\varphi}, \quad \xi(\eta_0, \omega_0) = \xi_0,$$

where $\hat{\varphi}$ is regular with respect to ω.

General conclusion reads: if the initial data $\hat{G}_0$ are already separated into the sum of singular and regular terms, then we may sequentially separate these terms in all formulas (9) – (15), and obtain singular and regular parts for shock disturbance, and pre-shock, and post-shock waves.

The source of singularity in algebraic formulas (9) – (15) is obviously the initial data $\hat{G}_0^{\pm}$. But there is an additional source of the singular terms, it is the equation for shock disturbance (11): the additional singular terms appear, when the coefficient $Y_1(\eta, \omega)$ has roots in analyticity region $\operatorname{Im} \omega > 0$ or in its boundary $\operatorname{Im} \omega = 0$ (see Appendix, Part 3, Subsection 6.2). The additional singular terms (plane waves) appearance, when Y_1 has real roots (neutral stability case, see below), is called spontaneous emission of waves by shock. Thus, the linear theory, in particular the singular terms separation, allows to compute spontaneously emitted waves and accordingly describe spontaneous emission. We consider this issue in third part.

So we have absolutely clear and rather simple method of singular and regular terms detachment in solution (9) – (15). This method is usable as for analytical investigations as for numerical calculations, in last case the singular terms are right a kind of plane waves with known (calculated) wave amplitudes and wavenumbers, and for regular terms we may use numerical inverse Fourier transform. In particular, this method allows to calculate wave amplitudes and wavenumbers of plane waves, generated either by the initial data, or by the roots of Y_1 in neutral stability case. We use it in second and third parts, when consider refraction and reflection of plane waves.

4.3. **Stability, neutral stability, instability.** Localization of the function Y_1 zeros is well known and has been repeatedly checked and rechecked, see e.g. [2, 7, 9, 10, 15, 16, 17, 18, 19, 20, 21, 22]; three cases are possible:

(1) The function $Y_1(\eta,\omega)$ has no roots at $\operatorname{Im}\omega \geqslant 0$, and so equation (11) doesn't provide additional singular terms. It is the case of stability (by D'yakov).
(2) The function $Y_1(\eta,\omega)$ has (at any real η) two real roots $\omega = \eta u_y \pm \alpha_1|\eta|\sqrt{(c^-)^2-(u_x^-)^2}$, $\alpha_1 > 1$. It is the case of the D'yakov-Kontorovich instability, now it is called the neutral stability. In this case, equation (11) gives two additional singular terms in form of plane waves. We consider this case in third part.
(3) The function $Y_1(\eta,\omega)$ has the only root in the upper half-plane $\omega_2 = \eta u_y + i\alpha_2|\eta|$, $\alpha_2 > 0$. It is the case of instability (by D'yakov).

The conditions dividing these cases, in terms of the shock adiabat, are well known too, they were first derived by D'yakov, specified by Kontorovich and then repeatedly checked and rechecked (see e.g. [2, 7, 9, 10, 15, 16, 17, 18, 19, 20, 21, 22]). We don't write them because they are not too important in our investigation.

We don't consider the instability case in the present work. We only mark that in this case the shock disturbance f has the form

$$f(y,t) = \int \frac{Z(\eta,\omega_2)}{(Y_1)'_\omega(\eta,\omega_2)} e^{i\eta(y-u_y t)} e^{\alpha_2|\eta|t}\, d\eta + \Phi(y,t)\,,$$

where $\Phi(y,t)$ is damped, i.e. in this case the linear theory predicts a temporally exponentiating shock disturbance (and post-shock waves as well). This conclusion is well known too.

5. Incidence, refraction and reflection

5.1. **Acoustic and entropy-vorticity waves.** Let us turn to the formulas (9), (14):

$$\hat G = \frac{F(\hat G_0 \mp \hat H)}{2\pi i PQ} = \frac{F\hat G_0}{2\pi i PQ} \mp \frac{F\hat H}{2\pi i PQ}\,.$$

We have two representations for the matrix F:

$$F = P(\Omega_{03}+\Omega_{04}) + Q((Q + c\sqrt{\xi^2+\eta^2})\Omega_{01} + (Q - c\sqrt{\xi^2+\eta^2})\Omega_{02})\,,$$

$$F = P u_x(\Omega_3+\Omega_4) + \frac{(u_x^2-c^2)Q}{u_x}((Q-Q_2)\Omega_1 + (Q-Q_1)\Omega_2)\,.$$

We substitute the first representation into the first summand of the formula for $\hat G$ and the second representation into the second summand. So we obtain the formulas

$$\hat G = \hat G_v + \hat G_e + \hat G_a\,,\quad \hat G_v = \hat G_3 = \frac{\Omega_{03}\hat G_0 \mp u_x\Omega_3\hat H}{2\pi i Q}\,,\quad \hat G_e = \hat G_4 = \frac{\Omega_{04}\hat G_0 \mp u_x\Omega_4\hat H}{2\pi i Q}\,,$$

$$\hat G_a = \hat G_1 + \hat G_2\,,$$

$$\hat G_1 = \frac{(Q + c\sqrt{\xi^2+\eta^2})\Omega_{01}\hat G_0 \mp (Q-Q_2)\Omega_1\hat H(u_x^2-c^2)/u_x}{2\pi i P}\,,$$

$$\hat{G}_2 = \frac{(Q - c\sqrt{\xi^2+\eta^2})\Omega_{02}\hat{G}_0 \mp (Q - Q_1)\Omega_2\hat{H}(u_x^2 - c^2)/u_x}{2\pi i P}\,.$$

We decompose the initial data according to Eq. (6)

$$\hat{G}_0 = \sum_{j=1}^{4}\Omega_{0j}\hat{G}_0 = \sum_{j=1}^{4}\hat{G}_{j0} = \hat{G}_{a0} + \hat{G}_{v0} + \hat{G}_{e0}\,,$$

$\hat{G}_{a0} = \hat{G}_{10} + \hat{G}_{20}$, $\hat{G}_{v0} = \hat{G}_{30}$, $\hat{G}_{e0} = \hat{G}_{40}$, and the boundary values according to Eq. (8):

$$\hat{G}_\Gamma = \sum_{j=1}^{4}\Omega_j A_0\hat{G}_\Gamma = \sum_{j=1}^{4}\hat{G}_{j\Gamma} = \hat{G}_{a\Gamma} + \hat{G}_{v\Gamma} + \hat{G}_{e\Gamma}\,,$$

$\hat{G}_{a\Gamma} = \hat{G}_{1\Gamma} + \hat{G}_{2\Gamma}$, $\hat{G}_{v\Gamma} = \hat{G}_{3\Gamma}$, $\hat{G}_{e\Gamma} = \hat{G}_{4\Gamma}$, and come to the representation

$$\hat{G}_v = \frac{\hat{G}_{v0} \mp u_x\hat{G}_{v\Gamma}}{2\pi i Q}\,,\quad \hat{G}_e = \frac{\hat{G}_{e0} \mp u_x\hat{G}_{e\Gamma}}{2\pi i Q}\,,$$

$$\hat{G}_1 = \frac{(Q + c\sqrt{\xi^2+\eta^2})\hat{G}_{10} \mp (Q - Q_2)\hat{G}_{1\Gamma}(u_x^2 - c^2)/u_x}{2\pi i P}\,, \tag{16}$$

$$\hat{G}_2 = \frac{(Q - c\sqrt{\xi^2+\eta^2})\hat{G}_{20} \mp (Q - Q_1)\hat{G}_{2\Gamma}(u_x^2 - c^2)/u_x}{2\pi i P}\,.$$

The summands $\hat{G}_v = \hat{G}_3$, $\hat{G}_e = \hat{G}_4$ are obviously the vorticity and entropy waves respectively. They are the solutions of the equation $iQ\hat{G}_j = (\hat{G}_{j0} \mp u_x\hat{G}_{j\Gamma})/(2\pi)$, $j = 3, 4$, or, in the physical variables (x, y, t), they satisfy the transfer equation $dG_j/dt = 0$ and have the initial values G_{j0} and the boundary values $G_{j\Gamma}$, $j = 3, 4$. Usually they together are called the entropy-vorticity wave.

The acoustic wave is the sum $G_a = G_1 + G_2$. Their summands G_k, $k = 1, 2$ are the solutions of the wave equation $dG_k/dt - c^2\Delta G_k = 0$, and they are generated by the initial data G_{k0} and boundary values $G_{k\Gamma}$ respectively, $k = 1, 2$. In the pre-shock zone, we call G_1^+ slow acoustic and G_2^+ fast acoustic waves in accordance to the usual terminology (e.g. [5]). In the post-shock zone, the situation is a bit more complex (e.g. the wave $\hat{G}_2^-$ is slow at $\omega < \omega_-^0(\eta)$ and fast at $\omega > \omega_+^0(\eta)$), we call the post-shock acoustic waves simply G_1^- first and G_2^- second.

5.2. **Initial, refracted and reflected post-shock waves.** Any pre-shock wave G^+ is unambiguously determined by the pre-shock initial data G_0^+ and its boundary value G_Γ^+ is also generated by initial data G_0^+. So there are no any pre-shock waves, generated by the post-shock initial data G_0^-, or, in other words, post-shock wave G^- doesn't go across the shock.

Let's turn to the post-shock zone (superscript minus on default). We decompose the boundary value G_Γ by the basis e_j (Eq. (8))

$$\hat{G}_\Gamma = \sum_{j=1}^{4}\hat{G}_{j\Gamma}\,;\quad \hat{G}_{j\Gamma} = \beta_j e_j\,,\quad \beta_j = \frac{g_j\hat{H}}{s_j}\,,\quad j = \overline{1,4}\,.$$

But since $g_1\hat{H} = -g_1\hat{G}_0(\xi_1)$ (solvability condition behind the shock), the formula (14) leads to representation

$$\hat{G} = \frac{F(\hat{G}_0 - A_0 e_1 g_1\hat{G}_0(\xi_1)/s_1)}{2\pi i PQ} + \frac{F\hat{H}_1}{2\pi i PQ} = \hat{G}_{00} + \hat{G}_{0\Gamma}\,,\quad \hat{H}_1 = A_0\sum_{j=2}^{4}\hat{G}_{j\Gamma}\,.$$

The first summand $\hat{G}_{00}$ is determined by the post-shock initial data G_0 immediately, we call it initial post-shock wave. This wave in general consists of all four kinds of waves – two acoustic, entropy and vorticity.

The second summand

$$\hat{G}_{0\Gamma} = \frac{F\hat{H}_1}{2\pi i PQ} = \sum_{j=2}^{4} \frac{FA_0\hat{G}_{j\Gamma}}{2\pi i PQ} = \sum_{j=2}^{4} \hat{G}_{j0\Gamma}$$

(boundary wave) has zero initial data and is a sum of only three kinds of waves: second acoustic $\hat{G}_{20\Gamma}$, vorticity $\hat{G}_{30\Gamma}$ and entropy $\hat{G}_{40\Gamma}$. Due to Eq. (16), these waves have a form

$$\hat{G}_{20\Gamma} = \frac{(Q-Q_1)\hat{G}_{2\Gamma}(u_x^2-c^2)}{2\pi i P u_x} = \frac{u_x(\xi-\xi_1)\hat{G}_{2\Gamma}(u_x^2-c^2)}{2\pi i(u_x^2-c^2)(\xi-\xi_1)(\xi-\xi_2)u_x} = \frac{\hat{G}_{2\Gamma}}{2\pi i(\xi-\xi_2)}\,,$$

$$\hat{G}_{j0\Gamma} = \frac{u_x\hat{G}_{j\Gamma}}{2\pi i Q} = \frac{\hat{G}_{j\Gamma}}{2\pi i(\xi-\xi_j)}\,, \quad j=\overline{3,4}\,,$$

or after inverse Fourier transform

$$G_{j0\Gamma}(x,y,t) = \iint \hat{G}_{j\Gamma}(\eta,\omega)e^{i(\xi_j(\eta,\omega)x+\eta y-\omega t)}\,d\eta d\omega\,, \quad j=\overline{2,4}\,. \tag{17}$$

In turn the post-shock boundary wave $G_{0\Gamma}$ one may decompose into the sum of waves generated by only pre-shock or only post-shock initial data. Namely, we present the shock disturbance $\hat{f}$ (Eqs. (11)–(13)) in a form $\hat{f} = \hat{f}^+ + \hat{f}^-$, where

$$Y_1\hat{f}^+ = Z^+ = -ig_1^-(A^-)^{-1}A^+\hat{H}^+\,,$$

i.e. $\hat{f}^+$ depends only on the pre-shock boundary value $\hat{H}^+ = A_0\hat{G}_\Gamma^+$ or finally on the pre-shock initial data G_0^+, $\hat{f}^+ = \hat{f}^+(G_0^+)$;

$$Y_1\hat{f}^- = Z^- = -ig_1^-\hat{G}_0^-(\xi_1^-)\,,$$

i.e. $\hat{f}^-$ depends only on the post-shock initial data G_0^-, $\hat{f}^- = \hat{f}^-(G_0^-)$. Then Eq. (15) gives

$$\hat{H}_1 = \hat{H}^- + A_0 e_1\frac{g_1\hat{G}_0^-(\xi_1)}{s_1} = \left[(A^-)^{-1}A^+\hat{H}^+ - i(A^-)^{-1}\hat{F}_0\hat{f}^+\right] +$$

$$+\left[-i(A^-)^{-1}\hat{F}_0\hat{f}^- + A_0e_1\frac{g_1\hat{G}_0^-(\xi_1)}{s_1}\right] = \hat{H}_+ + \hat{H}_-\,.$$

The boundary value

$$\hat{H}_+ = (A^-)^{-1}A^+\hat{H}^+ - i(A^-)^{-1}\hat{F}_0\hat{f}^+ = \hat{H}_+(G_0^+)$$

generates the post-shock wave $\hat{G}_+ = F\hat{H}_+/(2\pi i PQ) = \hat{G}_+(G_0^+)$ which is called refracted or transmitted wave. In turn the boundary value

$$\hat{H}_- = -i(A^-)^{-1}\hat{F}_0\hat{f}^- + A_0e_1\frac{g_1\hat{G}_0^-(\xi_1)}{s_1} = \hat{H}_-(G_0^-)$$

generates the post-shock wave $\hat{G}_- = F\hat{H}_-/(2\pi i PQ) = \hat{G}_-(G_0^-)$ which is called reflected wave.

The general conclusion reads: any post-shock wave is a sum of initial G_{00}^-, reflected G_-^- and transmitted (refracted) G_+^- waves. The initial post-shock wave

$$(18)\quad \hat{G}_{00}^-(\xi,\eta,\omega) = \frac{F^-(\hat{G}_0^-(\xi,\eta) - A_0^- e_1^-(\eta,\omega) g_1^-(\eta,\omega)\hat{G}_0^-(\xi_1^-(\eta,\omega),\eta)/s_1^-(\eta,\omega))}{2\pi i P^- Q^-}$$

is generated by the initial post-shock data G_0^- immediately. This wave interacts with shock and so generates the shock disturbance f^-, induces the post-shock boundary value $\hat{H}_-^-$, which in the end generates the post-shock reflected wave $\hat{G}_-^-$:

$$(19)\quad \begin{gathered} \hat{G}_-^-(\xi,\eta,\omega) = \frac{F^-(\xi,\eta,\omega)\hat{H}_-^-(\eta,\omega)}{2\pi i P^-(\xi,\eta,\omega) Q^-(\xi,\eta,\omega)}\,, \\ \hat{H}_-^- = -i(A^-)^{-1}\hat{F}_0 \hat{f}^- + A_0^- e_1^- \frac{g_1^- \hat{G}_0^-(\xi_1^-(\eta,\omega),\eta)}{s_1^-}\,, \\ Y_1(\eta,\omega)\hat{f}^-(\eta,\omega) = -i g_1^-(\eta,\omega)\hat{G}_0^-(\xi_1^-(\eta,\omega),\eta)\,. \end{gathered}$$

At last the pre-shock wave G^+ interacts with shock and so generates the shock disturbance f^+, induces the post-shock boundary value $\hat{H}_+^-$, which in the end generates the post-shock transmitted (refracted) wave $\hat{G}_+^-$:

$$(20)\quad \begin{gathered} \hat{G}_+^-(\xi,\eta,\omega) = \frac{F^-(\xi,\eta,\omega)\hat{H}_+^-(\eta,\omega)}{2\pi i P^-(\xi,\eta,\omega) Q^-(\xi,\eta,\omega)}\,, \\ \hat{H}_+^- = (A^-)^{-1}A^+\hat{H}^+ - i(A^-)^{-1}\hat{F}_0\hat{f}^+\,, \\ Y_1(\eta,\omega)\hat{f}^+(\eta,\omega) = -i g_1^-(\eta,\omega)(A^-)^{-1}A^+\hat{H}^+(\eta,\omega)\,. \end{gathered}$$

The pre-shock and initial post-shock waves consist in general of all four kinds of waves: both acoustic, entropy and vorticity. The transmitted and reflected post-shock waves consists only of three summands: second acoustic, entropy and vorticity waves.

5.3. **Pre-shock incidence and refraction.** The pre-shock wave is a sum of four summands (subsection 5.1) $G^+ = G_1^+ + G_2^+ + G_3^+ + G_4^+$ where G_1^+ and G_2^+ are the slow and fast acoustic waves, G_3^+ is vorticity and G_4^+ is an entropy wave. The same concerns to their initial data (decomposition (6))
$G_0^+ = G_{10}^+ + G_{20}^+ + G_{30}^+ + G_{40}^+$ and boundary values (decomposition (8))
$G_\Gamma^+ = G_{1\Gamma}^+ + G_{2\Gamma}^+ + G_{3\Gamma}^+ + G_{4\Gamma}^+$. The correspondence between according initial data, boundary values and waves themselves, follows from Eqs. (9), (10):

$$\hat{G}_{j\Gamma}^+(\eta,\omega) = \frac{\hat{G}_{j0}^+(\xi_j^+(\eta,\omega),\eta)}{s_j^+(\eta,\omega)} \text{ or } \hat{G}_{j0}^+(\xi,\eta) = s_j^+(\eta,\omega_j^+(\xi,\eta))\hat{G}_{j\Gamma}^+(\eta,\omega_j^+(\xi,\eta))\,,$$

$$\hat{G}_j^+(\xi,\eta,\omega) = \frac{F^+(\xi,\eta,\omega)(\hat{G}_{j0}^+(\xi,\eta) - A_0^+\hat{G}_{j\Gamma}^+(\eta,\omega))}{2\pi i P^+ Q^+}\,, \quad j = \overline{1,4}\,.$$

Further for simplicity, we consider the boundary pre-shock value
$\hat{G}_\Gamma^+ = \hat{G}_{1\Gamma}^+ + \hat{G}_{2\Gamma}^+ + \hat{G}_{3\Gamma}^+ + \hat{G}_{4\Gamma}^+$ as given.

The pre-shock wave is incident upon the shock, i.e. it generates shock disturbance f^+ and transmitted wave G_+^-. So we have a correspondence between four incident pre-shock waves and four results of interaction with shock: the shock disturbance and three transmitted waves. This correspondence is linear algebraic in spectral variables, and in the terms of boundary pre-shock and post-shock values $\hat{G}_\Gamma^\pm$ is given by following from (20) formulas:

$$
\begin{aligned}
&Y_1\hat{f}^+ = -ig_1^-(\eta,\omega)(A^-)^{-1}A^+A_0^+\hat{G}_\Gamma^+(\eta,\omega)\,,\\
&\hat{G}_\Gamma^- = (A_0^-)^{-1}\left[(A^-)^{-1}A^+A_0^+\hat{G}_\Gamma^+ - i(A^-)^{-1}\hat{F}_0\hat{f}^+\right]\,.
\end{aligned} \tag{21}
$$

The correspondence between pre-shock waves and results of interaction with shock is one-to one map. The inverse map has a form

$$
\hat{G}_\Gamma^+ = (A_0^+)^{-1}(A^+)^{-1}\left[i\hat{F}_0\hat{f}^+ + A^-A_0^-\hat{G}_\Gamma^-\right]\,. \tag{22}
$$

This inverse map allows us to decompose any incident pre-shock wave into the sum of waves, having physically different interaction with shock. Namely, we decompose transmitted wave $\hat{G}_\Gamma^-$ into the sum of transmitted acoustic $\hat{G}_{2\Gamma}^-$, vorticity $\hat{G}_{3\Gamma}^-$, and entropy $\hat{G}_{4\Gamma}^-$ waves, and inverse map (22) leads to the representation of incident wave

$$
\hat{G}_\Gamma^+ = \sum_{j=1}^{4}\hat{G}_{ij}^+\,,\quad \hat{G}_{i1}^+ = i(A_0^+)^{-1}(A^+)^{-1}\hat{F}_0\hat{f}^+\,;
$$

$$
\hat{G}_{ij}^+ = (A_0^+)^{-1}(A^+)^{-1}A^-A_0^-\hat{G}_{j\Gamma}^-,\quad j=\overline{2,4}.
$$

Here incident wave G_{i1}^+ generates shock disturbance f^+, but doesn't generate any post-shock waves $G_\Gamma^- \equiv 0$. All other incident summands G_{i2}^+, G_{i3}^+, G_{i4}^+ don't generate any shock disturbances $f^+ \equiv 0$, but each of them generates only one kind of transmitted wave: G_2^+ generates only transmitted acoustic wave, G_3^+ – only vorticity, and G_4^+ – only entropy transmitted wave.

So we find the peculiarity of refraction, which haven't been noted before: the decomposition of incident pre-shock wave into the sum of waves with physically different interaction with shock. Namely, one summand interacts with shock, i.e. generates shock disturbance, but doesn't generate any transmitted waves, or somehow "sticks" to the shock and spends all its energy on the shock deformation. Three other summands don't interact with shock, i.e. don't generate shock disturbance or don't spend energy on shock deformation, but generate the transmitted waves, and each of them generates only one kind of transmitted wave: sound, vorticity or entropy.

5.4. **Incidence of post-shock waves and reflection.** We consider post-shock waves only (superscript minus on default), i.e. the pre-shock initial data $G_0^+ \equiv 0$ and so there are no any pre-shock waves and no transmitted post-shock waves. So (subsection 5.2) any post-shock wave consists of two summands: the initial wave G_{00} (18), and the reflected wave G_- (19), and interaction between the initial data G_0 and shock is going on through the term (agent of incidence)
$\Phi(\eta,\omega) = g_1(\eta,\omega)\hat{G}_0(\xi_1(\eta,\omega),\eta)$:

$$
\begin{aligned}
&\hat{f}(\eta,\omega) = \Phi(\eta,\omega)\hat{f}_0\,,\quad Y_1(\eta,\omega)\hat{f}_0(\eta,\omega) = -i\,;\\
&\hat{H}_-(\eta,\omega) = \Phi(\eta,\omega)\hat{H}_0\,,\quad \hat{H}_0(\eta,\omega) = -iA^{-1}\hat{F}_0\hat{f}_0 + \frac{1}{s_1}A_0e_1\,;\\
&\hat{G}_-(\xi,\eta,\omega) = \Phi(\eta,\omega)\hat{G}_{0-}\,,\quad \hat{G}_{0-}(\xi,\eta,\omega) = \frac{F(\xi,\eta,\omega)\hat{H}_0(\eta,\omega)}{2\pi i P(\xi,\eta,\omega)Q(\xi,\eta,\omega)}\,.
\end{aligned} \tag{23}
$$

Decomposition of the reflected wave into the sum of reflected acoustic G_2, reflected vorticity G_3 and reflected entropy G_4 waves, and corresponding decomposition of its boundary value $\hat{H}_- = \hat{H}_2 + \hat{H}_3 + \hat{H}_4$ has a form

$$\begin{aligned} &\hat{H}_j(\eta,\omega) = \Phi(\eta,\omega)\hat{H}_{0j}\,, \quad \hat{H}_{0j}(\eta,\omega) = -iA_0 e_j \frac{Y_j \hat{f}_0}{s_j}\,, \quad Y_j = g_j A^{-1}\hat{F}_0\,, \\ &\hat{G}_j(\xi,\eta,\omega) = \Phi(\eta,\omega)\hat{G}_{0j}\,, \quad \hat{G}_{0j} = \frac{F\hat{H}_{0j}}{2\pi i PQ}\,, \quad j = \overline{2,4}\,. \end{aligned} \tag{24}$$

Assume $\Phi \equiv 0$, whence due to the form of g_1 and $\hat{G}_0$:

$$g_1 = \frac{1}{2}\left(\ 1; \quad -c\xi_1/Q_1; \quad -c\eta/Q_1; \quad -1\ \right), \quad \hat{G}_0(\xi,\eta) = \begin{pmatrix} \hat{\delta\rho_0} \\ \rho^0\delta\hat{U}_{x0}/c \\ \rho^0\delta\hat{U}_{y0}/c \\ r\hat{\delta s} \end{pmatrix},$$

we have

$$(Q(\hat{\delta\rho_0} - r\hat{\delta s_0}) - \rho^0(\xi\delta\hat{U}_{x0} + \eta\delta\hat{U}_{y0}))\Big|_{\xi=\xi_1} \equiv 0.$$

But $Q(\xi_1,\eta,\omega) = \pm c\sqrt{\xi_1^2+\eta^2}$, what due to the analyticity conditions and form of ξ_1 (section 3) leads to the condition

$$c\sqrt{\xi^2+\eta^2}(\hat{\delta\rho_0} - r\hat{\delta s_0}) - \rho^0(\xi\delta\hat{U}_{x0} + \eta\delta\hat{U}_{y0}) \equiv 0, \quad \operatorname{Im}\xi \leqslant 0,$$

where the function $\sqrt{\xi^2+\eta^2}$ is analytic with respect to ξ in the lower half-plane $\operatorname{Im}\xi < 0$ with the cut along the segment $\xi \in [-i|\eta|, 0]$ and limit values on this cut

$$\sqrt{\xi^2+\eta^2} \to \begin{cases} \sqrt{-(\operatorname{Im}\xi)^2+\eta^2}, & \operatorname{Re}\xi \to 0+, \\ -\sqrt{-(\operatorname{Im}\xi)^2+\eta^2}, & \operatorname{Re}\xi \to 0-, \end{cases}$$

and so at $\xi \in [-i|\eta|, 0]$ we have two conditions

$$\pm c\sqrt{-(\operatorname{Im}\xi)^2+\eta^2}(\hat{\delta\rho_0} - r\hat{\delta s_0}) - \rho^0(\xi\delta\hat{U}_{x0} + \eta\delta\hat{U}_{y0}) \equiv 0.$$

These conditions evidently mean

$$\hat{\delta\rho_0}(\xi,\eta) - r\hat{\delta s_0}(\xi,\eta) \equiv 0\,, \quad \xi\delta\hat{U}_{x0}(\xi,\eta) + \eta\delta\hat{U}_{y0}(\xi,\eta) \equiv 0, \quad \xi \in [-i|\eta|, 0],$$

and since the segment $\xi \in [-i|\eta|, 0]$ is located inside the analyticity region $\operatorname{Im}\xi < 0$, then

$$\hat{\delta\rho_0}(\xi,\eta) - r\hat{\delta s_0}(\xi,\eta) \equiv 0, \quad \xi\delta\hat{U}_{x0}(\xi,\eta) + \eta\delta\hat{U}_{y0}(\xi,\eta) \equiv 0\,, \quad \forall\,\xi,\eta.$$

In physical variables, last conditions mean $\delta p_0 = c^2(\delta\rho_0 - r\delta s_0) \equiv 0$, $\operatorname{div}\delta U_0 \equiv 0$, i.e. the initial data G_0 generate only entropy-vorticity wave.

General conclusion reads: $\Phi \equiv 0$ if and only if initial data G_0 correspond to entropy-vorticity wave only. So any initial post-shock entropy-vorticity wave doesn't generate shock disturbance and reflected waves ($f \equiv 0$, $G_- \equiv 0$), i.e. it isn't incident upon the shock or goes away from the shock; on the contrary any initial post-shock acoustic wave generates shock disturbance and reflected wave ($f \not\equiv 0$, $G_- \not\equiv 0$), i.e. it is incident upon the shock or comes up the shock. At that Eqs. (23), (24) show: if $\Phi(\eta,\omega) \not\equiv 0$ then shock disturbance f and three reflected waves G_j, $j = \overline{2,4}$ don't vanish too. So the second conclusion reads: each initial post-shock acoustic wave generates a four-wave configuration after reflection, regardless of the cases of stability/instability/neutral stability. This conclusion contradicts some statements of works by [11, 25], where the existence of a three-wave configuration in the cases of instability or neutral stability (or even stability by [25]) was stated. In fact, a three-wave configuration at reflection is impossible within the linear theory. The reason of misunderstanding is neglecting of some terms of solution. We return to this issue in second part.

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Evgeny Veniaminovich Semenko
Novosibirsk State Pedagogical University,
Viluiskaya Street 28,
630126, Novosibirsk, Russia
E-mail address: semenko54@gmail.com