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THE CONFERENCE «DYNAMICS IN SIBERIA» DEDICATED TO THE 90TH ANNIVERSARY OF B.V. CHIRIKOV, NOVOSIBIRSK, FEBRUARY 26 – MARCH 4, 2018

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Abstract. In this article abstracts of talks of the Conference «Dynamics in Siberia» held in Novosibirsk, February 26 – March 4, 2018 are presented.

The conference «Dynamics in Siberia» was organized by Sobolev Institute of Mathematics, Budker Institute of Nuclear Physics and Novosibirsk State University. It was held in House of Scientists and in the Sobolev Institute of Mathematics SB RAS (Novosibirsk) from February 26 to March 4, 2018.

The conference was dedicated to the 90th anniversary of Boris Valerianovich Chirikov, one of the founders of the theory of Classical and Quantum Dynamical Chaos.

Academician Boris Valerianovich Chirikov was the creator of a new direction in physics — the theory of dynamic classical and quantum chaos. B.V. Chirikov was born in 1928 in the city of Orel. In 1946 he entered the Faculty of Physics and Mathematics of the Moscow V.I. Lenin Pedagogical Institute and then transferred to the Physics and Technology Faculty of Moscow State University. Soon he was hired as a student trainee in the Thermal Engineering Laboratory of the USSR Academy of Sciences (TTL, now ITEP), where, after a brilliant diploma defense, he continued to work as an experimental physicist. In 1954, at the suggestion of G.I. Budker, who knew the student Boris Chirikov well, he went to work at LIPAN (now the Russian Research Center Kurchatov Institute). In 1958, on the basis of the Laboratory for

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Acceleration led by G.I. Budker there was created the Institute of Nuclear Physics of the Siberian Branch of the Academy of Sciences of the USSR, and two years later Boris Valerianovich was among his first employees to move to Novosibirsk. Starting his research as an experimental physicist, he created a new theoretical direction - the theory of classical dynamic chaos. Having made in the late 60s a choice from experiment to theory, Boris Valerianovich turned to a numerical experiment, displaying outstanding talent and a unique individuality of the researcher. His proposed model, the "standard map", which was later called Chirikov's map, made it possible to study the most important aspects of stochastic dynamics for a wide range of nonlinear physical systems, from particles in accelerators and plasma traps to astrophysical objects. Thanks to his high scientific prestige among specialists from different countries, he was able to organize international cooperation using advanced advances in computational technology. The culmination of such cooperation was the creation of a theory of dynamic quantum chaos. The Chirikov school developed and worked fruitfully. In addition to physicists from Novosibirsk, it included groups of scientists from the United States, Italy, and France, who have fruitfully collaborated with Boris Valerianovich for many years. Intense scientific activities B.V. Chirikov combined with teaching and educational activities. His bright and unconventionally written textbooks helped several generations of students of the Novosibirsk State University (NSU) to enter physics, where he lectured from the very foundation of the NSU.

The history of his life and scientific activities are described in the book

Boris Valerianovich Chirikov — legislator of chaos / editors: O.V. Zhirov, D.L. Shepelyansky, K.V. Epstein; editor in chief A.N. Skrinsky; Russian Aacademy of Sciences, Siberian Branch, Budker Institute of Nuclear Physics. - Novosibirsk: Publishing House of the SB RAS, 2014. - 282 p.
The book includes the memoirs of colleagues, students and friends of Boris Valerianovich, as well as a list of his scientific works and selected philosophical articles.

Members of the program committee were I.A. Dynnikov, A.A. Glutsyuk, G.N. Kulipanov, A.I. Milstein, A.E. Mironov, I.A. Taimanov and A.Yu. Vesnin.

More than 50 experts on dynamical systems, chaos theory, mathematical physics, geometry and topology participated in the conference. The conference program consisted of plenary talks, short talks and poster session. The talks were made by well-known experts from Moscow, St. Petersburg, Novosibirsk, Chelyabinsk, Krasnoyarsk, Kemerovo, Troitsk, Dubna, Ufa, Nizhny Novgorod, Vladivostok, Gorno-Altaisk and also by well-known mathematicians from Italy, France, Germany, Poland, Serbia and Slovenia. About 15 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

The conference was supported by the Regional Mathematical Center of Novosibirsk State University.

Program (Plenary talks)

February 26

10:10-11:00	G. Kulipanov (*Novosibirsk*). Experimental study of nonlinear resonances and stochastisity (tests Chirikov's criteria).
11:00-11:50	D. Shepelyansky (*Universite de Toulouse, France*). Interlinks of dynamical thermalization, Kolmogorov turbulence, KAM and Anderson localization.
12:10-13:00	G. Casati (*Universita' della Insubria, Italy*). That kind of motion we call heat.
13:00-13:50	M. Znidaric (*University of Ljubljana, Slovenia*). Nonequilibrium physics of quantum quasiperiodic systems.

February 27

9:30-10:20	V. Kozlov (*Moscow*). Symplectic geometry of linear Hamiltonian systems.
10:20-11:10	I. Guarneri (*Universita' della Insubria, Italy*). A model with chaotic scattering and reduction of wave packets.
11:30-12:20	D. Waltner (*Universitat Duisburg-Essen, Germany*). Semiclassical Classification of Periodic Orbits in Quantum Many-Body Systems.
12:20-13:10	S. Dobrokhotov (*Moscow*). Pairs of Lagrangian manifolds and semiclassical asymptotics of solutions of inhomogeneous stationary problems with localized right-hand sides.

February 28

9:30-10:20	D. Treschev (*Moscow*). Relative version of the Titchmarsh theorem and its applications in dynamics.
10:20-11:10	A. Gaifullin (*Moscow*). On the homology of Torelli groups.
11:50-12:40	P. Plotnikov (*Novosibirsk*). On the bellows conjecture in spaces of constant curvature.
12:40-13:30	A. Shafarevich (*Moscow*). Properties of harmonic functions and solutions of the wave equation on polyhedra.

March 1

9:30-10:20	N. Kuznetsov, G. Leonov (*Saint Petersburg*). Analytical and numerical methods for the study of attractors in dynamical systems: bifurcations, localization and dimension characteristics.
10:20-11:10	Yu. Kordyukov (*Ufa*). Magnetic Laplacians, generalized Bergman kernels and Berezin – Toeplitz quantization on symplectic manifolds.
11:30-12:20	Yu. Trakhinin (*Novosibirsk*). On application of the Nash – Moser method to weakly well-posed free boundary problems.
12:20-13:10	E. Kuznetsov (*Moscow*). Development of high vorticity structures and geometrical properties of the vortex line representation.

March 2

9:30-10:20	A. Odzijewicz (*University of Bialystok, Poland*). Connections on a principal G-bundle and related symplectic structures.
10:20-11:10	V. Grines (*Nizhny Novgorod*). On dynamics of cascades with surface dynamics on 3-manifolds.
11:30-12:20	Z. Rakic (*University of Belgrade, Serbia*). On modified non-local gravity.

March 3

9:30-10:20	A. Orlov. (*Moscow*) Hurwitz numbers, tau functions and ensembles of random matrices.
10:20-11:10	P. Akhmet'ev. (*Troitsk*) Linking coefficients for geodesic flow and applications for magnetic equilibrium.

Plenary talks

Short talks

Poster session

PLENARY TALKS

Development of high vorticity structures and geometrical properties of the vortex line representation

D. Agafontsev, E. Kuznetsov (Moscow), A. Mailybaev

According to the Kolmogorov-Obukhov theory of developed hydrodynamic turbulence [1], the velocity fluctuations at intermediate spatial scales l obey the power-law $\langle|\delta v|\rangle \propto \varepsilon^{1/3} l^{1/3}$, where ε is the mean energy flux. Consequently, fluctuations of the vorticity $\omega = \mathrm{rot}\,\mathbf{v}$ diverge at small scales as $\langle|\delta\omega|\rangle \propto \varepsilon^{1/3} l^{-2/3}$ and, thus, the Kolmogorov spectrum is linked with the small-scale structures of intense vorticity. The Kolmogorov arguments are based on the isotropy of the flow and locality of nonlinear interaction at intermediate scales. Then, the dynamics in the inertial interval can be described by the Euler equations.

In this talk we discuss our recent findings [2-4] concerning the general properties of the high-vorticity regions developing from generic large-scale initial conditions in 3D Euler equations. These findings are based on direct numerical simulations of the equations performed in the periodic box for more than 30 initial conditions. We use the pseudo-spectral method in adaptive anisotropic rectangular grid, which is uniform in each direction and adapted independently along each of the three coordinates, with up to 2048^3 total number of nodes.

As was first found by Brachet *et.al* [5], the regions of high vorticity in the generic case represent exponentially compressing pancake-like structures. Contrary to the previous studies, we show [2] that evolution of the pancakes is governed by two different exponents for thickness $\ell_1(t) \propto e^{-\beta_1 t}$ and maximal vorticity $\omega_{\max}(t) \propto e^{\beta_2 t}$ with the universal ratio $\beta_2/\beta_1 \approx 2/3$, respectively $\omega_{\max}(t) \propto \ell_1(t)^{-2/3}$.

For the asymptotic pancake evolution, we suggest [3] a novel *exact* solution of the Euler equations, which combines a shear flow aligned with an asymmetric straining flow, and is characterized by an arbitrary transversal vorticity profile.

The pancakes appear in increasing number with different scales and generate strongly anisotropic "jets"in the Fourier space. These jets dominate in the energy spectrum, where, for some initial flows, we observe clearly the gradual formation of the Kolmogorov spectrum $E_k \propto k^{-5/3}$, in fully inviscid system [2,4]. With the massive simulations, we examine [4] the influence of initial conditions on the processes of pancake formation and the Kolmogorov spectrum development. The initial flows are chosen as a superposition of the shear flow $\omega_x = \sin z$, $\omega_y = \cos z$, $\omega_z = 0$ and a random (not necessarily small) perturbation. The presence of the shear flow influences the orientation of emerging pancake structures, from fully random when the shear flow is absent to almost unidirectional close to z-axis when the perturbation is small. We observe that the 2/3-scaling holds universally, while initial conditions composed of the shear flow and a small perturbation develop the spectrum close to $E_k \propto k^{-5/3}$.

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Linking coefficients for geodesic flow and applications for magnetic equilibrium

P. Akhmet'ev (Troitsk)

Geodesics flows on the unit tangent bundle for hyperbolic orbifolds of type (p, q, ∞) is investigated in [1]. Applications for quadratic magnetic helicities, (see [2] for definition) are presented.

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That kind of motion we call heat a major societal problem for the 21st century

G. Casati (Universita' della Insubria, Italy)

Providing a sustainable supply of energy to the worlds population will become a major societal problem for the 21st century as fossil fuel supplies decrease and world demand and environmental concern increases. Thermoelectric phenomena, which involve the conversion between thermal and electrical energy, and provide a method for heating and cooling materials, are expected to play an increasingly important role in meeting the energy challenge of the future.

To this end it is important to understand the microscopic mechanism which determines the macroscopic laws of heat and particles transport and allows to control the heat current. Here we discuss a new approach, which is rooted in nonlinear dynamical systems, for increasing the efficiency of thermoelectric machines. The main focus will be on the physical mechanisms, unveiled by these dynamical models, which lead to high thermoelectric efficiency, approaching the Carnot limit.

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Пары лагранжевых многообразий и квазиклассические асимптотики решений неоднородных стационарных задач с локализованными правыми частями

С. Доброхотов (Москва)

В n-мерном пространстве с координатами $x = (x_1, ...x_n)$ рассматриваются дифференциальные и псевдодифференциальные уравнения вида $L'w = F$, где $L' = L(x, -ihgrad, h)$-дифференциальный или псевдодифференциальный оператор с гладким символом $L(x,p,h)$, заданный в 2n -мерном фазовом пространстве с координатами $(x,p) = (x_1,...x_n,p_1,...p_n)$, h-малый параметр. Предполагается, что функция F задана в виде канонического оператора Маслова на некотором лагранжевом многообразии M, примененного к функции A на M. Например, если M -вертикальная плоскость (x=0) в фазовом пространстве, то $F = f(x/h)$- функция локализованная в окрестности точки x=0 (если A=1, то функция F -дельта-функция Дирака). Если символ $L(x,p,h)$ не обращается в ноль на M, то задача является "эллиптической"и асимптотика решения выражается в виде стандартном для теории псевдодифференциальных операторов виде. Мы показываем, что если $L(x,p,h)$ обращается в ноль на некотором подмножестве N из M, (в общем положении N- (n-1)- мерное изоторпное многообразие), то при некоторых дополнительных условиях на L в асимптотике решения появляется "волновая"составляющая, связанная со вторым лагранжевым многообразием Λ, получаемым из N с помощью сдвигов по траекториям гамильтоновой системы, заданной функцией Гамильтона H(x,p)=L(x,p,0). Пара лагранжевых многообразий (Λ, M) определяет асимптотику решения исходной задачи в виде канонического оператора Маслова на (Λ, M). В других задачах пары лагранжевых многообразий появились в работах Мельроуза и Ульмана, Стернина и Шаталова. В качестве примеров рассматриваются уравнения Гельмгольца и линейной теории волн на воде. В случае уравнения Гельмгольца полученные формулы обобщают результаты Келлера, Бабича и Кучеренко.

Доклад основан на работах, совместных с А.Аникиным, В.Назайкинским и М.Руло.

О динамике каскадов с поверхностной динамикой на 3-многообразиях

В. Гринес (Нижний Новгород)

Доклад посвящен описанию топологии многообразий, допускающих структурно устойчивые каскады, обладающие поверхностной динамикой. Приводится полная топологическая классификация диффеоморфизмов рассматриваемого типа, при условии, что их неблуждающие множества состоят из двумерных базисных множеств. Для градиентно-подобных диффеоморфизмов дается точная оценка числа некомпактных гетероклинических кривых.

A model with chaotic scattering and reduction of wave packets.

I. Guarneri (Universita' della Insubria, Italy)

Some variants of Smilansky's model of a particle interacting with harmonic oscillators are examined in the framework of scattering theory. A dynamical proof is given of the existence of wave operators. Analysis of a classical version of the model provides a transparent picture for the spectral transition to which the quantum model owes its renown, and for the underlying dynamical behaviour. The model is thereby classified as an extreme case of chaotic scattering, with aspects related to wave packet reduction and irreversibility.

Analytical and numerical methods for the study of attractors: bifurcations, localization, and dimension characteristics

N. Kuznetsov (Saint Petersburg), G. Leonov (Saint Petersburg)

This lecture is devoted to recent results on the study of attractors in dynamical systems. Effective analytical and numerical methods for the study of transition to chaos, localization of attractors, and dimension characteristics of chaotic attractors are discussed.

Recently it was suggested to classify the attractors in dynamical systems as being hidden either self-excited [1, 2]: an attractor is called self-excited if its basin of attraction intersects with any vicinity of an equilibrium, otherwise it is called a hidden attractor. This allowed for combining the notions of transition processes in engineering systems, visualization in numerical mathematics, the basin of attraction, and the stability of dynamical systems. The classification, not only demonstrated difficulties of fundamental problems (e.g., the second part of Hilbert's 16th problem on the number and mutual disposition of limit cycles, Aizerman's and Kalman's conjecture on the monostability of nonlinear systems) and applied systems analysis, but also triggered the discovery of new hidden attractors in well-known physical and engineering models [1, 3, 4].

For the study of attractors and estimating the Hausdorff dimension the concept of the Lyapunov dimension was suggested by Kaplan and Yorke. Along with widely used numerical methods for computing the Lyapunov dimension it was developed an effective analytical approach, which is based on the direct Lyapunov method with special Lyapunov-like functions [5, 6]. The advantage of the method is that it allows one, in many cases, to estimate the Lyapunov dimension of an invariant set without localization of the set in the phase space, to prove Eden's conjecture for the self-excited attractors and get exact Lyapunov dimension formula for attractors of various well-known dynamical systems (e.g., such as the Chirikov, Henon, Lorenz, Shimizu-Morioka, and Glukhovsky-Dolzhansky systems). Also approaches for reliable numerical estimation of the finite-time Lyapunov exponents and finite-time Lyapunov dimension are discussed [7, 6].

The homoclinic orbits play an important role in the bifurcation theory and in scenarios of the transition to chaos. In the case of dissipative systems, the proof of the existence of homoclinic orbits is a challenging task. Recently it was developed an effective method, called Fishing principle, which allows one to obtain necessary and sufficient conditions of the existence of homoclinic orbits in various well-known dynamical systems. [2, 8, 9, 10].

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Partial spectral flow and the Aharonov-Bohm effect in graphene

V. Nazaikinskii (Moscow)

A mathematical model of the Aharonov-Bohm effect in a graphene sheet with holes was considered in my 2012 joint paper with M.I. Katsnelson (Radboud University, Nijmegen), where we give a formula for the spectral flow of a family of Dirac type self-adjoint operators with classical (local) boundary conditions on a compact Riemannian manifold with boundary and apply this formula to the family of graphene Dirac Hamiltonians arising as a magnetic field is switched on adiabatically. However, the Dirac Hamiltonian is only an approximation to the "true"graphene lattice Hamiltonian, which (for the case of a finite graphene sheet) acts on a finite-dimensional space and hence cannot produce a nontrivial spectral flow. In the present talk, which is based on ongoing joint work with M. I. Katsnelson and J. Bruning (Humboldt University, Berlin), we introduce the notion of partial spectral flow (spectral flow along a subspace), which is nontrivial even in the finite-dimensional case, give its main properties, and apply it to describe the Aharonov-Bohm effect in terms of the family of graphene lattice Hamiltonians. Namely, in the lattice Hilbert space we single out subspaces corresponding to the Dirac points K and K' of the reciprocal lattice, and it turns out that (a) the sum of partial spectral flows of the family of graphene lattice Hamiltonians along these subspaces is zero; (b) each of these partial spectral flows is equal to the spectral flow of the respective family of Dirac operators considered in our earlier paper and hence can be computed by the topological formula given there. The conclusion is that, as the magnetic field is switched on, electron and hole energy levels are created in pairs, and the total number of these pairs equals the number of magnetic flux quanta through selected holes on the graphene sheet.

Connections on a principal G-bundle and related symplectic structures

A. Odzijewicz (University of Bialystok, Poland)

We investigate the G-invariant symplectic structures on the cotangent bundle T^*P of a principal G-bundle $P(M, G)$ canonically related to connections on $P(M, G)$ as well as to the elements of the group $Aut_{TG}TP$ of automorphisms of the tangent bundle TP which covers the identity map of P and commute with the action of TG on TP. The classical reduction procedure for these symplectic structures is described and possible applications are discussed.

On application of the Nash-Moser method to weakly well-posed free boundary problems

Yu. Trakhinin (Novosibirsk)

We discuss the usage of the Nash-Moser method for the proof of the local-in-time existence of smooth solutions to free boundary problems whose linearizations are weakly well-posed problems. We mainly consider problems for hyperbolic systems of conservation laws, but our approach is also applicable for such systems as, for example, the incompressible Euler equations. Weak well-posedness means that the Kreiss-Lopatiski condition for the constant coefficients linearized problem holds only in a weak sense. In fact, weak well-posedness means neutral stability and usually implies the loss of derivatives phenomenon in a priori estimates for the linearized problem. The main idea of the Nash-Moser method is just the compensation of lost derivatives at each step of the iteration process for the nonlinear problem by using a sequence of smoothing operators. We briefly discuss peculiarities of the application of the Nash-Moser method to free boundary problems for the compressible Euler equations and the equations of ideal compressible magnetohydrodynamics (MHD). Our examples are the compressible liquid-vacuum problem, the plasma-vacuum interface problem and the free boundary problem for MHD contact discontinuities.

Nonequilibrium physics of quantum quasiperiodic systems

M. Znidaric (University of Ljubljana, Slovenia)

I will describe nonequilibrium physics of quantum quasiperiodic systems like the Aubry-Andre-Harper model. When a non-interacting critical system is driven out of equilibrium the steady state displays fractal spatial dependence of observables. Even more interesting is the behavior in the presence of interactions: infinitesimal interactions at half-filling cause a discontinuous (non-KAM) breakdown of localization to diffusion, as opposed to a smooth behavior for random potential.

Short talks

Hamiltonian approach for optimization of phase-sensitive double-pumped parametric amplifiers.

A. Bednyakova (Novosibirsk), S. Medvedev (Novosibirsk)

In this work we applied a Hamiltonian formalism to reduce the equations of non-degenerate nonlinear four-wave mixing to the one-degree-of-freedom Hamiltonian equations with a three-parameter Hamiltonian. Thereby, a problem of signal amplification in a phase-sensitive double-pumped parametric fiber amplifier was reduced to a geometrical study of the phase portraits of the one-degree-of-freedom Hamiltonian system. For a symmetric case of equal pump powers and equal signal and idler powers at the fiber input, it has been shown that the theoretical maximum gain occurs on the extremal trajectories. However, to reduce the nonlinear interaction of the waves, we suggested to choose the separatrix as the optimal trajectory on the phase plane. Analytical expressions were found for the maximum amplification, as well as the length of the optical fiber and the relative phase of the interacting waves allowing this amplification. Using the proposed approach, we performed optimization of the phase-sensitive parametric amplifier. As a result, the optimal parameters of the phase-sensitive amplifier were found and the maximum possible signal amplification was realized in a broad range of signal wavelengths.

Релятивистский особый вихрь и неявные дифференциальные уравнения

А. Чупахин (Новосибирск), А. Янченко (Новосибирск)

В работе найдено и исследуется точное решение уравнений релятивистской газовой динамики [1] частично инвариантное относительно группы вращений $SO(3)$ в пространстве $\mathbb{R}^6(\overrightarrow{x};\overrightarrow{u})$ координат-скоростей. Это релятивистский аналог вихря Овсянникова в классической газовой динамике [2, 3].

Доказана теорема о представлении факторсистемы, описывающей это точное решение, в виде объединения не инвариантной подсистемы для функции определяющей отклонение вектора скорости от меридиана и инвариантной, определяющей термодинамические параметры, фактор Лоренца и радиальную компоненту скорости.

Доказано, что инвариантная подсистема после введения обобщенного потенциала $h = h(R)$ сводится к неявному обыкновенному дифференциальному уравнению

$$F(R,h,p) = q^{3/2} - R^2p(3m_0 + s_0\tfrac{p^2}{1+h^2})q + 3m_0^2R^4p^2q^{1/2} - m_0^3R^6p^3 = 0, \quad (1)$$

где $q(R,h,p) = R^2(R^2-1)p^2 - (1+h^2)^2, p = dh/dR,\ m0,\ s0$ — постоянные, характеризующие интенсивность вихря и энергию системы. Геометрическая теория таких уравнений, восходящая к Пуанкаре, представлена в [4]. Все инвариантные функции — термодинамические параметры, модуль касательной компоненты скорости и ее радиальная компонента — представляются через h и $dh = dR$. Уравнение (1) определено на некоторой алгебраической поверхности в пространстве 1-струй $\mathbb{R}^3(R,h,p)$. Это уравнение имеет особенности на кривой — криминанте и, вдобавок, сложенную особую точку на ней. Доказано существование решений двух типов, первое определено на конечном интервале, второе продолжается неограниченно. Исследовано многообразие ветвления решений этого уравнения, поведение интегральных кривых в зависимости от параметров задачи (энергия и закрутка потока). Дана физическая интерпретация найденных решений. При анализе уравнения (1) существенно используются пакеты символьных вычислений.

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Dynamical symmetries, coherent states and nonlinear realizations: The SO(2, 4) case with applications to plasma physics

D. Cirilo-Lombardo (Dubna)

Nonlinear realizations of the SO(2,4) group are discussed from the point of view of symmetries. Dynamical symmetry breaking is introduced. One linear and one quadraticmodel in curvature are constructed. Coherent states of the Klauder–Perelomov type are defined for both cases taking into account the coset geometry. A new spontaneous compactification mechanism is defined in the subspace invariant under the stability subgroup. The physical implications of the symmetry rupture in the context of nonlinear realizations and direct gauging are analyzed and briefly discussed.

Combinatorics and toric topology of fullerenes and Pogorelov polytopes

N. Erokhovets (Moscow)

A *Pogorelov polytope* is a combinatorial simple 3-polytope realizable in the Lobachevsky (hyperbolic) space as a bounded right-angled polytope [1]–[3]. It has no 3- and 4-gons.

Theorem 1([1],[2],[5]). A Pogorelov polytope may have any prescribed numbers of k-gons, $k \geq 7$. Any simple 3-polytope with only 5-, 6- and at most one 7-gon is a Pogorelov polytope. For any other prescribed numbers of k-gons, $k \geq 7$, there is an explicit construction of a Pogorelov and a non-Pogorelov polytopes.

For any mapping Λ from the set of faces of a Pogorelov polytope P to $\mathbb{Z}_2$ (or $\mathbb{Z}$) satisfying the condition that for any vertex $v = F_i \cap F_j \cap F_k$ the vectors $\Lambda(F_i), \Lambda(F_j), \Lambda(F_k)$ form a basis in $\mathbb{Z}_2$ (respectively $\mathbb{Z}$) toric topology associates a 3-dimensional manifold $R(P, \Lambda)$ with an action of $\mathbb{Z}_2^3$ called a *small cover* (respectively a 6-dimensional manifold $M(P, \Lambda)$ with an action of the compact torus T^3 called a *quasitoric manifold*). Small covers over Pogorelov polytopes are also known in hyperbolic geometry (see [1]), since they admit a hyperbolic structure.

Theorem 2 ([2]). The $\mathbb{Z}_2$-cohomology ring of $R(P, \Lambda)$ (respectively the $\mathbb{Z}$-cohomology ring of $M(P, \Lambda)$) uniquely defines the pair (P, Λ) up to a natural equivalence of pairs.

An example of Pogorelov polytopes is given by any *(mathematical) fullerene* – a simple convex 3-polytope with all facets 5- and 6-gons. Another example is given by a *k-barrel* (also called a *Löbell polytope*)– a polytope with surface glued from two patches, each consisting of a k-gon surrounded by 5-gons. Results by T. Inoue [4] imply that any Pogorelov polytope can be combinatorially obtained from k-barrels by a sequence of (s,k)-*truncations* (cutting off s subsequent edges of a k-gon by a single plane), $2 \leq s \leq k-4$, and *connected sums along k-gonal faces* (combinatorial analog of gluing two polytopes along k-gons orthogonal to adjacent facets). k-barrels are irreducible with respect to these operations.

Theorem 3([2]). Any Pogorelov polytope except for k-barrels can be obtained from the 5- or the 6-barrel by a sequence of $(2,k)$-truncations, $k \geqslant 6$, and connected sums with 5-barrels along 5-gons.

In the case of fullerenes we prove a stronger result. Let $(2,k;m_1,m_2)$-truncation be a $(2,k)$-truncation that cuts off two edges intersecting an m_1-gon and an m_2-gon by vertices different from the common vertex. There is an infinite family of connected sums of 5-barrels along 5-gons surrounded by 5-gons called $(5,0)$-*nanotubes*.

Theorem 4 ([2]). Any fullerene except for the 5-barrel and the $(5,0)$-nanotubes can be obtained from the 6-barrel by a sequence of $(2,6;5,5)$-, $(2,6;5,6)$-, $(2,7;5,6)$-, $(2,7;5,5)$-truncations such that all intermediate polytopes are either fullerenes or Pogorelov polytopes with facets 5-, 6- and at most one additional 7-gon adjacent to a 5-gon.

This result can not be literally extended to the latter class of polytopes. We prove that it becomes valid if we additionally allow connected sums with the 5-barrel and 3 new operations, which are compositions of $(2,6;5,6)$-, $(2,7;5,6)$-, and $(2,7;5,5)$-truncations. We generalize this result to the case when the 7-gon may be isolated from 5-gons [5].

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О минимальных триангуляциях трехмерных многообразий

Е. Фоминых (Челябинск)

В докладе будут представлены различные подходы к установлению минимальности триангуляций трехмерных многообразий.

Conformal invariance in 2D turbulence — the proof of Polyakov's conjecture

V. Grebenev (Novosibirsk), M. Waclawczyk, M. Oberlack

We study the statistical properties of the vorticity field in two-dimensional turbulence. The field is described in terms of the infinite Lundgren–Monin– Novikov (LMN) chain of equations for multi-point probability density functions (pdf's) of vorticity. We perform a Lie group analysis of the first equation in this chain using the direct method based on the canonical Lie- Backlund transformations devised for integro-differential equations. We analytically show that the conformal group is broken for the first LMN equation i.e. for the 1-point pdf at least for the inviscid case but the equation is still conformally invariant on the associated characteristic with zero-vorticity or the boundary of vortex clusters. Then, we demonstrate that this characteristic (the isoline of vorticity) is conformally transformed. We find this outcome coincides ith the numerical results about the conformal invariance of the statistics of zero-vorticity isolines, see e.g. Falkovich (2007 Russian Math. Surv. 63 497–510). We also show that the probability measure itself is conformally invariant. The conformal symmetry can be understood as a 'local scaling' and its traces in two-dimensional turbulence were already discussed in the literature, i.e. it was conjectured more than twenty years ago in Polyakov (1993 Nucl. Phys. B 396 367–85) and clearly validated experimentally in Bernard et al (2006 Nat. Phys. 2 124–8).

The results presented above are published in Grebenev et al (2017 J. Phys. A: Math. Theor. Vol. 50(43) 435502).

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On Fourier's law for linear chain of particles

M. Guzev (Vladivostok)

This report is linked with investigation of temperature distribution and heat flux in a one-dimensional chain of particles. The problem formulation of the work is motivated by investigations (see, e.g. [1, 2]) in which a homogeneous harmonic chain of particles was proposed to describe thermal effects in an ideal crystalline system. This model showed that in the stationary state the Fourier's law is not obeyed.

In our work we analyze thermal effects in a one-dimensional chain of particles for arbitrary time on the foundation of the exact solution of linear equations with initial stochastic conditions. It is shown that the spectral characteristics of the basis matrix are calculated through the Chebyshev polynomials. The constructed fundamental solution is written in the terms of the Bessel functions and generalizes Schrodinger solution for a harmonic infinite chain [3]. Different integral representations of the solution are obtained on the complex plane and with the help of the Laplace transformation. The exact solution is used to calculate temperature distribution and heat flux in the chain. We demonstrated the breakdown of the Fourier's law.

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Обобщение конструкции Смейла растягивающихся аттракторов

Н. Исаенкова (Нижний Новгород)

Смейл в 1967 году предложил метод построения диффеоморфизмов, имеющих растягивающиеся гиперболические аттракторы, с помощью растягивающих эндоморфизмов. Эта конструкция была реализована в явном виде с помощью растягивающего эндоморфизма окружности так, что в результате получается диффеоморфизм трехмерного полнотория в себя с соленоидальным одномерным растягивающимся аттрактором, который локально гомеоморфен произведению отрезка на канторово множество. Мы предлагаем обобщение этой конструкции, когда вместо растягивающего эндоморфизма окружности рассматривается А-эндоморфизм (вообще говоря, многомерного) тора. Изучаются типы транзитивных инвариантных множеств соответствующих диффеоморфизмов.

From analytic iteration to turbulence models

O. Kaptsov (Krasnoyarsk)

I will speak about finite groups acting by automorphisms of compact complex surfaces. We will see that these groups have certain boundedness properties similar to properties of finite subgroups of general linear groups. We will also consider in details automorphism groups of several types of non-projective surfaces, including Inoue and Kodaira surfaces.

This report consists of two parts. The first part deals with a problem of the analytic iteration. Given an analytic diffeomorphism f of a neighborhood of $0 \in \mathbb{R}^n$, the problem of analytic iteration is to find a analytic map $F : \mathbb{R} \times D \longrightarrow D$, where D is a open set in $\mathbb{R}^n$, which satisfies the following conditions:

$$F(t,0) = 0, \qquad F(0,x) = x, \qquad F(1,x) = f(x), \qquad F(t+s,x) = F(t,F(s,x)).$$

The history of this problem is presented in [1],[2]. We gave sufficient conditions for existence of a solution of the problem of analytic iteration in [3]. Moreover, we presented in [4] an area preserving analytic mapping f for which there is no formal series g such that

$$g \circ g = f.$$

This is an counterexample to theorem of Moser's about interpolation in [5].

At the second part of the report, we say about self-similar structures in the turbulent wake. Two-dimensional and axisymmetric models of the turbulent far wake are considered. The full symmetry groups for this models are derived. In particular, the models are invariant under two dilatations. Using the symmetry subgroup we reduce the correspondent systems of partial differential equations to the ordinary differential equations and obtain solutions with natural boundary conditions. Some scale-invariant solutions agree qualitatively with the experimental data.

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Subtwistor And Almost Hermitian Structures On Six-dimensional Sphere

E. Kornev (Kemerovo)

In the talk we define the Subtwistor Structures on homogeneous spaces, and using these structures obtain the three-parametric family of Almost Hermitian Structures on six-dimensional sphere. We discuss the integrability of these Almost Hermitian structures and state the conjecture of their non-integrability.

Finite-time stability in randomly driven classical and quantum systems.

D. Makarov (Vladivostok)

Low-dimensional nonlinear dynamical systems subjected to random driving basically exhibit ergodic chaos. However, regular motion does not cease suddenly, and remnants of stability can persist for relatively long timescales. In the present talk we consider the one-step Poincare map that allows one to identify phase space domains of finite-time Lyapunov stability under stochastic driving. It is shown that location of regular domains in the space of the action variable is almost the same for different realizations of the stochastic drive. We introduce the finite-time evolution operator being a quantum equivalent of the one-step Poincare map. Finite-time stability is reflected in spectral statistics of the operator. Various models demonstrating finite-time stability under random driving are considered.

Probabilistic boundaries of graphs, groups, and semigroups

A. Malyutin (Saint Petersburg)

Modern mathematics uses several dozen various concepts of asymptotic type boundaries (boundaries at infinity): Bowditch boundary, Dynkin exit-boundary, Feller boundaries, Floyd boundaries, Freudenthal's space of ends, Gromov boundary, Kuramochi boundary, Martin boundary, Morse boundary, Poisson–Furstenberg boundary, Roller boundary, etc. A prominent example here is geometric group theory, where the study of boundaries of this kind forms a central topic. We will discuss basic notions, examples, and recent developments in the theory of asymptotic boundaries of random walks.

An illustration of Action-angle duality arising from Hamiltonian reduction

I. Marshall (Moscow)

I will show how Hamiltonian reduction may often be used to generate pairs of systems in Action-Angle duality with one another, and present an example. This example is a pair of systems of Ruijsenaars type with BC_n symmetry. Both systems can be identified as limiting cases of the van Diejen system, and their flows can be shown to be complete by means of the reduction procedure.

Characteristic Lie algebras of the Klein-Gordon PDE

D. Millionshchikov (Moscow)

Consider the Klein-Gordon equation $u_{tt} - u_{zz} = f(u)$. It can be rewritten as $u_{xy} = f(u)$. Introduce a Lie algebra $\mathcal{L}$ of first-order differential operators of the form

$$X = \sum_{i=1}^{+\infty} P_i(u, u_1, u_2, \dots)\frac{\partial}{\partial u_i},\ P_i(u, u_1, u_2, \dots) \in C^\omega(\Omega)[u_1, u_2, \dots],$$

acting on the algebra $C^\omega(\Omega)[u_1, u_2, \dots]$ of polynomials in an infinite number of variables $u_1, u_2, \dots, u_n, \dots$ with coefficients in the algebra of analytic functions $C^\omega(\Omega)$ of the variable u on some open interval (domain) Ω. An operator $D = \frac{\partial}{\partial x}$ "total derivative with respect to x"is

$$D = u_1\frac{\partial}{\partial u} + u_2\frac{\partial}{\partial u_1} + u_3\frac{\partial}{\partial u_2} + \dots + u_{n+1}\frac{\partial}{\partial u_n} + \dots,$$

Definition ([3],[2]). Characteristic Lie algebra $\chi(f)$ of the Klein-Gordon equation is called a Lie algebra, generated by two operators X_0, X_1

$$X_0 = \frac{\partial}{\partial u}, X_1 = X(f) = f\frac{\partial}{\partial u_1} + D(f)\frac{\partial}{\partial u_2} + D^2(f)\frac{\partial}{\partial u_3} + \dots + D^{n-1}(f)\frac{\partial}{\partial u_n} + \dots$$

Theorem 1. The characteristic Lie algebra $\chi(\sinh u)$ of the *sinh*-Gordon equation $u_{xy}=\sinh u$ is a pro-solvable Lie algebra generated by three operators X_0', X_1', X_2' with relations

(1)
$$[X_0', X_1'] = X_1',\ [X_0, X_2'] = -X_2',\ [X_1', [X_1, [X_1', X_2']]] = 0,\ [X_2', [X_2', [X_2', X_1']]] = 0.$$

This in particular means that its commutant $\chi(\sinh u)^+$ is generated by X_1' and X_2' is isomorphic to the maximal nilpotent Lie subalgebra $N(A_1^{(1)})$ of the Kac-Moody $A_1^{(1)}$.

Theorem 2. The characteristic Lie algebra $\chi(e^u+e^{-2u})$ of the Tzitzeica equation $u_{xy} = e^u+e^{-2u}$ is pro-solvable and it can be defined by generators Y_0', Y_1', Y_2' and relations

$$\begin{gathered}[Y_0', Y_1'] = Y_1',\ [Y_0, Y_2'] = -2Y_2',\\ [Y_1', [Y_1', [Y_1', [Y_1', [Y_1', Y_2']]...] = 0,\ [Y_2', [Y_2', Y_1']] = 0.\end{gathered} \tag{2}$$

Its Lie commutant $\chi(e^u+e^{-2u})^+$ generated by Y_1', Y_2' is isomorphic to the maximal nilpotent subalgebra $N(A_2^{(2)})$ of the twisted Kac-Moody algebra $A_2^{(2)}$.

Other bases and commutation relations describing the structure of the characteristic Lie algebras of the sinh-Gordon and Tzitzeica equations were found in [1], [4], but the most important link with the Kac-Moody algebras $A_1^{(1)}, A_2^{(2)}$ was missed there.

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Foliations arising from configurations of vectors, and topology of nondegenerating leaf spaces

T. Panov (Moscow)

Let $V \cong \mathbb{R}^k$ be a k-dimensional real vector space, and let $\Gamma = \{\gamma_1, \ldots, \gamma_m\}$ be a sequence (a *configuration*) of m vectors in the dual space V^*. We consider the action of V on the complex space $\mathbb{C}^m$ given by

$$V \times \mathbb{C}^m \to \mathbb{C}^m,$$
$$(v, z) \mapsto (z_1 e^{\langle \gamma_1, v\rangle}, \ldots, z_m e^{\langle \gamma_m, v\rangle}).$$

This is a very classical dynamical system taking its origin in the works of Poincare. There is a well-known relationship between linear properties of the vector configuration Γ and the topology of the foliation of $\mathbb{C}^m$ by the orbits of (1). We systematise the existing knowledge on this relationship and proceed by analysing the topology of the nondegenerate leaf space using some recent constructions of toric topology.

The properties of Van der Pol - Duffing haemodynamics mathematical model for clinical applications.

D. Parshin (Novosibirsk), A. Cherevko (Novosibirsk), A. Chupakhin (Novosibirsk)

Current study concerns Van der Pol - Duffing (VDP) mathematical model of a cerebral haemodynamics with an aneurysm as well without one. The stop points of dynamical system for this equation were found, and their relation to the presence of an aneurysm in a vessel observed. The damping compartment of VDP equation was analysed and the role of this compartment for estimation of treatment quality has been discussed.

Miquel dynamics on circle patterns

S. Ramassamy (Ecole normale Superieure de Lyon, France)

Circle patterns are one of the ways to uniformize graphs on surfaces, by embedding them in such a way that every face admits a circumcircle. In this talk I will describe a discrete-time dynamical system on circle patterns with the combinatorics of the square grid, called Miquel dynamics. It is based on the classical Miquel's six circles theorem. I will present some properties of this dynamics which suggest its integrability.

Partly joint work with Alexey Glutsyuk (Ecole normale superieure de Lyon / Higher School of Economics).

Asymptotic solution for the linear one-dimensional surface waves with surface tension.

S. Sergeev (Moscow)

In the recent paper by S. Yu. Dobrokhotov and V. E. Nazaikinskii [1] the new approach for the asymptotic problems was made. This approach is based on the special type of the Lagrangian manifold and non-standart characteristics. As an example of such approach the Cauchy problem for the wave equation was considered.

In the present talk we implement this approach for the Cauchy problem with localized initial date for the one-dimensional surface waves over variable depth [2],[3]. The asymptotc solution of this problem can be built with the help of the Maslov's canonical operator [4]. The dispersion effects was compared with the surface tension and their mutual influnce on the wave profile.

Considered the case of significant effects of the surface tension and dispersive effects. It is shown that in this case the surface tension can crucially influence on the wave profile even if the dispersive effects prevalent over the surface tension.

Also following S. Yu. Dobrokhotov and V. E. Nazaikinskii the asymptotic profile of the head wave was studied. For the special case of the initial function in the form of the gaussian exponential such profile can be represented via Airy function.

The author wishes to express gratitude to S. Yu. Dobrokhotov for constant interest in the paper and valuable advice.

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Certain integrable systems of algebraic origin. Reductions and degenerations of Hitchin systems

O. Sheinman (Moscow)

A plane algebraic curve whose Newton polygone contains d integer points is completely determined by giving d points of the plane, the curve is passing through. Then its coefficients, regarded as functions of sets of coordinates of the points, are Poisson commuting with respect to any pair of coordinates corresponding to the same point. This has been observed by Babelon and Talon (2002). They reduced the statement to a special version of separation of variables. A result, more general in some respects, and less general in the others, is obtained by Enriquez and Rubtsov (2003). It follows as a particular case that the coefficients of the interpolation polynomial are Poisson commuting with respect to the interpolation data. We prove a general statement in frame of separation of variables explaining all these facts. It is as follows: any (non-degenerate) system of n smooth functions of n+2 variables yields an integrable system with n degrees of freedom. Apart from already mentioned, the examples include a version of Hermit interpolation polynomial, systems related to Weierstrass models of curves (= miniversal deformations of singularities). If the time admits, I'll explain how these questions arise in reductions and degenerations of Hitchin systems on hyperelliptic curves.

Automorphisms of complex surfaces.

C. Shramov (Moscow)

I will speak about finite groups acting by automorphisms of compact complex surfaces. We will see that these groups have certain boundedness properties similar to properties of finite subgroups of general linear groups. We will also consider in details automorphism groups of several types of non-projective surfaces, including Inoue and Kodaira surfaces.

Примеры модифицированных многообразий модулей специальных бор-зоммерфельдовых подмногообразий

Н. Тюрин (Москва)

Для любого односвязного компактного гладкого многообразия X с очень обильным расслоением $L \to X$ оказывается можно построить модифицированное многообразие модулей специальных бор - зоммерфельдовых подмногообразий $\tilde{M}_{SBS}(L)$. Такое модифицированное многообразие модулей всегда является конечномерным кэлеровым многообразием. Однако известные примеры показывают, что на самом деле такое многообразие имеет вид $P \backslash D$ где P — алгебраическое многообразие, а D — очень обильный дивизор. Цель доклада — проиллюстрировать двумя примерами нашу главную гипотезу, утверждающую что $\tilde{M}_{SBS}$ всегда имеет такой вид.

On the Riemann-Hilbert Problem for Difference and q-Difference Systems

I. Vyugin (Moscow)

We study an analogue of the classical Riemann-Hilbert problem stated for the classes of difference and q-difference systems. A generalization of Birkhoff's existence theorem is presented. We prove that for any admissible set of characteristic constants there exists a system $Y(z+1) = A(z)Y(z)$ or $Y(qz) = Q(z)Y(z)$, which has the given constants.

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Poster session

Стохастические динамические системы с инвариантами

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В теории динамических систем представляет значительный интерес задача построения дифференциальных уравнений исходя из траектории движения системы или известного множества ее первых интегралов.

В работе [1] были предложены методы решения таких задач для детерминированных, а также для стохастических динамических систем при наличии возмущений. В работе [2] были построены стохастические дифференциальные уравнения (СДУ) с винеровскими возмущениями, решения которых с вероятностью 1 находятся на заданном гладком многообразии. Была предложена методика тестирования численных методов на системах СДУ, явное аналитическое решение которых неизвестно, но известен первый интеграл. На построенных примерах было проведено сравнение восьми численных методов, в том числе обобщенных методов типа Розенброка, рассмотренных в статьях [3, 4].

В данной работе найдены аналитические решения построенных в [2] трех систем СДУ, траектории которых с вероятностью 1 находятся на заданном гладком многообразии (эллиптическом, гиперболическом и параболическом цилиндрах).

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An inverse problem for one nonlinear dynamical system of gene network modeling

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Аннотация. We show, that an inverse problem of determination of some parameters for one piecewise linear 3D dynamical system which describes gene networks functioning has a unique solution. Existence and uniqueness of a cycle in the phase portrait of this dynamical system is shown as well.

1. Direct problem Consider nonlinear dynamical systems which simulates functioning of some simple gene networks.

$$\frac{dx_1}{dt} = L(x_3) - kx_1; \quad \frac{dx_2}{dt} = L(x_1) - kx_2; \quad \frac{dx_3}{dt} = L(x_2) - kx_3. \qquad (1)$$

Here the non-negative variable x_j denotes concentration of the j-th substance in the gene network, negative terms kx_j correspond to the natural degradation of the substances, k is velocity of these degradations, same for all these species. The function $L(w)$ is monotinically decrasing, it describes the synthesis of the substanses and is defined as follows:

$$L(w) = Ak \quad \text{for} \quad 0 \leqslant w < \alpha; \quad L(w) = 0 \quad \text{for} \quad \alpha \leqslant w;$$

the positive coefficients A, k, and $\alpha < A$ here are constant. Similar dynamical systems in gene networks modeling were studied earlier in [1,2], see also references therein.

Lemma. $D^3 = [0, A] \times [0, A] \times [0, A]$ *is an invariant domain of the system* (1).

The planes $x_1 = \alpha$, $x_2 = \alpha$, $x_3 = \alpha$ decompose this invariant cube D^3 to 8 blocks $\{\varepsilon_*\}$ which we enumerate by binary multi-indices $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ as follows: for a given block ε, we put $\varepsilon_j = 0$ if for its points we have $x_j \leqslant \alpha$, and $\varepsilon_j := 1$ if for its points the opposite inequality $x_j > \alpha$ holds, see [2,3].

We say that the block ε has valency k, if trajectories of its points leave it through k of its faces exactly, see [3]. In our 3-dimensional case (1), the blocks $\{000\}$ and $\{111\}$ have valency 3, and all other blocks have valency 1, i.e., for each of these 6 blocks, trajectories of its points leave this block to exactly one of its incident blocks according to the diagram

$$\ldots \to \{001\} \to \{011\} \to \{010\} \to \{110\} \to \{100\} \to \{101\} \to \{001\} \to \ldots \quad (2)$$

For all interior points of D^3, their trajectories of the system (1) are piecewise linear with vertices on the planes $x_1 = \alpha$, $x_2 = \alpha$, $x_3 = \alpha$. These trajectories can be described explicitly. Denote by W the union of the blocks listed in the diagram (2), this is also an invariant domain of the system (1).

Theorem 1. *If $A > 0$, then W contains a unique cycle C which is symetric with respect to the cyclic permutation of the coordinates $x_1 \to x_2 \to x_3 \to x_1$.*

This theorem solves the direct problem, i.e. if we know the parameters A, α, and k, then W contains a unique cycle of the system (1), and this cycle can be described explicitly. Because of symmetry of the system (1), in order to find this cycle, it is sufficient to consider only two first steps in the diagram (2). The period τ of this cycle satisfies the equation $\tau = 3(t_1 + t_2)$, where t_1 is the travel time of the cycle through the block $\{001\}$, similarly, t_2 is its travel time in the block $\{011\}$.

2. Inverse problem. Let τ be the period of this cycle C, which can be measured in experiments, and let the parameters A and k be known as well. Also, we assume that we can measure the times t_1 and t_2, i.e., we can measure the times between the peacks of the graphs of the functions $x_1(t)$, $x_2(t)$, $x_3(t)$. At the same time these functions $x_j(t)$ are not assumed to be known.

Is it possible to find the unknown parameter α?

Theorem 2. *Let the parameter A and the times τ, t_1 in the system* (1) *be known, and $\alpha \in (0, A)$ is unknown. Then the inverse problem of determination of α has unique solution.*

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On existence of cycles in some asymmetric dynamical systems

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Аннотация. We find sufficient conditions of existence of cycles in phase portraits of nonlinear dynamical systems which describe functioning of some asymmetric circular gene networks.

We study phase portraits of dynamical systems which simulate functioning of circular gene networks described by circular schemes of the following type:

$$\ldots \Rightarrow p_{n,s_n} \cdots\blacktriangleleft m_1 \Rightarrow p_{1,1} \Rightarrow \ldots \to p_{1,s_1} \cdots\blacktriangleleft m_2 \Rightarrow p_{2,1} \Rightarrow \ldots \Rightarrow p_{2,s_2} \cdots\blacktriangleleft m_3 \Rightarrow \ldots$$

Here $\cdots\blacktriangleleft$ denote negative feedbacks, and $\Rightarrow$ correspond to positive feedbacks, m_j denote concentrations of mRNA, and $p_{j,s}$ are concentrations of protein components corresponding to intermdiate stages in a circular gene network. Consider as an example of such gene network model the following asymmetric dynamical system:

$$\frac{dx_1}{dt} = -k_1x_1 + f_1(x_9);\ \frac{dx_2}{dt} = \mu_2x_1 - k_2x_2;\ \frac{dx_3}{dt} = \mu_3x_2 - k_3x_3;$$
$$\frac{dx_4}{dt} = \mu_4x_3 - k_4x_4;\ \frac{dx_5}{dt} = -k_5x_5 + f_5(x_4);\ \frac{dx_6}{dt} = \mu_6x_5 - k_6x_6; \quad (1)$$
$$\frac{dx_7}{dt} = \mu_7x_6 - k_7x_7;\ \frac{dx_8}{dt} = -k_8x_8 + f_8(x_7);\ \frac{dx_9}{dt} = \mu_9x_8 - k_9x_9.$$

Here $f_1,\ f_5,\ f_8$ are smooth positive monotonically decreasing functions, which describe negative feedbacks, equations 2,3,4,6,7, and 9 correspond to positive feedbacks; μ_j, ν_j, k_j are positive coefficients, $j = \overline{1,9}$. Some particular 6D cases of these systems with $s_1 = s_2 = s_3 = 1$, $n = 3$ were studied in [1-3].

Let $A_j := \frac{f_j(0)}{k_j}$, if $j = 1,5,8$; $A_j := \frac{\mu_j}{\nu_j}A_{j-1}$, if $j \neq 1,5,8$; and $Q^9 := \prod_{j=1}^{j=9}[0, A_j] \subset \mathbb{R}^9_+$.

Lemma 1. a. Q^9 *is an invariant domain of the system* (1).

b. *The system* (1) *has one and only one equilibrium point* $S_0 \in Q^9$.

Denote the coordinates of this point as follows:

$$S_0 = (x_1^0; x_2^0; x_3^0; x_4^0; x_5^0; x_6^0; x_7^0; x_8^0; x_9^0),$$

and consider 9 hyperplanes parallel to the coordinates ones, and containing the point S_0. So, Q^9 is decomposed by these hyperplanes to 2^9 blocks which we shall enumerate by binary multi-indices $\{\varepsilon_1, \ldots \varepsilon_9\}$. Here $\varepsilon_j = 0$ if in this block $x_j \le x_j^0$, and $\varepsilon_j = 1$ if for the points of this block the opposite inequality $x_j > x_j^0$ holds.

Lemma 2. *For any pair* E_1, E_2 *of adjacent blocks, trajectories of all points of their common face* $F = E_1 \cap E_2$ *pass either from* E_1 *to* E_2*, or from* E_2 *to* E_1.

We denote these transitions as $E_1 \to E_2$, respectively $E_2 \to E_1$, and we say that the *valency of a block* $E = \{\varepsilon_1, \ldots \varepsilon_9\}$ equals ℓ if the number of its ajacent blocks E_j such that $E \to E_j$, equals ℓ.

The following circular diagram is composed by all the blocks whose valency equals 1, and the arrows in this diagram show possible transitions from block to block.

$$\{000011101\} \to \{000011100\} \to \{100011100\} \to \{110011100\} \to \{111011100\} \to$$
$$\{111111100\} \to \{111101100\} \to \{111100100\} \to \{111100000\} \to \{111100010\} \to$$
$$\{111100011\} \to \{011100011\} \to \{001100011\} \to \{000100011\} \to \{000000011\} \to$$
$$\{000010011\} \to \{000011011\} \to \{000011111\} \to \{000011101\} \to \dots \qquad (2)$$

Denote by W the union of the blocks listed in the diagram (2). This is an invariant domain of the system (1). Consider the linearization matrix of the system (1) at the point S_0

$$M_0 = \begin{pmatrix} -k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -q_1 \\ \mu_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_4 & -k_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -q_5 & -k_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_6 & -k_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_7 & -k_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -q_8 & -k_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_9 & -k_9 \end{pmatrix}$$

Here $-q_j = \frac{df_j}{dp_{j-1}}$ for $j = 1, 5, 8$. The characteristic polynomial of M_0 has the form

$$P(\lambda) = \prod_{j=1}^{j=9} (k_j + \lambda) + a^9, \quad \text{where} \quad a^9 := \prod_{j=1,5,8} q_j \prod_{j \neq 1,5,8} \mu_j.$$

Lemma 3. *For sufficiently large values of the parameter a, the equilibrium point S_0 is hyperbolic.*

Theorem. *If S_0 is a hyperbolic point of the system* (1)*, then the invariant domain W contains at least one cycle of the cyctem* (1)*, and this cycle travels from block to block according to the diagram* (2)*.*

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Twofold Cantor sets

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If a self-similar set does not possess weak separation property (WSP) it can have unpredicted and surprising properties, especially if it satisfies some additional regularity conditions.

As it was shown in 2006 by Tetenov [1], a self-similar structure $(\gamma, \mathcal{S})$ on a Jordan arc γ in $\mathbb{R}^2$, which does not satisfy WSP, is possible only if γ is a line segment, and two self-similar structures $(\gamma_1, \mathcal{S}_1)$ and $(\gamma_2, \mathcal{S}_2)$, which do not satisfy WSP, are isomorphic iff the homeomorphism $\varphi: \gamma_1 \to \gamma_2$, which induces the isomorphism of these structures, is a linear map. So the question arises, does such rigidity phenomenon occur for self-similar sets whose dimension is smaller than 1?

A system $\mathcal{S} = \{S_1, \ldots, S_m\}$ of contraction similarities satisfy the WSP iff $\mathrm{Id} \notin \overline{\mathrm{G}^{-1}\mathrm{G} \setminus \mathrm{Id}}$, where G is a semigroup generated by system $\mathcal{S}$.

Let K and K' be the attractors of systems $\mathcal{S} = \{S_1, \ldots, S_m\}$ and $\mathcal{S}' = \{S'_1, \ldots, S'_m\}$ of contraction similarities. We say that a homeomorphism $f: K \to K'$ realises the isomorphism of self-similar structures $(K, \mathcal{S})$ and $(K', \mathcal{S}')$, if $f(S_i(x)) = S'_i(f(x))$ for any $x \in K$, $i \in \{1, ..., m\}$.

We define a system $\mathcal{S}_{pq} = \{S_1, S_2, S_3, S_4\}$ of contraction similarities on $[0,1]$ by the equations $S_1(x) = px$, $S_2(x) = qx$, $S_3(x) = px + 1 - p$, $S_4(x) = qx + 1 - q$, where the contraction ratios $p, q \in (0, 1/2)$. Let K_{pq} be the attractor of the system $\mathcal{S}_{pq}$. Denote $A = S_3(K_{pq}) \cup S_4(K_{pq})$.

Notice that:

$$K_{pq} \setminus \{0\} = \bigcup_{m,n=0}^{\infty} S_1^m S_2^n(A). \tag{1}$$

If for any $m, n \in \mathbb{N}$, $S_1^m(A) \cap S_2^n(A) = \varnothing$, then the union in (1) is disjoint, and we call K_{pq} a *twofold Cantor set.*

First, we prove the following properties of twofold Cantor sets:

Theorem 1. *Let K_{pq} be twofold Cantor set. Then:*
(i) its Hausdorff dimension d satisfies the equation $p^d + q^d - (pq)^d = 1/2$.
(ii) there is a topological limit $\lim\limits_{t \to +\infty} tK_{pq} = [0, +\infty)$.
(iii) the system $\mathcal{S}_{pq}$ does not have WSP.

Theorem 2. *Let K_{pq}, $K_{p'q'}$ be twofold Cantor sets. Then:*
(i) There is a homeomorphism $f: K_{pq} \to K_{p'q'}$, which realises the isomorphism of self-similar structures $(K_{pq}, \mathcal{S}_{pq})$ and $(K_{p'q'}, \mathcal{S}_{p'q'})$.
(ii) If $(p, q) \neq (p', q')$, then f cannot be extended to a homeomorphism of $[0,1]$ to itself.
(iii) f has an extension to a homeomorphism $\tilde{f}: \mathbb{C} \to \mathbb{C}$, but if $\tilde{f}(S_i(z)) = S'_i(\tilde{f}(z))$ for any $z \in \mathbb{C}$, $i \in I$, then $(p, q) = (p', q')$.

Finally we want to show that such sets do exist. Our problem is how to find those p, q, for which $S_1^m(A) \cap S_2^n(A) = \varnothing$ for all $m, n \in \mathbb{N}$, so we analyse how large is the set of those pairs (p, q) which do not possess such property.

First we consider the set $\Delta_{mn}(p) = \{q \in (0, 1/16) : S_1^m(A) \cap S_2^n(A) \neq \varnothing\}$ for $m, n \in \mathbb{N}$, $p \in (0, 1/16)$. We prove that $\dim_H \Delta_{mn}(p) < 1$ using a bunch of two statements, General Position Theorem and Displacement Theorem, which are initially used in [2]. Therefore for any $p \in (0, 1/16)$ the set $\bigcup\limits_{m,n=0}^{\infty} \Delta_{mn}(p)$ has zero 1-dimensional Lebesgue measure in $\{p\} \times (0, 1/16)$. This allows us to show the following:

Theorem 3. *The set $\mathcal{K}$ of those $(p, q) \in \mathcal{V} = (0, 1/16)^2$, for which K_{pq} is a twofold Cantor set, has full measure in $\mathcal{V}$, and its complement is uncountable and dense in $\mathcal{V}$.*

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On chaos theory application in intrusion detection system

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The problem of intrusion detection in sensor networks is considered. We observe that chaotic behavior can be inherent in traffic dynamics in the normal case. Chaos occurs not only from collisions produced by traffic sources behavior but also from the propagation medium behavior. Some intrusions decrease the chaos degree (traffic flooding, black hole), while others increase this one (selective forwarding, burst injections). Assuming that a concrete intrusion is present, it is reasonable to demand that a chaos degree deviates from the normal case. As a chaos degree metrics, the greatest Lyapunov exponent is computed. In this research we focus on sensor systems, however the offered approach can be also adopted for other types of networks. The research was supported by the National Research Foundation of Korea (NRF-2017R1D1A3B03030386).

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