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SOME CLASSICAL NUMBER SEQUENCES IN CONTROL SYSTEM DESIGN

A.V. CHEKHONADSKIKH

Abstract. Algebraic tools of LTI control systems design need graphical and analytical structures which depend on dimension of their control parameter space. Essential elements for optimal low-order control systems are the least stable system poles, i.e. the rightmost on the complex plane characteristic roots. Their mutual location is described by critical root diagrams; the algebraic design procedure uses the root polynomials, i.e. factors of characteristic polynomials, which involve only the rightmost poles. From a theoretical point of view it is important to know the dependence between control space dimension and numbers of arising object sets and their asymptotics; they are represented by Fibonacci numbers and partial sums of Euler partitions. From a practical design point of view we need complete lists of required diagrams and polynomials; so we specify the recursive procedure to build a root polynomial list for each control parameter dimension.

Keywords: LTI control systems, system pole, relative stability, Hurwitz function, critical root diagram, root polynomial, Fibonacci numbers, Euler partitions.

1. Introduction

Linear time invariant control systems (LTI CS) are described by ordinary differential equations, and after the Laplace transform they take a form of matrix equations with polynomial elements, which include transform matrices $N(s)D^{-1}(s)$ and $X^{-1}(s)Y(s)$ of a plant and a controller correspondingly. Matrices $D(s)$ and $N(s)$ are considered to be known, and matrices $X(s)$ and $Y(s)$ need to be found, providing desired properties of a closed loop system. Matrix order corresponds to a number of

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control channels; in particular, for a single-channel (SISO) system all relations are scalar polynomial ones. In CS design its characteristic matrix (*system denominator*) plays an important role; it is defined by Diophantine relation $C(s) = X(s)D(s) + Y(s)N(s)$. In this case many basic requirements for closed loop system transients take form of restrictions on a location of system poles, i.e. roots $z_1, ..., z_n$ of characteristic equation $\det C(s) = 0$. Thus, the Hurwitz stability means location of all poles in the left half-plane: $\mathrm{Re} z_i < 0$; limited oscillations reduce to the inequality $|\mathrm{Im} z_i/\mathrm{Re} z_i| < \Theta$, oscillation-free suppression of disturbances requires domination of negative real pole, etc. In full-order systems controller structure has approximately the same or higher complexity than a controlled plant. The latter allows to reach any pole placement, which a designer deems optimal for the closed loop system properties and its transients; the appropriate techniques for different cases are developed in details [1]. However, low-order controllers prevail in industry and technology [2], their free parameter number or a way of their entry into characteristic equation do not allow to achieve any pole location. Most of the technical construction types offer traditional low-order controllers and well proven tuning methods [3]. Theoretically speaking, the problem of low-order controllers design allows many approaches [4, 5]. One of the most natural version is a condition of CS poles location in a fixed domain D (the so-called D-stability), which guarantees a satisfactory quality of transients in a real technical system. A classic example of this type is D-decomposition,which recently have allowed to get several important results [6, 7].

In 2004-06 Prof. A.A. Voevoda and the author offered an optimization approach to low-order CS design. In the most widespread case the goal is the highest relative stability of closed loop system, so Hurwitz function

$$H(p_1, ..., p_m) = \max(\mathrm{Re} z_1, ..., \mathrm{Re} z_n)$$

is to be minimized in the control parameter space $P = \{(p_1, ..., p_m) | p_k \in \mathbf{R}\}$, where $m < n$. We take Hurwitz function as objective one; being the function of controller parameters, it characterizes the rightmost, i.e., the least stable of the system poles and the right border of pole location on the complex plane. Moreover, comparison of pole real parts defines preordering on characteristic root set. The general case of root preorder $\leq_\alpha$ and *root graduation* $H(p_1, ..., p_m)$ is presented in [8]; of course, for real roots it coincides with usual order on reals: $x_k \leq_\alpha x_{k+1} \Leftrightarrow x_k \leq x_{k+1}$. We use also two important particular cases more (Fig. 1):

— comparison of oscillation considering stability:

$$z_k \leq_\alpha z_{k+1} \Leftrightarrow \mathrm{Re} z_k + |\mathrm{Im} z_k| \leq \mathrm{Re} z_{k+1} + |\mathrm{Im} z_{k+1}| \ ;$$

it is used for minimizing an oscillatory component with prevention of a system exit to a stability border, it corresponds to minimization of the conic function

$$K(p_1, ..., p_m) = \max(\mathrm{Re} z_k + |\mathrm{Im} z_k|) \ ;$$

— comparison of stability considering oscillations:

$$z_k \leq_\alpha z_{k+1} \Leftrightarrow \mathrm{Re} z_k + \sqrt{L^2 + \mathrm{Im}^2 z_k} \leq \mathrm{Re} z_{k+1} + \sqrt{L^2 + \mathrm{Im}^2 z_{k+1}} \ ;$$

it is used to maximize stability with limited oscillations; it corresponds to minimization of the hyperbolic function (its domain family is close to widespread in CS design truncated cones):

$$G(p_1, ..., p_m) = \max(\mathrm{Re} z_k + \sqrt{L^2 + \mathrm{Im}^2 z_k}) - L \ .$$

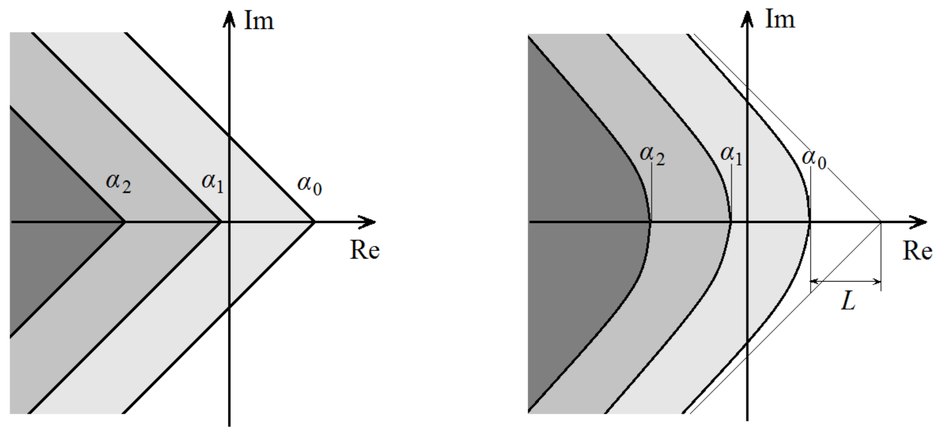

FIG. 1. Root optimization principle: reaching pole location in the leftmost possible domain $D_{\alpha*}$ in a family of enclosed domains $\{D_\alpha | D_\alpha \subset D_\beta \Leftrightarrow \alpha < \beta\}$ of some fixed type. On the left – the family of conic type associated with conic graduation $K(p_1, ..., p_m)$. On the right – a family of hyperbolic type associated with hyperbolic graduation $G(p_1, ..., p_m)$.

2. Root Simplexes and Symplectic Graphs

Due to the root preorder $\leq_\alpha$ orderless sets of characteristic real roots and complex pairs can be enumerated in α-ascending: $z_1 \leq_\alpha ... \leq_\alpha z_n$. Presenting complex pairs with using real and imaginary parts, we'll see different tuples like, for example,

$$\langle x_1; x_2 \pm iy_3; x_4; x_5 \pm iy_6; ...; x_{n-1} \pm iy_n \rangle \ ,$$

where $x_1 \leq_\alpha x_2 \pm iy_3 \leq_\alpha x_4 \leq_\alpha x_5 \pm iy_6 \leq_\alpha ... \leq_\alpha x_{n-1} \pm iy_n$; (the latter in the Hurwitz case is equal to $x_1 \leq x_2 \leq x_4 \leq x_5 \leq ... \leq x_{n-1}; y_j > 0$).

If characteristic polynomial is separable, a *coefficients*↔*roots* correspondence is differentiable [9], and while the strict inequality (for example, $x_1 <_\alpha x_2 \pm iy_3 <_\alpha x_4 <_\alpha x_5 \pm iy_6 <_\alpha ... <_\alpha x_{n-1} \pm iy_n$) holds, it is the diffeomorphism between some domain in polynomial coefficient space and the corresponding segment in ordered root space, and root enumeration remains the same, so we can speak of *piecewise real coordinatization* of root tuples, whether they are real or complex. In Hurwitz case such segment is defined by strict inequalities as above, which in our example are equal to $x_1 < x_2 < x_4 < x_5 < ... < x_{n-1}; y_j > 0$. These segments are bordering on with each other along the boundaries defined by tuples with α-equalities, e.g. the two segments $... \leq_\alpha x_4 \leq_\alpha x_5 \pm iy_6 \leq_\alpha ...$ and $... \leq_\alpha x_4 \pm iy_5 \leq_\alpha x_6 \leq_\alpha ...$ are bordering on along the boundary with the two possible enumerations:

$$... \leq_\alpha x_4 =_\alpha x_5 \pm iy_6 \leq_\alpha ... \cong ... \leq_\alpha x_4 \pm iy_5 =_\alpha x_6 \leq_\alpha ...$$

Segment↔boundary correspondence in a root simplex of polynomial of degree n can be represented by an unoriented graph H_n, whose vertices represent root segments and edges represent their boundaries [9]. For a cubic polynomial its root simplex and symplectic graph are presented on Fig.2; it is similar to root distribution in classic Vyshnegradsky diagram. However, for higher polynomial degrees it is less foreseeable, because simplex root construction and the symplectic graph structure grows exponentially. For polynomials of high degrees their root simplex construction is facilitated by the recurrence between their graphs: $H_{n+1} \cong H_n \sqcup H_{n-1}$, where the sign $\sqcup$ denotes uncomplete graph juxtaposing [10]. As the easy conclusion from this statement we get the following:

Proposition 1. Symplectic graph power dependence of the polynomial degree n is represented by Fibonacci numbers: $|H_n| = \varphi_{n+1}$.

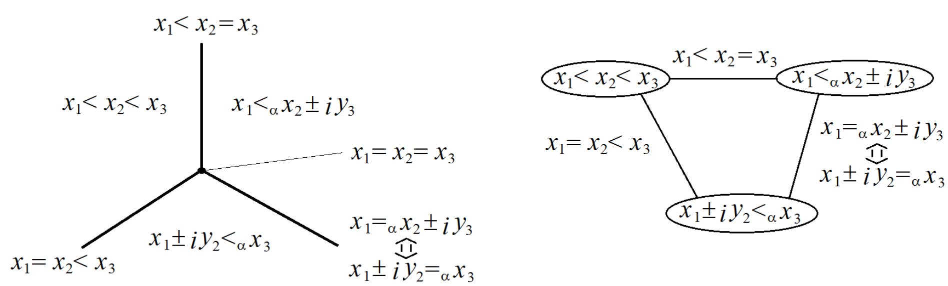

FIG. 2. Root simplex of a cubic polynomial (leftwards) and its symplectic graph (rightwards).

Since some zones in control parameter space, defined by strict root α-inequalities, can be empty, *segment↔boundary* correspondence in a parameter space can be somewhat simpler than adequate one in a root simplex. Nevertheless the proposition provides an estimate of complexity of different root distributions in a parameter space, arising in numeric design procedures [10].

3. Critical Root Diagrams

Root preorder and related root numbering let one schematically represent a mutual location of CS poles on the complex plane as a root diagram, i.e., as a picture, which represents pole α-ordering and α-equalities. Obviously only one real root (maybe multiple) can be placed on the right boundary of all poles location. Poles with larger α-values are shown to the right of poles with lower ones; poles with equal α-graduation values are shown on the same vertical line so that a complex root pair $x_3 \pm iy_4$ is further apart than an α-equal to it pair $x_1 \pm iy_2$ with $0 < y_2 < y_4$; we specify multiplicities of real roots and complex pairs near their depicting points, see Fig. 3.

Since values of functions of a graduation type are defined by α-rightmost roots, equality (multiplicity) or α-equality of such roots leads to a non-differentiable objective function [8] and, consequently, to a critical set represented by a *critical root zone* in control parameter space. Dimension of a critical zone is defined by number of corresponding root multiplicities and α-equalities. Each α-equality bounds one degree of freedom in a parameter space, whether it is multiplicity of a real root or α-equality of complex conjugate pairs; duplicity of complex conjugate pair bounds two degrees of freedom; n-plicity of complex pairs bounds $2n-2$ degrees of freedom.

Thus, rightmost roots multiplicities or α-equalities define a manifold of lower dimension in the parameter space. If this dimension is positive, it is possible to minimize objective function further along this manifold until we add a few more equalities (i.e., until several more roots fall onto the right boundary of their common location), and the manifold dimension vanishes.

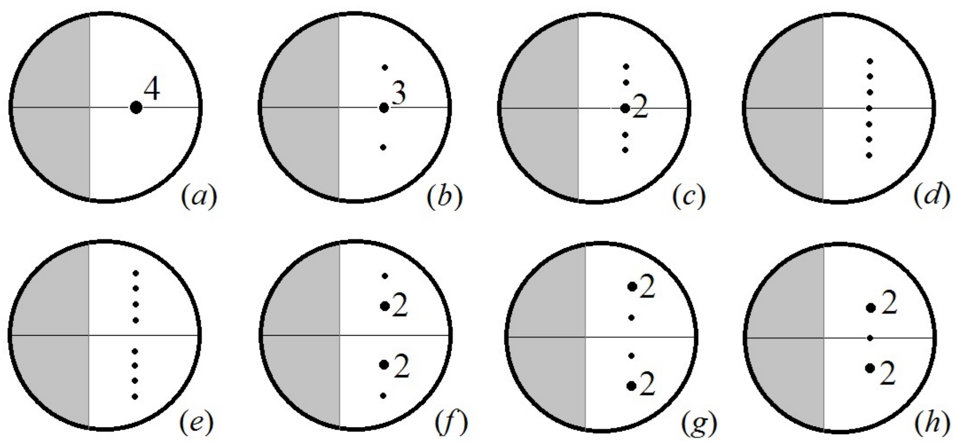

FIG. 3. Critical root diagram list for a 3-parameter control space. A pole multiplicity is labeled near a point which depicts this pole. Vertical point placement schematically shows location of the rightmost poles on a border of a graduation set domain. Other poles are represented as a grey segment leftwards.

Therefore, a three-dimensional parameter space has eight possibilities (see Fig. 3) for which from 4 to 8 roots are located on the right boundary; roots, which are smaller with respect to α-preordering, are located in a grey segment in the left part of each diagram. All such root locations appear to be zero-dimensional, i.e., critical points in the usual meaning. For control parameter number $m > 3$ the same diagrams define $(m-3)$-dimensional critical manifolds.

Practical effectiveness of critical diagram concept depends on how quickly a critical root diagram number increases while control space dimension grows. The stated below theorem specifies the exact growth rate [8]. Let us assume that the degree of a characteristic polynomial is large enough, so that the number of roots doesn't influence the realization of critical diagrams.

Theorem 1. *Number of different critical root diagrams for a polynomial, whose coefficients depend on m control parameters, equals to $(m+3)$-d Fibonacci number φ_{m+3}.*

Proposition 1 and Theorem 1 lead to the following statement:

Corollary. *Numbers of different symplectic graphs and critical root diagrams grow asymptotically as* $\sim (\frac{1+\sqrt{5}}{2})^m$, *where* m *is a number of control parameters included in the polynomial's coefficients.*

Proof of Theorem 1 shows a recursive way to construct critical diagram list for an arbitrary number m [8].

4. Root Polynomials

As LTI control system design involves *root polynomials*, which include only the α-rightmost poles [8], the estimate of such polynomial number deserve attention as well as effective procedure of polynomial list drawing for each control parameter dimension.

Particular techniques close to root polynomials were used in numerous series of papers by A.M. Shubladze and his colleagues (e.g. [11-13]), though the general notion wasn't formulated there. For the sake of simplicity we will illustrate it with the following examples (for general notion see [8]).

Let us denote the index of all roots as $l = 1, ..., n$, the index of the rightmost roots as $j = 1', ..., k'$, so that a real root $z_{1'} = x_{1'}$ of multiplicity $a_{1'}$ and complex pairs $z_{2',3'} = x_{2'} \pm iy_{2'}, ..., z_{(2k-2)',(2k-1)'} = x_{k'} \pm iy_{k'}$ of multiplicities $a_{2'}, ..., a_{k'}$ are placed on the right boundary of all roots location.

— Root polynomial for the Hurwitz graduation has the form

$$r(s) = (s-H)^{a_{1'}}(s^2 - 2Hs + H^2 + y_{2'}^2)^{a_{2'}} \cdot ... \cdot (s^2 - 2Hs + H^2 + y_{k'}^2)^{a_{k'}},$$

because here $H = \max(\mathrm{Re} z_1, ..., \mathrm{Re} z_n) = x_{1'} = ... = x_{k'}$.

— For conic graduation root polynomial has the form

$$r(s) = (s - x_{1'})^{a_{1'}} \prod_{j=2'}^{k'} (s^2 - 2x_{j'}s + x_{j'}^2 + y_{j'}^2)^{a_{j'}} =$$

$$= (s-K)^{a_{1'}} \prod_{j=2'}^{k'} (s^2 - 2x_{j'}s + 2x_{j'}^2 - 2Kx_{j'} + K^2)^{a_{j'}},$$

because here $y_{j'}^2 = (K - x_{j'})^2$ and

$$K = \max_{l=1,...,n}(\mathrm{Re} z_l + |\mathrm{Im} z_l|) = x_{1'} = x_{2'} + y_{2'} = ... = x_{k'} + y_{k'}\ .$$

— For hyperbolic graduation root polynomial has the form

$$r(s) = (s - x_{1'})^{a_{1'}} \prod_{j=2'}^{k'} (s^2 - 2x_{j'}s + x_{j'}^2 + y_{j'}^2)^{a_{j'}} =$$

$$= (s-G)^{a_{1'}} \prod_{j=2'}^{k'} (s^2 - 2x_j s + x_j^2 - (x_j - G - L)^2 + L^2)^{a_j},$$

because here $y_j^2 = (x_j - x_C)^2 - L^2$ (where $x_C = G + L$ is abscissa of hyperbola center), $G = \max(x_l + \sqrt{L^2 + y_l^2} - L) = x_{1'} = x_j + \sqrt{L^2 + y_j^2} - L$, see Fig. 1.

The important feature of root polynomial construction is algebraic dependence of root polynomial coefficients on the real and imaginary parts of the roots. Due to it root polynomials allow to establish links between control parameters and root coordinates (like y_j and x_j above) and to minimize objective functions in difficult design problems[15, 16]. Obviously, each critical root diagram allows us to write down corresponding root polynomial. But because we needn't specify a comparison of root imaginary parts, a few different diagrams can have the same root polynomial, e.g. the two diagrams (f) and (g) in Fig. 3 correspond to the root polynomial

$$r(s) = (s^2 - 2Hs + H^2 + y_{2'}^2)^2(s^2 - 2Hs + H^2 + y_{3'}^2) \ ,$$

Therefore, some practical design relief may be caused by a slower growth of root polynomial number. It may be clarified with the help of tuples of the rightmost roots multiplicities.

Let us define *codes of root polynomials*, which are universal for all graduation types. They differ from the critical root diagram codes [8] due to the feature, that only multiplicities but not the size imaginary parts of complex root pairs are taken into account. So we can dispose complex pair multiplicities of some root set in nonascending order. As above, *generating index* for each code set is the control space dimension m, or equal to it number of connections between the real and imaginary parts of characteristic roots. For the sake of simplicity (without loss of generality) let us consider the case of Hurwitz objective function.

Definition. Code of root polynomial

$$r(s) = (s - H)^{a_{1'}}(s^2 - 2Hs + H^2 + y_{2'}^2)^{a_{2'}} \cdot ... \cdot (s^2 - 2Hs + H^2 + y_{k'}^2)^{a_{k'}}$$

have the form of a row with natural elements $(a_{1'}\ a_{2'} ...\ a_{k'})$, where $a_{1'} \geq 0$; $a_p \geq a_q \geq 1$ for $1 < p < q$; a number k' is the code length.

Lemma. *Dimension of control parameter space is connected with a polynomial code by equation* $m = a_{1'} + 2a_{2'} + ... + 2a_{k'} - k'$.

Proof. As above, we can assume, that in order to reach a pole location corresponding to critical root diagram, m free control parameters allow to obtain m equalities including real or imaginary parts of characteristic roots: $\mathrm{Re}z_p = \mathrm{Re}z_q$ or $\mathrm{Im}z_p = \mathrm{Im}z_q$ (the latter for multiple complex pairs). Real root multiplicity $a_{1'}$ means $a_{1'} - 1$ equalities $x_{1'} = ... = x_{a_{1'}}$; an appearance of a simple complex pair on the same boundary results in one more equality, but if the complex pair $x_{a_{1'}+1} \pm iy_{a_{1'}+1}$ has a multiplicity $a_{2'} > 1$, then there are achieved $2a_2 - 2$ equalities more: $x_{1'} = ... = x_{a_{1'}} = x_{a_{1'}+1} = ... = x_{a_{1'}+a_{2'}}$ and $y_{a_{1'}+1} = ... = y_{a_{1'}+a_{2'}}$, so the total number of equalities is $(a_{1'} - 1) + 1 + (2a_{2'} - 2) = (a_{1'} - 1) + (2a_{2'} - 1)$. The same is true for an appearance of a few complex pairs, therefore finally we get

$$m = (a_{1'} - 1) + (2a_{2'} - 1) + ... + (2a_{k'} - 1) = a_{1'} + 2a_{2'} + ... + 2a_{k'} - k'. \ \square$$

Hence we can regard dimension m of control parameter space as *generating index* for root polynomial code set.

Theorem 2. *The number $N(m)$ of different root polynomials of index m is equal to partial sum $\sum_{m+2}$ of Euler partitions sequence for natural numbers.*

Proof. We have to find the number of different polynomial codes corresponding to the fixed index m. At first, the number of Euler partitions of zero is considered to be equal to 1; it is the first element of Euler partitions sequence and it equals to the sum $\sum_1$.

Second, for $m = 0$ we have two possible polynomials: $s - x_{1'}$ with code (1) and $s^2 - 2x_{1'}s + x_{1'}^2 + y_{1'}^2$ with code (0 1); taking into account the beginning of Euler partitions sequence 1, 1, 1, 2, 2, 3, 4, 5, 6,... we see in that case $\sum_2 = 2$, and the statement of Theorem holds.

Third, the equality of lemma can be rewritten as

$$m - a_{1'} + 1 = (2a_{2'} - 1) + ... + (2a_{k'} - 1) \ .$$

Finally, the real root multiplicity $a_{1'}$ can take all values from 0 to $m+1$, i.e. the number of distinct root polynomial codes for each index m is the sum of all of these cases without gaps of summands. □

Remark. The beginning of sequence A036469 is the following [14]: 1, 2, 3, 5, 7, 10, 14, 19, 25, 33, 43, 55, 70, 88, 110, 137, 169, 207, 253, 307, 371, 447, 536,... As stated ibid., in February 2015 Vaclav Kotesovec gave the estimate of the growth rate of the sequence, which shows that the number of root polynomials is growing slower, than the number of critical root diagrams: $A_m \sim \frac{e^{\pi\sqrt{m/3}}}{2\pi\sqrt[4]{m/3}}$.

Listing of root polynomials is carried out by double recursion — on root polynomial index m and on maximum multiplicity $a_{2'}$ of complex root pairs.

As we have mentioned above, root polynomial code list of index $m=0$ consists of two polynomials, whose codes are (1) and (0 1).

A list of index $m>0$ starts with single element row $(m+1)$, and further the following:

for $a_{2'}=1$ (i.e. for simple complex roots) it has the form

$$(0\underbrace{1\dots1}_{m+1}),(1\underbrace{1\dots1}_{m}),(2\underbrace{1\dots1}_{m-1}),...,(m\ 1);$$

the code list of index m with an element $a_{2'}=q>1$ (i.e. for complex roots of maximal multiplicity $q\le[m/2]+1$) consists of code lists of forms $(a^-_{1'}\ q\ a^-_{2'}\ \dots a^-_{k'})$, where $(a^-_{1'}\ a^-_{2'}\ \dots a^-_{k'})$ are code lists of indexes $l^-=m-2q+1$ with $a^-_{1'}\le q$.

Finally, if the index m is even and $q=m/2+1$ (i.e. $m-2q+1=-1$), then the list includes the single code $(0\ q)$.

5. Conclusion

The current state of the control theory doesn't offer a universal approach to the low-order control system design. This is one of the reasons why the design of three-parameter PID systems remains a topical area of control theory and practice. Slightly more complicated systems are extremely hard for the standard tools of numerical analysis and design. The algebraic approach allows to carry on low-order CS design for the parameter space dimensions more than three (in particular, the root polynomial technique proved to be effective for CS design with 6-7 control parameters, [15, 16]), but its practical implementation also depends on the number of emerging options.

Elements of number theory and recursion became rather useful means of assessing the applicability of this approach in higher dimension parameter spaces. We found Fibonacci numbers and Euler partitions partial sums as the laws of the growth rate of emerging objects; the latter is sufficiently lower than the first one. Therefore, the exponential asymptotics of the critical diagram and root polynomial numbers stimulates the development of exploration methods for each case and automation of the entire design process.

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Alexander Vasil'evich Chekhonadskikh
Novosibirsk State Technical University,
pr. K. Marx, 20,
630073, Novosibirsk, Russia
E-mail address: alcheh@ngs.ru