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THE SHOCK FRONT ASYMPTOTICS IN THE LINEAR PROBLEM OF SHOCK WAVE

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Abstract. The article is a direct continuation of the previous papers of the authors, devoted to the linear problem of shock wave disturbance. The asymptotics of shock front disturbance in time in the stability case at the presence of the pre-shock initial perturbations is studied. The two principal terms of the asymptotics are determined and studied. We want to point out that the first term of the asymptotics of first degree was not discovered before.

Keywords: shock wave, shock disturbance, shock front, Fourier transform.

1. Introduction

This article is a direct continuation of the papers [1, 2, 3] of the authors, devoted to the classical linear problem of shock wave disturbance, i.e. to the propagation of small perturbations of hydrodynamical quantities and small deformations of the shock wave front against the background of the basic (piecewise-constant) flow. In those papers, we constructed the solution of initial value problem and inspected some of its properties.

This paper is devoted to the asymptotics of the shock front disturbance in time. On the one hand, this asymptotics depends on the problem's stability/instability/neutral stability: in the case of instability, the shock front disturbance tends to infinity; in the case of neutral stability, it oscillates in time; and in the case of stability, it, generally speaking, tends to zero. Here we consider the asymptotics just in stability case. On the other hand, even in stability case the shock front asymptotics, in general, depends on the initial value's asymptotics at $x \to \infty$, where x is the variable, orthogonal to the undisturbed shock front, see below. But

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the question is: what is (and is there) the maximum velocity (maximum degree) of the shock front tendency to zero? In other words, what are (and are there) the first terms of shock front asymptotics, the same (by degree) for any initial values tending to zero at $x \to \infty$ sufficiently fast?

A series of papers is devoted to this problem, see e.g. [4, 5, 6, 7]. The general conclusion is: this degree exists and it is equal to 3/2, i.e. the shock front tends to zero as $O(t^{-3/2})$. But the exact asymptotic terms of this degree weren't founded, and, moreover, some contradictions to the experimental data were noticed. We note, the asymptotics was studied for specific form of initial perturbation, usually for plane wave with respect to the tangent to the undisturbed shock variable y.

The works [8, 9], devoted to the problem of flow around the wedge (Courant – Friedrichs hypothesis), should be marked additionally, where the idea of exact solutions construction were applied for the first time in the shock wave problem.

In this paper, we present principal asymptotic terms of the shock front, namely, asymptotic terms up to the second degree, i.e. up to the terms, tending to zero as $O(t^{-2})$. There are two of them: the first term of the first degree $O(t^{-1})$ (we note: this term was not found out before), and the second term just of the degree 3/2. Here we consider the shock front perturbations, generated by the pre-shock perturbations of initial data only.

This paper is organized as follows. In section 2, we formulate the problem and give necessary terms, notations and formulas. In section 3, we derive the representation of shock front disturbance, convenient for further study. The section 4 is devoted to the asymptotics itself. In section 5, we formulate the obtained results, discuss them and in particular analyze some examples. At last, in Summary (section 6), we shortly state the general results of paper, and in Appendix (section 7), we bring and prove some necessary auxiliary statements.

2. Formulation of the problem, notations, formulas

Here we remind the conventional terminology and notations, relating to the classical problem of shock wave disturbance, see e.g. the widely known monograph [10] and the references therein.

We consider the sought functions of variables (x, y, t): density ρ, velocity $U = (U_x, U_y)$, pressure p, entropy s, specific (per unit volume) internal energy ϵ, temperature T, enthalpy w, and specific energy e. These functions are involved into the functional relations $\rho = \rho(p, s)$ (equation of state), $\epsilon = \epsilon(\rho, s)$, $w = \epsilon + p/\rho$, $e = \rho\epsilon + \rho|U|^2/2$, and the Euler equations: continuity equation

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho U) = 0,$$

momentum conservation law

$$\rho\left(\frac{\partial U}{\partial t} + (U, \nabla)U\right) + \nabla p = 0,$$

and energy conservation law

$$\frac{\partial e}{\partial t} + \mathrm{div}(U(e + p)) = 0.$$

We consider discontinuous solutions. Let the surface of the discontinuity (shock wave or shock front or simply shock) has the form $x = f(y, t)$, where f is the sought function. At $x = f(y,t)$, we have the standard Rankine-Hugoniot conditions

$$[\rho U_x] = f'_t[\rho] + f'_y[\rho U_y]\,,$$
$$[\rho U_x^2] + [p] = f'_t[\rho U_x] + f'_y[\rho U_y U_x]\,,$$
$$[\rho U_x U_y] = f'_t[\rho U_y] + f'_y[\rho U_y^2] + f'_y[p]\,,$$
$$[U_x(e+p)] = f'_t[e] + f'_y[U_y(e+p)]\,,$$

where, as usual, the square brackets denote the jump in a quantity across the discontinuity.

We linearize the problem on a partial (basic) solution. As the basic solution, we use a piecewise-constant solution with a jump on the plane $x = 0$, i.e. the basic shock front is $x = f^0 \equiv 0$ (the superscript 0 denotes the basic solution),

$$\rho^0 = \begin{cases} \rho^+, & x < 0, \\ \rho^-, & x > 0, \end{cases} \qquad \rho^\pm = \text{const}, \quad [\rho^0] = \rho^- - \rho^+,$$

similarly, $U^0 = (u_x^\pm, u_y)$ ($u_y^+ = u_y^-$ according to Rankine-Hugoniot conditions), p^0, s^0. We accept some assumptions having clear physical grounds (see [1, 2, 3, 4, 5, 6, 7, 10]):

$$s^- > s^+, \quad p^- > p^+, \quad \rho^- > \rho^+, \quad u_x^+ > c^+, \quad u_x^- < c^-,$$

where $c^\pm = \sqrt{1/\rho'_p(p^\pm, s^\pm)}$ is the sound velocity for the basic solution. It is these assumptions, that ensures the well-posedness of linearized Cauchy problem.

We introduce the pre- and post-shock Mach numbers $M^\pm = u_x^\pm / c^\pm$ and the ratio of sound velocities $R = c^-/c^+$. According to conditions above, $[\rho^0] > 0$, $M^+ > 1$, $M^- < 1$. Further we (for short) often denote M^-, c^-, u_x^- as simply M, c, u_x respectively.

After the linearization of the original problem (Euler equations and Rankine-Hugoniot conditions) on the basic solution, we obtain for the variations of sought quantities

$$\delta\rho = \delta\rho^\pm\,, \quad \delta U = \delta U^\pm = (\delta U_x^\pm, \delta U_y^\pm)\,, \quad \delta s = \delta s^\pm\,, \quad \pm x \leqslant 0,$$

and for the shock front disturbance $f(y,t)$ the system of linear differential equations at $x \neq 0$ and the boundary conditions at $x = 0$. In papers [1, 2], we constructed the solution of the initial value problem for this system of differential equations and boundary conditions, in particular obtain the expression for shock front disturbance $f(y,t)$.

Our goal in this paper is to determine the principal terms of the shock front disturbance $f(y,t)$ asymptotics at $t \to \infty$. The obtained in [1, 2] formula for $f(y,t)$ (Eq. (3) in this paper) is a starting point in our considerations. So, we have to introduce and describe all terms, that have been used in mentioned formula. We note that the absolutely similar notations, formulas and considerations one may find in authors works [11, 12], devoted to other (non-classical) shock wave problems.

We introduce the sought vector in a form

$$G = G^\pm(x,y,t) = \begin{pmatrix} \delta\rho \\ \rho^0 \delta U_x / c \\ \rho^0 \delta U_y / c \\ r\delta s \end{pmatrix}$$

i.e., we reduce all sought quantities to the dimension of density. We denote the initial values as $G_0(x,y) = G_0^\pm(x,y) = G^\pm(x,y,0)$. In this paper, we consider only the effect of the pre-shock initial perturbations $G_0^+(x,y)$ at $x<0$, i.e. we assume $G_0^-(x,y) = G_0(x,y)\Big|_{x>0} \equiv 0$, and so we denote G_0^+ as simply G_0.

For solving the problem, we employ the so called one-sided Fourier transform [1, 2, 3, 11, 12]:

$$\hat{G} = \hat{G}^{\pm}(\xi,\eta,\omega) = \frac{1}{(2\pi)^3} \iiint\limits_{\substack{t>0\\ \pm x<0}} G^{\pm}(x,y,t) e^{-i(\xi x + \eta y - \omega t)}\, dxdydt\,,$$

accordingly,

$$\hat{G}_0(\xi,\eta) = \frac{1}{(2\pi)^2} \iint\limits_{x<0} G_0(x,y) e^{-i(\xi x+\eta y)}\, dxdy\,, \tag{1}$$

and inverse transform

$$G_0(x,y) = \iint\limits_{\mathbb{R}^2} \hat{G}_0(\xi,\eta) e^{i(\xi x+\eta y)}\, d\xi d\eta\,. \tag{2}$$

The properties of the introduced Fourier transform are described in articles [3] and [11] (in Appendix). In particular, the vector $\hat{G}_0(\xi,\eta)$ is analytic with respect to ξ in upper half-plane $\operatorname{Im}\xi > 0$.

In works [1, 2, 3] and similarly in [11, 12], we compute the solution $\hat{G}(\xi,\eta,\omega)$ from the initial data $\hat{G}_0$ and the basic solution. Thus, introducing the variable $\alpha = (\omega - \eta u_y)/\sqrt{c^2-u_x^2}$ instead of ω, we actually in [1, 2] obtain the formula for shock front disturbance

$$f(y,t) = f(Y,T) = \tag{3}$$

$$= -\frac{i}{[\rho^0]} \iint\limits_{\mathbb{R}^2} \varphi(\alpha, \operatorname{sign}(\eta)) A \sum_{j=1}^{4} \Omega_j(\alpha,\operatorname{sign}(\eta)) \hat{G}_0(|\eta|\xi_j(\alpha),\eta) e^{i\eta Y}\, e^{-i\alpha|\eta|T}\, d\eta d\alpha,$$

$$Y = y - tu_y, \quad T = t\sqrt{c^2-u_x^2};$$

where

- the vector-row $\varphi(\alpha,\pm1)$ is

$$\varphi(\alpha,\pm1) = \frac{R}{M^+}\cdot\frac{g_{11}\alpha + g_{12}\sqrt{\alpha^2-1} \pm g_{13}}{Y_{11}\alpha^2 + Y_{12}\alpha\sqrt{\alpha^2-1}+Y_{13}},$$

$$g_{11} = (1\,;\,-M\,;\,0\,;\,-1)\,,\quad g_{12} = (M\,;\,-1\,;\,0\,;\,-M)\,,\quad g_{13} = \sqrt{1-M^2}(0\,;\,0\,;\,1\,;\,0)\,,$$

$$Y_{11} = \frac{R}{M^+}\left(MD_0\left(M - \frac{M^+}{R}\right) - 1 - M^2\right),$$

$$Y_{12} = \frac{RM}{M^+}\left(MD_0\left(M - \frac{M^+}{R}\right) - 2\right),$$

$$Y_{13} = M, \quad D_0 = r^-(c^-)^2/(\rho^- T^-);$$

- the constant matrix A is $A = (A^-)^{-1}A^+A_0^+$,

$$A^{\pm} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ u_x^{\pm} & c^{\pm} & 0 & 0 \\ 0 & 0 & c^{\pm} & 0 \\ 0 & u_x^{\pm}c^{\pm} & 0 & \rho^{\pm}T^{\pm}/r^{\pm} \end{pmatrix}, \quad A_0^+ = \begin{pmatrix} u_x^+ & c^+ & 0 & 0 \\ c^+ & u_x^+ & 0 & -c^+ \\ 0 & 0 & u_x^+ & 0 \\ 0 & 0 & 0 & u_x^+ \end{pmatrix},$$

where $r^{\pm} = \rho_s'(p^{\pm}, s^{\pm})$ is the isobaric derivative of density and $T^{\pm} = e_s'(\rho^{\pm}, s^{\pm}, U^{\pm})/\rho^{\pm}$ is the temperature for basic solution;

- the functions $\xi_j(\alpha)$, $j = \overline{1,4}$ are

$$\xi_1(\alpha) = \frac{M^+\sqrt{1-M^2}R\alpha + \sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{(M^+)^2 - 1},$$

$$\xi_2(\alpha) = \frac{M^+\sqrt{1-M^2}R\alpha - \sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{(M^+)^2 - 1},$$

$$\xi_3(\alpha) = \xi_4(\alpha) = \frac{\sqrt{1-M^2}\,R\alpha}{M^+};$$

- the matrices $\Omega_j(\alpha, \pm 1)$, $j = \overline{1,4}$ are

$$\Omega_j(\alpha, s) = e_j(\alpha, s) \otimes g_j(\alpha, s)/s_j(\alpha), \quad j = \overline{1,4}, \quad s = \pm 1,$$

$$e_1(\alpha, s) = \begin{pmatrix} 1 \\ -\xi_1/Q_1 \\ -s/Q_1 \\ 0 \end{pmatrix}, \quad e_2(\alpha, s) = \begin{pmatrix} 1 \\ -\xi_2/Q_2 \\ -s/Q_2 \\ 0 \end{pmatrix},$$

$$e_3(\alpha, s) = \frac{1}{\sqrt{\xi_3^2 + 1}} \begin{pmatrix} 0 \\ s \\ -\xi_3 \\ 0 \end{pmatrix}, \quad e_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$g_1(\alpha, s) = \frac{1}{2}\left(1\,;\, -\xi_1/Q_1\,;\, -s/Q_1;\, -1\right),$$

$$g_2(\alpha, s) = \frac{1}{2}\left(1\,;\, -\xi_2/Q_2\,;\, -s/Q_2\,;\, -1\right),$$

$$g_3(\alpha, s) = \frac{1}{\sqrt{\xi_3^2 + 1}}\left(0\,;\, s\,;\, -\xi_3\,;\, 0\right),\; g_4 = (0\,;\, 0\,;\, 0\,;\, 1),$$

$$s_1(\alpha) = \frac{\sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{RQ_1},$$

$$s_2(\alpha) = -\frac{\sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{RQ_2},$$

$$s_3 = s_4 = u_x^+,$$

$$Q_1 = \frac{\sqrt{1-M^2}R\alpha + M^+\sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{R((M^+)^2 - 1)},$$

$$Q_2 = \frac{\sqrt{1-M^2}R\alpha - M^+\sqrt{(1-M^2)R^2\alpha^2 + (M^+)^2 - 1}}{R((M^+)^2 - 1)}.$$

We should describe some properties of used in basic formula (3) quantities, which have been established in [1, 2, 3]. So:

- the function $\sqrt{\alpha^2 - 1}$ is analytic in complex plane with the cut along the segment $[-1, 1]$, where $\sqrt{\alpha^2 - 1} = \alpha + O(1/\alpha)$, $\alpha \to \infty$, and on the sides of the cut $\sqrt{\alpha^2 - 1} = \pm i\sqrt{1 - \alpha_0^2}$, $\alpha = \alpha_0 \pm i0$, $|\alpha_0| < 1$;
- in the expression of $\varphi(\alpha, \pm 1)$ the function $\sqrt{\alpha^2 - 1}$ at $|\alpha| < 1$ has the value from the upper half-plane $\sqrt{\alpha^2 - 1} = i\sqrt{1 - \alpha^2}$;
- in the case of stability the denominator of φ doesn't have zeroes, i.e. the function $\varphi(\alpha, \pm 1)$ is analytic in the upper half-plane $\operatorname{Im}\alpha > 0$;

- at real α $\varphi(-\alpha,-1)=-\overline{\varphi(\alpha,1)}$ and $\varphi(\alpha,1)$ is real at $|\alpha|>1$;
- at real α $\xi_2(-\alpha)=-\xi_1(\alpha)$ and $\xi_j(-\alpha)=-\xi_j(\alpha)$, $j=\overline{3,4}$, accordingly $\Omega_1(-\alpha,-1)=\Omega_2(\alpha,1)$, $\Omega_2(-\alpha,-1)=\Omega_1(\alpha,1)$ and $\Omega_j(-\alpha,-1)=\Omega_j(\alpha,1)$, $j=\overline{3,4}$;
- the vector
$$\sum_{j=1}^{4}\Omega_j(\alpha,\mathrm{sign}(\eta))\hat{G}_0(|\eta|\xi_j(\alpha),\eta)$$
is analytic with respect to α in upper half-plane $\mathrm{Im}\,\alpha>0$.

3. The shock front disturbance representation

In this section, we give the representation of shock front disturbance $f(y,t)$, more convenient for asymptotic investigation. We substitute the expression of Fourier transform (1) in the basic formula (3):

$$f(Y,T)=-\frac{i}{[\rho^0]}\iint\limits_{\mathbb{R}^2}\varphi(\alpha,\mathrm{sgn}(\eta))A\sum_{j=1}^{4}\Omega_j(\alpha,\mathrm{sgn}(\eta))$$
$$\times\left[\frac{1}{4\pi^2}\iint\limits_{x<0}G_0(x,y)e^{-i(|\eta|\xi_j(\alpha)x+\eta y)}\,dxdy\right]e^{i\eta Y}\,e^{-i\alpha|\eta|T}\,d\eta d\alpha$$
$$=-\frac{i}{4\pi^2[\rho^0]}\iint\limits_{x<0}\Phi(x,y,Y,T)G_0(x,y)\,dxdy,$$

where

$$\Phi=\iint\limits_{\mathbb{R}^2}\varphi(\alpha,\mathrm{sgn}(\eta))A\sum_{j=1}^{4}\Omega_j(\alpha,\mathrm{sgn}(\eta))e^{-i|\eta|(\alpha T+\xi_j(\alpha)x)}e^{i\eta(Y-y)}\,d\eta d\alpha$$
$$=\int\limits_{\alpha\in\mathbb{R}}\varphi(\alpha,1)A\sum_{j=1}^{4}\Omega_j(\alpha,1)\left[\int\limits_{0}^{\infty}e^{-i\eta(\alpha T+\xi_j(\alpha)x+y-Y)}\,d\eta\right]d\alpha$$
$$+\int\limits_{\alpha\in\mathbb{R}}\varphi(\alpha,-1)A\sum_{j=1}^{4}\Omega_j(\alpha,-1)\left[\int\limits_{-\infty}^{0}e^{-i\eta(-\alpha T-\xi_j(\alpha)x+y-Y)}\,d\eta\right]d\alpha=\Phi_1+\Phi_2.$$

But according to the theorem 2 (Appendix, section 7) the equation $\alpha T+\xi_j(\alpha)x=Y-y$ doesn't have complex roots at $j=\overline{3,4}$ and may have no more than one complex root in upper or lower half-plane at $j=\overline{1,2}$, we denote this root (if any) as $\alpha_j^{\pm}(T,x,Y-y)$, $j=\overline{1,2}$, $\pm\mathrm{Im}\,\alpha_j^{\pm}>0$. Since $\varphi(\alpha,1)$ and $\sum\Omega_j(\alpha,1)e^{-i\eta\xi_j(\alpha)x}$ are analytic in upper half-plane [1, 2], then according to the theorem 3 (Appendix, section 7), we obtain

$$\Phi_1=2\pi\int\limits_{\alpha\in\mathbb{R}}\varphi(\alpha,1)A\sum_{j=1}^{4}\Omega_j(\alpha,1)\delta(\alpha T+\xi_j(\alpha)x+y-Y)\,d\alpha$$
$$+2\pi\sum_{j=1}^{2}\mathrm{Res}\frac{\varphi(\alpha,1)A\Omega_j(\alpha,1)}{\alpha T+\xi_j(\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^{+}}.$$

In integral Φ_2 we make the change of variable from α to $-\alpha$, and since evidently $\varphi(-\alpha,-1)$ and $\sum \Omega_j(-\alpha,-1)e^{i\eta\xi_j(-\alpha)x}$ are analytic in lower half-plane, then according to the theorem 3 (Appendix, section 7) again, we obtain

$$\Phi_2 = \int\limits_{\alpha\in\mathbb{R}} \varphi(-\alpha,-1)A\sum_{j=1}^{4}\Omega_j(-\alpha,-1)\left[\int\limits_{-\infty}^{0} e^{-i\eta(\alpha T-\xi_j(-\alpha)x+y-Y)}\,d\eta\right]d\alpha$$

$$= 2\pi\int\limits_{\alpha\in\mathbb{R}} \varphi(-\alpha,-1)A\sum_{j=1}^{4}\Omega_j(-\alpha,-1)\delta(\alpha T-\xi_j(-\alpha)x+y-Y)\,d\alpha$$

$$-2\pi\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(-\alpha,-1)A\Omega_j(-\alpha,-1)}{\alpha T-\xi_j(-\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^-}.$$

Using the properties of $\varphi(\alpha,s)$ and $\Omega_j(\alpha,s)$, $\xi_j(\alpha)$ at real α, we come to the representation

$$\Phi_2 = -2\pi\int\limits_{\alpha\in\mathbb{R}} \overline{\varphi(\alpha,1)}A\sum_{j=1}^{4}\Omega_j(\alpha,1)\delta(\alpha T+\xi_j(\alpha)x+y-Y)\,d\alpha$$

$$-2\pi\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(-\alpha,-1)A\Omega_j(-\alpha,-1)}{\alpha T-\xi_j(-\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^-},$$

and eventually

$$\Phi = \Phi_1+\Phi_2 = 2\pi\int\limits_{\alpha\in\mathbb{R}} (\varphi(\alpha,1)-\overline{\varphi(\alpha,1)})A\sum_{j=1}^{4}\Omega_j(\alpha,1)\delta(\alpha T+\xi_j(\alpha)x+y-Y)\,d\alpha$$

$$+2\pi\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(\alpha,1)A\Omega_j(\alpha,1)}{\alpha T+\xi_j(\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^+} -2\pi\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(-\alpha,-1)A\Omega_j(\alpha,1)}{\alpha T+\xi_j(\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^-}.$$

But the function $\varphi(\alpha,1)$ is real at $|\alpha|>1$, so finally

$$\Phi = 4\pi i\int\limits_{-1}^{1}(\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)\delta(\alpha T+\xi_j(\alpha)x+y-Y)\,d\alpha$$

$$+2\pi\left[\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(\alpha,1)A\Omega_j(\alpha,1)}{\alpha T+\xi_j(\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^+} -\sum_{j=1}^{2}\operatorname{Res}\frac{\varphi(-\alpha,-1)A\Omega_j(\alpha,1)}{\alpha T+\xi_j(\alpha)x+y-Y}\Big|_{\alpha=\alpha_j^-}\right]$$

$$= \Phi_0(x,y,Y,T)+\Psi_0(x,y,Y,T),$$

and

$$f(Y,T) = -\frac{i}{4\pi^2[\rho^0]}\iint\limits_{x<0}\Phi_0 G_0(x,y)\,dxdy - \frac{i}{4\pi^2[\rho^0]}\iint\limits_{x<0}\Psi_0 G_0(x,y)\,dxdy = I+J.$$

In the first term I we return to the variables (ξ,η), i.e. we substitute the inverse Fourier transform (2), hence

$$I = -\frac{i}{4\pi^2[\rho^0]}\iint\limits_{\mathbb{R}^2}\left[\iint\limits_{x<0}\Phi_0 e^{i(x\xi+y\eta)}\,dxdy\right]\hat{G}_0(\xi,\eta)\,d\xi d\eta,$$

but

$$\iint\limits_{x<0} \Phi_0 e^{i(x\xi+y\eta)}\,dxdy = 4\pi i \int\limits_{-1}^{1} (\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)$$

$$\times \left[\iint\limits_{x<0} \delta(\alpha T + \xi_j(\alpha)x + y - Y)e^{i(x\xi+y\eta)}\,dxdy\right] d\alpha$$

$$= 4\pi i \int\limits_{-1}^{1} (\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)\left[\int\limits_{-\infty}^{0} e^{i(x\xi-\alpha\eta T-\xi_j^+(\alpha)\eta x+\eta Y)}\,dx\right] d\alpha$$

$$= 4\pi i \int\limits_{-1}^{1} (\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)e^{i\eta(Y-\alpha T)}\left[\int\limits_{0}^{\infty} e^{-ix(\xi-\xi_j^+(\alpha)\eta)}\,dx\right] d\alpha,$$

therefore

$$I = \frac{1}{\pi[\rho^0]}\int\limits_{\eta\in\mathbb{R}}\int\limits_{-1}^{1}(\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)e^{i\eta(Y-\alpha T)}$$

$$\times\left[\int\limits_{\xi\in\mathbb{R}}\hat{G}_0(\xi,\eta)\left(\int\limits_{0}^{\infty} e^{-ix(\xi-\xi_j(\alpha)\eta)}\,dx\right)d\xi\right]d\alpha d\eta,$$

and since $\hat{G}_0(\xi,\eta)$ is analytic with respect to ξ in upper half-plane, and the equation $\xi_j(\alpha) = \xi \in \mathbb{R}$ doesn't have complex roots, then according to the theorem 3 (Appendix, section 7) again, we obtain the final representation

$$f(Y,T)$$

$$(4)\qquad = \frac{2}{[\rho^0]}\int\limits_{\eta\in\mathbb{R}}\int\limits_{-1}^{1}(\operatorname{Im}\varphi(\alpha,1))A\sum_{j=1}^{4}\Omega_j(\alpha,1)e^{i\eta(Y-\alpha T)}\hat{G}_0(\xi_j(\alpha)\eta,\eta)d\alpha d\eta$$

$$+\iint\limits_{x<0}\Psi_0 G_0(x,y)\,dxdy = I + J.$$

Note that: first,

$$\operatorname{Im}\varphi(\alpha,1) = \sqrt{1-\alpha^2}\,\frac{R}{M^+}\,\frac{(Y_{11}\alpha^2+Y_{13})g_{12} - Y_{12}\alpha(\alpha g_{11}+g_{13})}{(Y_{11}\alpha^2+Y_{13})^2 + Y_{12}^2\alpha^2(1-\alpha^2)} = \sqrt{1-\alpha^2}\varphi_0(\alpha),$$

and second, since $\varphi(\alpha,\pm1)$ is bounded, then $|\Psi_0(x,y,Y,T)| \leqslant M = \text{const}$, and it follows from the theorem 2 (Appendix, section 7), that $\Psi_0 \equiv 0$ at $|x| < \beta T$ or $|Y-y| > \beta_0 T$, $\beta,\beta_0 = \text{const}$ (there are no complex roots $\alpha^\pm$ at these x and y).

4. Asymptotic conclusion

Hereinafter the M are definite constants independent of the G_0.

First we consider the second term J in representation (4). According to the remark at the end of previous section, we have

$$J = \int_{-\infty}^{-\beta T} \int_{|y-Y|<\beta_0 T} \Psi_0 G_0 \, dy \, dx \Longrightarrow |J| \leqslant M \int_{-\infty}^{-\beta T} \int_{|y-Y|<\beta_0 T} |G_0(x,y)| \, dy \, dx.$$

The first condition C1. Let $|G_0(x,y)| \leqslant M/|x|^4$ at $x \to -\infty$.

Under the first condition C1 fulfilment, we obtain

$$|J| \leqslant M \int_{-\infty}^{-\beta T} \frac{\beta_0 T}{x^4} \, dx \leqslant \frac{M}{T^2}.$$

So, $f(Y,T) = I$ up to the $O(1/T^2)$.

We present

$$I = \frac{2}{[\rho^0]} \int_{\eta \in \mathbb{R}} \int_{-1}^{1} \sqrt{1-\alpha^2} \varphi_0(\alpha) A \sum_{j=1}^{4} \Omega_j(\alpha,1) e^{i\eta(Y-\alpha T)} \hat{G}_0(0,0) d\alpha d\eta$$

$$+ \frac{2}{[\rho^0]} \int_{\eta \in \mathbb{R}} \int_{-1}^{1} \sqrt{1-\alpha^2} \varphi_0(\alpha) A \sum_{j=1}^{4} \Omega_j(\alpha,1) e^{i\eta(Y-\alpha T)} \left[\hat{G}_0(\xi_j(\alpha)\eta, \eta) - \hat{G}_0(0,0) \right] d\alpha d\eta$$

$$= I_0 + I_1.$$

The first summand I_0 we compute immediately: as [1]

$$\sum_{j=1}^{4} \Omega_j(\alpha,1) = E$$

and [14]

$$\int_{\eta \in \mathbb{R}} e^{i\eta(Y-\alpha T)} \, d\eta = 2\pi \delta(Y - \alpha T),$$

whence

$$I_0 = \frac{4\pi}{[\rho^0]} \int_{-1}^{1} \sqrt{1-\alpha^2} \varphi_0(\alpha) A \delta(Y-\alpha T) d\alpha \cdot \hat{G}_0(0,0)$$

$$= \frac{1}{\pi T [\rho^0]} \varphi_{00}(Y/T) A \widetilde{G}_0, \tag{5}$$

where

$$\varphi_{00}(\alpha) = \begin{cases} \varphi_0(\alpha)\sqrt{1-\alpha^2}, & |\alpha| < 1, \\ 0, & |\alpha| > 1, \end{cases}$$

$$\widetilde{G}_0 = 4\pi^2 \hat{G}_0(0,0) = \iint_{x<0} G_0(x,y) \, dx dy.$$

For the second summand I_1, we use the asymptotic formula (12) in the theorem 4 (Appendix, section 7). First we estimate the residue term.

The second condition C2. Let

$$M(G_0) = \sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}\\ j=\overline{1,4}}} \left(\left| \int\limits_{\eta\in\mathbb{R}} \frac{\hat{G}_0(\xi_j(\alpha)\eta,\eta) - \hat{G}_0(0,0)}{\eta^2} e^{i\eta B} d\eta \right| \right.$$

$$\left. + \left| \int\limits_{\eta\in\mathbb{R}} \frac{(\hat{G}_0)'_\xi(\xi_j(\alpha)\eta,\eta)}{\eta} e^{i\eta B} d\eta \right| + \left| \int\limits_{\eta\in\mathbb{R}} (\hat{G}_0)''_{\xi\xi}(\xi_j(\alpha)\eta,\eta) e^{i\eta B} d\eta \right| \right) < \infty.$$

Lemma 1 (the equivalent form of the condition C2)**.** *In initial variables* (x, y) *the condition C2 has a form*

$$M(G_0) = \sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}\\ j=\overline{1,4}}} \left(\left| \iint\limits_{x<0} G_0(x,y)\left(|B| - |B - \xi_j(\alpha)x - y|\right) dxdy \right| \right.$$

$$\left. + \left| \iint\limits_{x<0} xG_0(x,y)\operatorname{sign}(B - \xi_j(\alpha)x - y)\, dxdy \right| + \left| \int\limits_{-\infty}^{0} x^2 G_0(x, B - \xi_j(\alpha)x)\, dx \right| \right) < \infty.$$

Proof. Indeed, by substituting the Fourier transform (1) in condition C2, and using the well-known formulas [14]

$$\int\limits_{\eta\in\mathbb{R}} e^{i\eta Z}\, d\eta = 2\pi\delta(Z), \qquad \int\limits_{\eta\in\mathbb{R}} \frac{e^{i\eta Z}}{\eta}\, d\eta = \pi i \operatorname{sgn}(Z), \qquad \int\limits_{\eta\in\mathbb{R}} \frac{e^{i\eta Z}}{\eta^2}\, d\eta = -\pi|Z|,$$

we have

$$\int\limits_{\eta\in\mathbb{R}} \frac{\hat{G}_0(\xi_j(\alpha)\eta,\eta) - \hat{G}_0(0,0)}{\eta^2} e^{i\eta B} d\eta$$

$$= \frac{1}{4\pi^2} \iint\limits_{x<0} G_0(x,y) \left[\int\limits_{\eta\in\mathbb{R}} \frac{e^{i\eta(B-\xi_j(\alpha)x-y)} - e^{i\eta B}}{\eta^2}\, d\eta \right] dxdy$$

$$= \frac{1}{4\pi} \iint\limits_{x<0} G_0(x,y)\left(|B| - |B - \xi_j(\alpha)x - y|\right) dxdy;$$

$$\int\limits_{\eta\in\mathbb{R}} \frac{(\hat{G}_0)'_\xi(\xi_j(\alpha)\eta,\eta)}{\eta} e^{i\eta B} d\eta$$

$$= -\frac{i}{4\pi^2} \iint\limits_{x<0} xG_0(x,y) \left[\int\limits_{\eta\in\mathbb{R}} \frac{e^{i\eta(B-\xi_j(\alpha)x-y)}}{\eta}\, d\eta \right] dxdy$$

$$= \frac{1}{4\pi} \iint\limits_{x<0} xG_0(x,y)\operatorname{sgn}(B - \xi_j(\alpha)x - y)\, dxdy;$$

$$\int\limits_{\eta\in\mathbb{R}} (\hat{G}_0)''_{\xi\xi}(\xi_j(\alpha)\eta,\eta) e^{i\eta B} d\eta$$

$$= -\frac{1}{4\pi^2} \iint\limits_{x<0} x^2 G_0(x,y) \left[\int\limits_{\eta\in\mathbb{R}} e^{i\eta(B-\xi_j(\alpha)x-y)}\,d\eta \right] dxdy$$
$$= -\frac{1}{2\pi} \iint\limits_{x<0} x^2 G_0(x,y)\delta(B-\xi_j(\alpha)x-y)\,dxdy$$
$$= -\frac{1}{2\pi} \int\limits_{-\infty}^{0} x^2 G_0(x, B-\xi_j(\alpha)x)\,dx.$$

Lemma 2. *Let*

$$\varphi(\alpha,\eta) = \frac{2}{[\rho^0]}\varphi_0(\alpha)A\sum_{j=1}^{4}\Omega_j(\alpha,1)\left[\hat{G}_0(\xi_j(\alpha)\eta,\eta) - \hat{G}_0(0,0)\right],$$

$V(\eta) = \eta T$ *and (see theorem 4, Appendix, section 7)*

$$M(\varphi,V) = \sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}}} \left(\left| \int\limits_{\eta\in\mathbb{R}} \frac{\varphi'_\alpha(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta \right| + \left| \int\limits_{\eta\in\mathbb{R}} \frac{\varphi''_{\alpha\alpha}(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta \right| \right.$$
$$\left. + \left| \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta \right| \right).$$

If the condition C2 is fulfilled, then $M(\varphi,V) \leqslant M\cdot M(G_0)/T^2$.

Indeed, evidently we have $|\varphi_0(\alpha)|+|\varphi'_0(\alpha)|+|\varphi''_0(\alpha)| \leqslant M$, $|\alpha|\leqslant 1$, and the same relates to the $\Omega_j(\alpha,1)$, $\xi_j(\alpha)$, $j=\overline{1,4}$ [1], and so the lemma statement obviously follows from the condition C2.

Now we may apply the theorem 4 (Appendix, section 7) to

$$I_1 = \frac{2}{[\rho^0]} \int\limits_{\eta\in\mathbb{R}} \int\limits_{-1}^{1} \sqrt{1-\alpha^2}\varphi_0(\alpha)A\sum_{j=1}^{4}\Omega_j(\alpha,1)e^{i\eta(Y-\alpha T)}$$
$$\times \left[\hat{G}_0(\xi_j(\alpha)\eta,\eta) - \hat{G}_0(0,0)\right] d\alpha d\eta =$$
$$= \int\limits_{\eta\in\mathbb{R}} \int\limits_{-1}^{1} \sqrt{1-\alpha^2}\varphi(\alpha,\eta)e^{iV(\eta)(B-\alpha)}d\alpha d\eta, \quad V(\eta)=\eta T, \quad B = Y/T,$$

and obtain the asymptotic formula (12):

$$I_1 = \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B+1)}e^{-i\,\mathrm{sign}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta$$
$$- \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(1,\eta)e^{iV(\eta)(B-1)}e^{i\,\mathrm{sign}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta + O\left(\frac{1}{T^2}\right)$$
$$= \frac{f_{-1}(Y+T) + f_1(Y-T)}{T\sqrt{T}} + O\left(\frac{1}{T^2}\right), \tag{6}$$

where

$$f_{\pm1}(Z) = \mp\sum_{j=1}^{4}\frac{\sqrt{2\pi}\psi_j(\pm1)}{i[\rho^0]}\int\limits_{\eta\in\mathbb{R}}\frac{\left[\hat{G}_0(\xi_j(\pm1)\eta,\eta)-\hat{G}_0(0,0)\right]e^{i\eta Z}e^{\pm i\,\mathrm{sign}(\eta)\pi/4}}{\eta\sqrt{|\eta|}}d\eta,$$

$\psi_j(\pm1) = \varphi_0(\pm1)A\Omega_j(\pm1,1)$, $j=\overline{1,4}$.

We compute the functions $f_{\pm1}(Z)$ in terms of initial variables (x,y). We start from the $f_1(Z)$. Substituting the Fourier transform (1) in

$$J_j(Z) = \int\limits_{\eta\in\mathbb{R}}\frac{\left[\hat{G}_0(\xi_j(1)\eta,\eta)-\hat{G}_0(0,0)\right]e^{i\eta Z}e^{i\,\mathrm{sign}(\eta)\pi/4}}{\eta\sqrt{|\eta|}}d\eta,\quad j=\overline{1,4},$$

and using the formula (11) (lemma 3, Appendix, section 7), we have

$$\begin{aligned}J_j(Z) &= \frac{1}{4\pi^2}\iint\limits_{x<0}G_0(x,y)\left[e^{-i\pi/4}\int\limits_{-\infty}^{0}\frac{e^{-i\eta(\xi_j(1)x+y)}-1}{\eta\sqrt{-\eta}}e^{i\eta Z}\,d\eta\right.\\ &\quad\left.+e^{i\pi/4}\int\limits_{0}^{\infty}\frac{e^{-i\eta(\xi_j(1)x+y)}-1}{\eta\sqrt{\eta}}e^{i\eta Z}\,d\eta\right]dxdy\\ &= \frac{1}{4\pi^2}\iint\limits_{x<0}G_0(x,y)\\ &\quad\times\left[2i\,\mathrm{sgn}(Z-\xi_j(1)x-y)\sqrt{\pi|Z-\xi_j(1)x-y|}\right.\\ &\quad\times\left(e^{i(1+\mathrm{sgn}(Z-\xi_j(1)x-y))\pi/4}+e^{-i(1+\mathrm{sgn}(Z-\xi_j(1)x-y))\pi/4}\right)\\ &\quad\left.-2i\,\mathrm{sgn}(Z)\sqrt{\pi|Z|}\left(e^{i(1+\mathrm{sgn}(Z))\pi/4}+e^{-i(1+\mathrm{sgn}(Z))\pi/4}\right)\right]dxdy.\end{aligned}$$

But

$$\mathrm{sgn}(Z)\left(e^{i(1+\mathrm{sgn}(Z))\pi/4}+e^{-i(1+\mathrm{sgn}(Z))\pi/4}\right) = -2\theta(-Z),$$

where $\theta(Z)$ is a Heaviside function, i.e. if we denote (for simplicity)

$$\sqrt{Z} = \theta(Z)\sqrt{|Z|} = \begin{cases}\sqrt{Z}, & Z>0,\\ 0, & Z<0,\end{cases}$$

then

$$J_j(Z) = -\frac{i\sqrt{\pi}}{\pi^2}\iint\limits_{x<0}G_0(x,y)\left[\sqrt{\xi_j(1)x+y-Z}-\sqrt{-Z}\right]dxdy,\quad j=\overline{1,4},$$

whence

$$\begin{aligned}f_1(Z) &= -\sum_{j=1}^{4}\frac{\sqrt{2\pi}\psi_j(\pm1)}{i[\rho^0]}J_j(Z)\\ &= \frac{\varphi_0(1)\sqrt{2}}{\pi[\rho^0]}A\iint\limits_{x<0}\left[\sum_{j=1}^{4}\Omega_j(1,1)\sqrt{\xi_j(1)x+y-Z}-\sqrt{-Z}\right]G_0(x,y)\,dxdy.\end{aligned}$$

Absolutely similar

$$f_{-1}(Z) = \frac{\varphi_0(-1)\sqrt{2}}{\pi[\rho^0]} A \iint\limits_{x<0} \left[\sum_{j=1}^{4} \Omega_j(-1,1)\sqrt{Z - \xi_j(-1)x - y} - \sqrt{Z} \right] G_0(x,y)\, dxdy.$$

5. General results, discussion, examples

Now we summarize the results of previous section in general statement. We substitute the variables $Y = y - u_y t$, $T = t\sqrt{c^2 - u_x^2}$ in formulas (5), (6).

Theorem 1 (general asymptotic theorem)**.** *Let $G_0(x,y)$ satisfy the conditions:*

C1: $|G_0(x,y)| \leqslant M/|x|^4$ *at* $x \to -\infty$.

C2: $$M(G_0) = \sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}\\ j=\overline{1,4}}} \left(\left| \iint\limits_{x<0} G_0(x,y)\left(|B| - |B - \xi_j(\alpha)x - y|\right) dxdy \right| \right.$$

$$\left. + \left| \iint\limits_{x<0} xG_0(x,y)\,\mathrm{sign}(B - \xi_j(\alpha)x - y)\, dxdy \right| + \left| \int\limits_{-\infty}^{0} x^2 G_0(x, B - \xi_j(\alpha)x)\, dx \right| \right) < \infty,$$

or in equivalent form

$$M(G_0) = \sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}\\ j=\overline{1,4}}} \left(\left| \int\limits_{\eta\in\mathbb{R}} \frac{\hat{G}_0(\xi_j(\alpha)\eta, \eta) - \hat{G}_0(0,0)}{\eta^2} e^{i\eta B} d\eta \right| \right.$$

$$\left. + \left| \int\limits_{\eta\in\mathbb{R}} \frac{(\hat{G}_0)'_\xi(\xi_j(\alpha)\eta,\eta)}{\eta} e^{i\eta B} d\eta \right| + \left| \int\limits_{\eta\in\mathbb{R}} (\hat{G}_0)''_{\xi\xi}(\xi_j(\alpha)\eta,\eta) e^{i\eta B} d\eta \right| \right) < \infty.$$

Then the shock front disturbance has a form

$$f(y,t) = \frac{I_0(y,t)}{t} + \frac{I_1(y,t)}{t\sqrt{t}} + O\left(\frac{1}{t^2}\right),$$

where the first term of asymptotics is

$$I_0 = \frac{1}{2\pi[\rho^0]\sqrt{c^2 - u_x^2}}\, \varphi_{00}\left(\frac{y/t - u_y}{\sqrt{c^2 - u_x^2}} \right) A \iint\limits_{x<0} G_0(x,y)\, dxdy, \tag{7}$$

$$\varphi_{00}(\alpha) = \begin{cases} \varphi_0(\alpha)\sqrt{1-\alpha^2}, & |\alpha| < 1, \\ 0, & |\alpha| > 1; \end{cases}$$

and the second term is

$$I_1 = \frac{\sqrt{2}\left(f_1(y - u_y t - t\sqrt{c^2 - u_x^2}) + f_{-1}(y - u_y t + t\sqrt{c^2 - u_x^2}) \right)}{\pi[\rho^0](c^2 - u_x^2)^{3/4}}, \tag{8}$$

$$f_{\pm1}(Z) = \varphi_0(\pm1) A \iint\limits_{x<0} \left[\sum_{j=1}^{4} \Omega_j(\pm1,1)\sqrt{\xi_j(1)x \pm (y - Z)} - \sqrt{\mp Z} \right] G_0(x,y)\, dxdy,$$

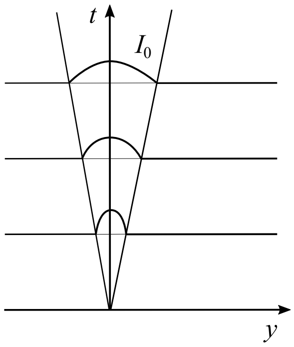

Fig. 1: The first term I_0 plots at different t for subsonic post-shock basic flow, $u_y^2 + u_x^2 < c^2$, $u_y > 0$.

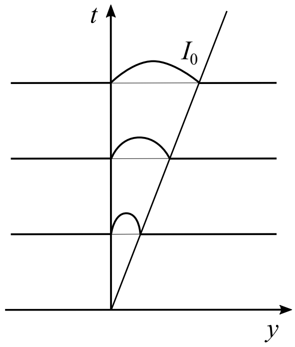

Fig. 2: The first term I_0 plots for sonic post-shock basic flow, $u_y^2 + u_x^2 = c^2$, $u_y > 0$.

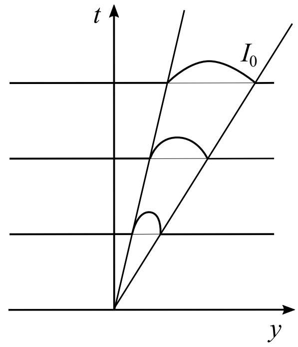

Fig. 3: The first term I_0 plots for supersonic post-shock basic flow $u_y^2 + u_x^2 > c^2$, $u_y > 0$.

where

$$\sqrt{Z} = \theta(Z)\sqrt{|Z|} = \begin{cases} \sqrt{Z}, & Z > 0, \\ 0, & Z < 0. \end{cases} ;$$

or in equivalent form

$$f_{\pm 1}(Z) = \pm i\pi^{3/2}\varphi_0(\pm 1)A \times$$
$$\times \sum_{j=1}^{4} \Omega_j(\pm 1, 1) \int\limits_{\eta \in \mathbb{R}} \frac{\left[\hat{G}_0(\xi_j(\pm 1)\eta, \eta) - \hat{G}_0(0,0)\right] e^{i\eta Z} e^{\pm i\, sign(\eta)\pi/4}}{\eta\sqrt{|\eta|}} d\eta.$$

Let us discuss the physical sense of the found asymptotic terms. The first term (7) is of the first degree $O(t^{-1})$, it almost doesn't depend on the initial data G_0, only through the multiplier (the vector-column)

$$\widetilde{G}_0 = \iint\limits_{x<0} G_0(x, y)\, dxdy,$$

and the function (vector-row) φ_{00} depends on one variable y/t only and doesn't vanish only on the segment $y/t \in [u_y - \sqrt{c^2 - u_x^2}, u_y + \sqrt{c^2 - u_x^2}]$ or $y \in [u_y t - t\sqrt{c^2 - u_x^2}, u_y t + t\sqrt{c^2 - u_x^2}]$. The ends of this segment at increasing t are moving in opposite directions, if the post-shock basic flow as a whole is subsonic $u_y^2 + u_x^2 < c^2$, and in the same direction, if the post-shock basic flow is supersonic $u_y^2 + u_x^2 > c^2$. The general form of I_0 plots for different t is conditionally depicted in fig. 1, 2, 3.

The second term is of the degree $3/2$, it is a sum of two one-dimensional waves $f_1(y - u_y t - t\sqrt{c^2 - u_x^2})$ and $f_{-1}(y - u_y t + t\sqrt{c^2 - u_x^2})$, moving along the y-axis with the speed $u_y \pm \sqrt{c^2 - u_x^2}$, again in opposite directions at subsonic post-shock flow and in the same direction at supersonic post-shock flow. The form of these waves depends on the initial data $G_0(x, y)$ essentially.

The general sense of the conditions C1 and C2 in theorem 1 is that the initial data $G_0(x, y)$ decreases (tends to zero) sufficiently fast at $x \to -\infty$. Thus, it is easy to see, that (for example) it is enough, if $|G_0(x, y)| < M/((1 + |x|^p)(1 + |y|^q))$, $p \geqslant 4$, $q > 2$; or even $G_0(x, y) = \delta(x - x_0)G_0(y)$, $|G_0(y)| \leqslant M/(1 + |y|^p)$, $p > 2$. But, generally speaking, the condition $G_0(x, y) \to 0$ at $y \to \infty$ is not necessary.

Thus, as separate example we consider the plane wave with respect to y: $G_0(x,y) = G_0(x)e^{i\eta_0 y}$, $\eta_0 \neq 0$. It is the form of the initial perturbation, that has been usually investigated before. For these examples the sufficient condition for C1 and C2 fulfilment is $|G_0(x)| \leqslant M/(1+x^4)$. But in this case

$$\iint\limits_{x<0} G_0(x,y)\,dxdy = 0,$$

here we assume the generalized meaning of the integral:

$$\int\limits_{-\infty}^{\infty} e^{i\eta_0 y}\,dy = \delta(\eta_0) = 0, \quad \eta_0 \neq 0.$$

Therefore the first term of asymptotics vanishes, and it is the reason, why this first term has not been found out before. The second term of asymptotics for these functions is the sum of plane waves:

$$I_1 = H_1 e^{i\eta_0(y-u_y t - t\sqrt{c^2-u_x^2})} + H_{-1} e^{i\eta_0(y-u_y t + t\sqrt{c^2-u_x^2})},$$

$$H_{\pm 1} = \mp \frac{\sqrt{2\pi}\varphi_0(\pm 1)e^{\pm i\operatorname{sgn}(\eta_0)\pi/4}}{i[\rho^0]\eta_0\sqrt{|\eta_0|}} A \sum_{j=1}^{4} \Omega_j(\pm 1, 1) \int\limits_{-\infty}^{0} e^{\mp i\xi_j(1)\eta_0 x} G_0(x)\,dx = \text{const}.$$

6. Summary

In this paper, we investigate the asymptotics in time of the shock front disturbance, generated by the initial pre-shock perturbations of the basic (piecewise-constant) solution, when the initial perturbations decrease (tend to zero) sufficiently fast at $x \to -\infty$. We find out that there are two principal asymptotic terms up to the terms of second degree $O(t^{-2})$, namely, the first term of the first degree $O(t^{-1})$ and the second term of degree 3/2, i.e. $O(t^{-3/2})$.

The first term is the function of one variable $\psi(y/t)$, defined (non-vanished) in the angle $y/t \in [u_y - \sqrt{c^2-u_x^2}, u_y + \sqrt{c^2-u_x^2}]$. It depends on the initial data $G_0(x,y)$ only through the multiplier

$$\widetilde{G}_0 = \iint\limits_{x<0} G_0(x,y)\,dxdy.$$

This term has not been found out before, perhaps because the examples studied before gave zero (vanishing) term $\widetilde{G}_0 = 0$.

The second term is a sum of two one-dimensional waves $f_1(y - u_y t - t\sqrt{c^2-u_x^2})$ and $f_{-1}(y - u_y t + t\sqrt{c^2-u_x^2})$, where the functions $f_{\pm 1}$ essentially depends on the initial data.

7. Appendix

Theorem 2. *We consider the roots of equation*

$$\alpha T + x\xi_j(\alpha) = Y - y, \quad j = \overline{1,4}. \tag{9}$$

At $j = \overline{3,4}$ the equation (9) *has only one real root. At $j = \overline{1,2}$ this equation may have as real, as complex roots, but complex roots exist only at $|x| > T\beta$ and $|Y-y| < T\beta_0$, and each equation* (9) *for $j=1$ or $j=2$ has no more than one* *complex root in upper or lower half-plane. Here the constants β, β_0 depend only on basic solution.*

Proof. For $j=\overline{3,4}$ the statement is obvious since $\xi_j(\alpha)=h\alpha$, $h=\sqrt{1-M^2}\,R/M^+$.

For $j=\overline{1,2}$ we have $\xi_j(\alpha)=A\alpha\pm\sqrt{B^2\alpha^2+C}$,

$$A=\frac{M^+R\sqrt{1-M^2}}{(M^+)^2-1}>\frac{R\sqrt{1-M^2}}{(M^+)^2-1}=B>0,\quad C=\frac{1}{(M^+)^2-1}>0,$$

and equation (9) reduces to the quadratic equation

$$\alpha^2((T+xA)^2-B^2x^2)-2\alpha Y_0(T+xA)+Y_0^2-x^2C=0,\quad Y_0=Y-y.$$

This equation has complex conjugated roots, when the discriminant is negative, i.e.

$$Y_0^2<\frac{C(B^2x^2-(T+Ax)^2)}{B^2}.$$

First, it follows that $B^2x^2-(T+Ax)^2>0$, so $-T/(A-B)<x<-T/(A+B)$ and $T/(A+B)<|x|<T/(A-B)$. Hence $\beta=1/(A+B)$. Second,

$$|Y_0|<|x|\sqrt{C}<\beta_0T,\quad \beta_0=\frac{\sqrt{C}}{A-B}.$$

At last, as roots are conjugated, then each equation (9) for $j=\overline{1,2}$ has no more that one root in upper or lower half-plane.

Theorem 3. *Let the functions $\varphi(z)$, $\psi(z)$ be analytic in upper half-plane and $\varphi(z)\to 0$, $\psi(z)\to\infty$, $\psi'(z)\to const\neq 0$ at $z\to\infty$, and let $\psi(z)$ is real at real z and has the finite number of roots z_k, $k=\overline{1,n}$ in the upper half-plane. Then*

$$\int\limits_{-\infty}^{\infty}\varphi(z)\left[\int\limits_0^{\infty}e^{-i\psi(z)\eta}d\eta\right]dz=2\pi\int\limits_{-\infty}^{\infty}\varphi(z)\delta(\psi(z))\,dz+2\pi\sum_k \operatorname{Res}\frac{\varphi(z)}{\psi(z)}\Big|_{z=z_k}.$$

Similarly for analytic in lower half-plane functions $\varphi(z)$, $\psi(z)$ we have

$$\int\limits_{-\infty}^{\infty}\varphi(z)\left[\int\limits_{-\infty}^{0}e^{-i\psi(z)\eta}d\eta\right]dz=2\pi\int\limits_{-\infty}^{\infty}\varphi(z)\delta(\psi(z))\,dz-2\pi\sum_k \operatorname{Res}\frac{\varphi(z)}{\psi(z)}\Big|_{z=z_k},$$

$\psi(z_k)=0$, $\operatorname{Im}z_k<0$.

Proof. Using the well-known formulas [14]

$$\int\limits_0^{\infty}e^{-i\eta x}\,d\eta=-\frac{i}{x}+\pi\delta(x),\quad \int\limits_{-\infty}^{0}e^{-i\eta x}\,d\eta=\frac{i}{x}+\pi\delta(x),\quad \operatorname{Im}x=0,$$

we obtain

$$\int\limits_{-\infty}^{\infty}\varphi(z)\left[\int\limits_0^{\infty}e^{-i\psi(z)\eta}d\eta\right]dz=2\pi\left(\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{\varphi(z)}{\psi(z)}dz+\frac{1}{2}\int\limits_{-\infty}^{\infty}\varphi(z)\delta(\psi(z))\,dz\right).$$

But according to Cauchy's integral formula and Plemelj formulas [13, p.414]

$$\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{\varphi(z)}{\psi(z)}dz=\frac{1}{2}\int\limits_{-\infty}^{\infty}\varphi(z)\delta(\psi(z))\,dz+\sum_k \operatorname{Res}\frac{\varphi(z)}{\psi(z)}\Big|_{z=z_k}.$$

Indeed, if all real roots z_0 of the function $\psi(z)$ are single, then

$$\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{\varphi(z)}{\psi(z)}dz=\sum_k \operatorname{Res}\frac{\varphi(z)}{\psi(z)}\Big|_{z=z_k}+\frac{1}{2}\sum_{z_0}\frac{\varphi(z_0)}{\psi'(z_0)}$$

$$=\sum_k \operatorname{Res}\frac{\varphi(z)}{\psi(z)}\Big|_{z=z_k}+\frac{1}{2}\int\limits_{-\infty}^{\infty}\varphi(z)\delta(\psi(z))\,dz,$$

the other cases one may obtain through the limit transition from the mentioned case of finite number of single real zeroes.

The proof for the analytic in lower half-plane functions is absolutely similar.

Lemma 3.

$$\int\limits_0^{\infty}\frac{e^{i\alpha V}}{\sqrt{\alpha}}\,d\alpha=\frac{\sqrt{\pi}}{\sqrt{|V|}}e^{i\,\operatorname{sign}(V)\pi/4}, \tag{10}$$

$$\int\limits_0^{\infty}\frac{e^{i\alpha V}-1}{\alpha\sqrt{\alpha}}\,d\alpha=2i\,\operatorname{sign}(V)\sqrt{\pi|V|}e^{i\,\operatorname{sign}(V)\pi/4}. \tag{11}$$

Proof. Using the Cauchy's integral formula [13], we obtain

$$\int\limits_0^{+\infty}\frac{e^{-z}}{\sqrt{z}}\,dz=\int\limits_0^{i(+\infty)}\frac{e^{-z}}{\sqrt{z}}\,dz,$$

and so

$$\sqrt{\pi}=\Gamma(1/2)=\int\limits_0^{\infty}\frac{e^{-t}}{\sqrt{t}}\,dt=\int\limits_0^{i\infty}\frac{e^{-z}}{\sqrt{z}}\,dz=e^{i\pi/4}\int\limits_0^{\infty}\frac{e^{-it}}{\sqrt{t}}\,dt,$$

and by making the change of variable $t=-V\alpha$, $V<0$, we obtain

$$\sqrt{\pi}=e^{i\pi/4}\sqrt{|V|}\int\limits_0^{\infty}\frac{e^{iV\alpha}}{\sqrt{\alpha}}\,d\alpha,$$

what gives the formula (10) at $V<0$. In turn, at $V>0$ we have

$$\int\limits_0^{\infty}\frac{e^{iV\alpha}}{\sqrt{\alpha}}\,d\alpha=\overline{\int\limits_0^{\infty}\frac{e^{i(-V)\alpha}}{\sqrt{\alpha}}\,d\alpha}=\overline{\frac{\sqrt{\pi}}{\sqrt{|V|}}e^{-i\pi/4}}=\frac{\sqrt{\pi}}{\sqrt{|V|}}e^{i\pi/4}\,.$$

The formula (11) follows from (10) by means of integrating with respect to V along the segment $[0,V]$.

Theorem 4. *Let the function $\varphi(\alpha,\eta)$ is doubly continuously differentiable with respect to α on the segment $\alpha\in[-1,1]$ and*

$$M(\varphi,V)=\sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}}}\left(\left|\int\limits_{\eta\in\mathbb{R}}\frac{\varphi'_\alpha(\alpha,\eta)}{V^2(\eta)}e^{iV(\eta)B}d\eta\right|+\left|\int\limits_{\eta\in\mathbb{R}}\frac{\varphi''_{\alpha\alpha}(\alpha,\eta)}{V^2(\eta)}e^{iV(\eta)B}d\eta\right|\right.$$

$$+\left|\int\limits_{\eta\in\mathbb{R}} \frac{\varphi(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta\right|\Bigg) < \infty.$$

Then the asymptotic formula takes place

$$I = \int\limits_{\eta\in\mathbb{R}} \int\limits_{-1}^{1} \sqrt{1-\alpha^2}\varphi_(\alpha,\eta) e^{iV(\eta)(B-\alpha)} d\alpha d\eta$$

$$= \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B+1)}e^{-isgn(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta \tag{12}$$

$$- \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(1,\eta)e^{iV(\eta)(B-1)}e^{isgn(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta + O(M(\varphi,V)),$$

$|O(M(\varphi,V))| \leqslant M\cdot M(\varphi,V)$.

Proof. First we note:

$$\left|\int\limits_{\eta\in\mathbb{R}} \frac{\varphi(\alpha,\eta)-\varphi(\alpha_0,\eta)}{V^2(\eta)(\alpha-\alpha_0)} e^{iV(\eta)B} d\eta\right|$$

$$= \left|\frac{1}{\alpha-\alpha_0}\int\limits_{\alpha_0}^{\alpha}\left[\int\limits_{\eta\in\mathbb{R}} \frac{\varphi'_\alpha(\beta,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta\right] d\beta\right|$$

$$\leqslant \frac{1}{|\alpha-\alpha_0|}\left|\int\limits_{\alpha_0}^{\alpha} M(\varphi,V)\, d\beta\right| M(\varphi,V),$$

i.e. we have additional conditions

$$\sup_{\substack{\alpha\in[-1,1]\\ B\in\mathbb{R}}} \left|\int\limits_{\eta\in\mathbb{R}} \frac{\varphi(\alpha,\eta)-\varphi(\alpha_0,\eta)}{V^2(\eta)(\alpha-\alpha_0)} e^{iV(\eta)B} d\eta\right| \leqslant M(\varphi,V), \quad \alpha_0=\pm1. \tag{13}$$

The integral I we first integrate by parts with respect to α:

$$I = -\iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,1]}} \frac{\alpha\varphi(\alpha,\eta)}{iV(\eta)\sqrt{1-\alpha^2}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta$$

$$+ \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,1]}} \frac{\sqrt{1-\alpha^2}\varphi'_\alpha(\alpha,\eta)}{iV(\eta)} e^{iV(\eta)(B-\alpha)} d\alpha d\eta = I_1 + I_2.$$

The integral I_2 we integrate by parts once more:

$$I_2 = \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,1]}} \frac{\alpha\varphi'_\alpha(\alpha,\eta)}{V^2(\eta)\sqrt{1-\alpha^2}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta$$

$$- \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,1]}} \frac{\sqrt{1-\alpha^2}\varphi''_{\alpha\alpha}(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)(B-\alpha)} d\alpha d\eta,$$

hence

$$|I_2| \leqslant \int\limits_{-1}^{1} \frac{|\alpha|}{\sqrt{1-\alpha^2}} \left| \int\limits_{\eta\in\mathbb{R}} \frac{\varphi'_{\alpha}(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)(B-\alpha)} d\eta \right| d\alpha$$

$$+ \int\limits_{-1}^{1} \sqrt{1-\alpha^2} \left| \int\limits_{\eta\in\mathbb{R}} \frac{\varphi''_{\alpha\alpha}(\alpha,\eta)}{V^2(\eta)} e^{iV(\eta)(B-\alpha)} d\eta \right| d\alpha$$

and finally $|I_2| \leqslant M \cdot M(\varphi, V)$.

In turn,

$$I_1 = \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,0]}} \frac{\psi_+(\alpha,\eta)}{iV(\eta)\sqrt{1+\alpha}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta$$

$$+ \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[0,1]}} \frac{\psi_-(\alpha,\eta)}{iV(\eta)\sqrt{1-\alpha}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta = I_+ + I_-,$$

$$\psi_+ = -\frac{\alpha\varphi(\alpha,\eta)}{\sqrt{1-\alpha}}, \quad \psi_- = -\frac{\alpha\varphi(\alpha,\eta)}{\sqrt{1+\alpha}}.$$

First we consider

$$I_+ = \int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)e^{iV(\eta)B}}{iV(\eta)} \left[\int\limits_{-1}^{0} \frac{1}{\sqrt{1+\alpha}} e^{-iV(\eta)\alpha} d\alpha \right] d\eta$$

$$+ \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,0]}} \frac{\psi_+(\alpha,\eta)-\psi_+(-1,\eta)}{iV(\eta)\sqrt{1+\alpha}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta = I_+^0 + I_+^1.$$

The integral I_+^1 we integrate by parts with respect to α, according to the theorem conditions the function ψ_+ is differentiable with respect to α in point $\alpha = -1$, and since $\psi_+(0,\eta) = 0$, we obtain

$$I_+^1 = -\int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)}{V^2(\eta)} e^{iV(\eta)B} d\eta + \iint\limits_{\substack{\eta\in\mathbb{R}\\ \alpha\in[-1,0]}} \frac{\psi_+^1(\alpha,\eta)}{V^2(\eta)\sqrt{1+\alpha}} e^{iV(\eta)(B-\alpha)} d\alpha d\eta,$$

$$\psi_+^1 = \frac{\psi_+(\alpha,\eta)-\psi_+(-1,\eta)}{2(\alpha+1)} - (\psi_+)'_{\alpha}(\alpha,\eta)$$

$$= \frac{-\frac{\alpha\varphi(\alpha,\eta)}{\sqrt{1-\alpha}} + \frac{-1\varphi(-1,\eta)}{\sqrt{2}}}{2(\alpha+1)} - \left(-\frac{\alpha\varphi(\alpha,\eta)}{\sqrt{1-\alpha}}\right)'_{\alpha}$$

$$= -\frac{\alpha(\varphi(\alpha,\eta)-\varphi(-1,\eta))}{2(\alpha+1)\sqrt{1-\alpha}} - \frac{\varphi(-1,\eta)(1/2-\alpha)}{\sqrt{2(1-\alpha)}(\sqrt{1-\alpha}-\alpha\sqrt{2})}$$

$$+ \left(\frac{\alpha}{\sqrt{1-\alpha}}\right)' \varphi(\alpha,\eta) + \frac{\alpha}{\sqrt{1-\alpha}} \varphi'_{\alpha}(\alpha,\eta),$$

and the conclusion $|I_+^1| \leqslant M \cdot M(\varphi, V)$ evidently follows from the theorem conditions, together with additional condition (13).

Now we turn to

$$I_+^0 = \int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)e^{iV(\eta)B}}{iV(\eta)} \left[\int\limits_{-1}^{0} \frac{1}{\sqrt{1+\alpha}} e^{-iV(\eta)\alpha} d\alpha \right] d\eta$$

$$= \int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)e^{iV(\eta)B}}{iV(\eta)} \left[\int\limits_{-1}^{\infty} \frac{1}{\sqrt{1+\alpha}} e^{-iV(\eta)\alpha} d\alpha - \int\limits_{0}^{\infty} \frac{1}{\sqrt{1+\alpha}} e^{-iV(\eta)\alpha} d\alpha \right] d\eta$$

$$= H_1 + H_2.$$

The integral H_2 we integrate by parts with respect to α:

$$H_2 = -\int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)e^{iV(\eta)B}}{iV(\eta)} \left[\frac{1}{iV(\eta)} + \frac{1}{2iV(\eta)} \int\limits_{0}^{\infty} \frac{e^{-iV(\eta)\alpha}}{(\alpha+1)\sqrt{1+\alpha}} d\alpha \right] d\eta$$

$$= \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)B}}{V^2(\eta)\sqrt{2}} d\eta + \int\limits_{0}^{\infty} \frac{1}{2(\alpha+1)\sqrt{2(1+\alpha)}} \left[\int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B-\alpha)}}{V^2(\eta)} d\eta \right] d\alpha,$$

and, evidently, $|H_2| \leqslant M \cdot M(\varphi, V)$.

It remains to analyse H_1. But (lemma 3)

$$\int\limits_{-1}^{\infty} \frac{1}{\sqrt{1+\alpha}} e^{-iV(\eta)\alpha} d\alpha = e^{iV(\eta)} \int\limits_{0}^{\infty} \frac{e^{-iV(\eta)\alpha}}{\sqrt{\alpha}} d\alpha = \frac{\sqrt{\pi}}{\sqrt{|V(\eta)|}} e^{i(V(\eta) - \mathrm{sgn}(V(\eta))\pi/4)},$$

whence

$$H_1 = \int\limits_{\eta\in\mathbb{R}} \frac{\psi_+(-1,\eta)e^{iV(\eta)B}}{iV(\eta)} \frac{\sqrt{\pi}}{\sqrt{|V(\eta)|}} e^{i(V(\eta) - \mathrm{sgn}(V(\eta))\pi/4)} d\eta$$

$$= \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B+1)} e^{-i\mathrm{sgn}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta.$$

Thus,

$$I_+ = \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B+1)} e^{-i\mathrm{sgn}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta + O(M(\varphi, V)).$$

Absolutely similar we have

$$I_- = -\frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(1,\eta)e^{iV(\eta)(B-1)} e^{i\mathrm{sgn}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta + O(M(\varphi, V)),$$

and finally

$$I = I_+ + I_- + I_2$$

$$= \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(-1,\eta)e^{iV(\eta)(B+1)} e^{-i\mathrm{sgn}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta$$

$$- \frac{\sqrt{\pi}}{i\sqrt{2}} \int\limits_{\eta\in\mathbb{R}} \frac{\varphi(1,\eta)e^{iV(\eta)(B-1)} e^{i\mathrm{sgn}(V(\eta))\pi/4}}{V(\eta)\sqrt{|V(\eta)|}} d\eta + O(M(\varphi, V)).$$

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