S©MR ISSN 1813-3304

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru

Том 16, стр. 591–599 (2019) УДК 517.956.3
DOI 10.33048/semi.2019.16.037 MSC 86-08, 86A17

SIMOULATION OF THE SEISMIC WAVE PROPAGATION IN POROUS MEDIA DESCRIBED BY THREE ELASTIC PARAMETERS

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Abstract. An algorithm based on the spectral-difference method for numerical solution of the dynamic problem for porous media is proposed. We consider a linear two-dimensional problem in the form of dynamic equations in terms of displacement components described by three elastic parameters. The governing equations are based on conservation laws and consistent with the thermodynamics conditions. The medium is assumed to be isotropic and two-dimensional-inhomogeneous with respect to the spatial coordinates. To numerically solve the problem, we propose a method based on the joint use of the Laguerre integral transformation with respect to time and the finite difference approximation with respect to spatial coordinates. A description of the numerical implementation of the proposed method is given and its features are analyzed in the calculations. The efficiency of applying the Laguerre transformation and its difference from the Fourier transform for solving the direct dynamic seismic problems is discussed. Numerical results of the simulation of the seismic wave propagation fields for the test medium model are presented.

Keywords: Laguerre transform, porous media, wave field, numerical modeling, difference scheme.

Imomnazarov, Kh.Kh., Mikhailov, A.A., Rakhmonov, T.T., Simoulation of the seismic wave propagation in porous media described by three elastic parameters.

The work is supported by RFFI (grant 16-01-00729) and the grant of Ministry of Education and Science of the Republic of Kazakhstan (grant number AP05131026).

Received August, 9, 2018, published April, 23, 2019.

1. Introduction

The simulation of the physical properties of a porous medium and their related investigations of fluid flows in porous structures conventionally occupy one of important places among modern problems of computational mathematics and mathematical modeling.

On the one hand, this is due to the fact that porous can be a structure of a variety of natural and artificial materials: soils and soil, plant and animal tissues, fibrous, powder and foamed metal, ceramic, polymer and composite materials. On the other hand, this is due to the complexity both of theoretical, and experimental analysis of the internal structure of a porous medium. Without taking into account such a complexity it is impossible to predict and assess the effectiveness of applying porous materials in the new and modernized technological processes.

The use of models of porous structures strongly affected the development of many areas of scientific research: the theory of filtration and energy, mechanics and materials sciences, medicine and biology, agriculture and earths sciences. As mathematical models the Frenkel-Biot type models are generally used [1, 2]. A characteristic feature of the latter is the availability of additional secondary longitudinal wave. In the Frenkel-Biot type theory, velocities of the propagation of such waves is a function of four elastic parameters for given values of the physical parameters of a medium [1, 2]. In 1989, V.N. Dorovsky [3], based on the first common physical principles, constructed a nonlinear mathematical model for porous media. Just as in the Frenkel-Biot theory, in the Dorovsky model there are three types of sound oscillations: transverse and two types of longitudinal oscillations. In contrast to models of the Frenkel-Bio type, in the Dorovsky linearized model a medium is described by three elastic parameters [4, 5]. These elastic parameters are in a one-to-one manner expressed by three velocities of elastic vibrations. This circumstance is important for the numerical modeling of the propagation of elastic waves in porous media, when velocity distributions of acoustic waves, the relations of the physical density of the enclosing medium to the liquid saturating it and the value of the porosity coefficient are known.

In this paper, we solve a system of linearized dynamic equations for the two-dimensional problem of the seismic waves propagation in porous media [4-7]. The initial system is written down in terms of the matrix displacements, the displacements of a saturating liquid. In the numerical solution of the specified problem, the method of combining the Laguerre analytical transform with respect to time and a finite difference method with respect to space is employed. This method of solving the dynamic problems of the elasticity theory was first considered in [8, 9], and then developed for the viscoelasticity problems [10, 11]. The proposed method of solution can be regarded as an analog to the well-known spectral-difference method based on the Fourier spectral transforms, only instead of the frequency ω we have the parameter $\boldsymbol{m}$, i.e. the degree of the Laguerre polynomials. However, in contrast to the Fourier transform, the use of the Laguerre integral transform with respect to time makes it possible to reduce the original problem to solving a system of equations in which the separation parameter is present only in the right-hand side of the equations and has a recurrent dependence. As opposed to the finite difference method, when using the spectral method with the analytical transformation we can reduce the original problem to the solution of a differential system of equations in which there are derivatives only with respect to the spatial coordinates. This allows us to apply well-known stable difference schemes for a subsequent solution to similar systems. This approach is effective in solving non-stationary dynamic problems for porous media. However the presence of the secondary wave with a low velocity results in an increase in the amount of calculation when using explicit difference schemes.

2. Statement of the problem

Let the half-plane $x_2 > 0$ be filled with a porous medium of a saturated liquid. The propagation of elastic oscillations in a porous medium saturated with liquid in the reversible hydrodynamic approximation is described by a system of equations [12, 13]:

$$\ddot{\mathbf{U}} - c_s^2\,\Delta\mathbf{U} - a_1\nabla\mathrm{div}\mathbf{U} + a_2\nabla\mathrm{div}\mathbf{V} = \mathbf{F}, \tag{1}$$

$$\ddot{\mathbf{V}} - a_4\nabla\mathrm{div}\mathbf{V} + a_3\nabla\mathrm{div}\mathbf{U} = \mathbf{F}, \tag{2}$$

where $\rho = \rho_l + \rho_s$; ρ_s is the partial density of a solid of an enclosing medium with elastic vibrations by a given displacement vector $\mathbf{U}$; ρ_l is the partial density of the saturating fluid with oscillations by a given displacement vector $\mathbf{V}$; $\mathbf{F}$ is the mass force; c_s is the velocity of transverse waves; a_k (k = 1,2,3,4) are the coefficients determined from the equation of state which are the functions of wave propagation velocities [5, 13-16]:

$$a_1 = \alpha\rho_s + \frac{K}{\rho_s} + \frac{1}{3}\frac{\mu}{\rho_s} - \frac{2K}{\rho}, \qquad a_2 = \frac{\rho_l}{\rho}\left(\frac{K}{\rho_s} - \alpha\rho\right),$$

$$a_3 = \frac{K}{\rho} - \alpha\rho_s, \qquad a_4 = \alpha\rho_l, \qquad \alpha = \alpha_3\rho + K/\rho^2, \qquad \mu = \rho_s\, c_t^2,$$

$$K = \frac{\rho}{2}\frac{\rho_s}{\rho_l}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_l}{\rho}c_s^2 - \sqrt{\left(c_{p_1}^2 - c_{p_2}^2\right)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right),$$

$$\alpha_3 = \frac{1}{2\rho^2}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_s}{\rho}c_s^2 + \sqrt{\left(c_{p_1}^2 - c_{p_2}^2\right)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right),$$

c_{p_1} and c_{p_2} are the velocities of fast and slow longitudinal waves, respectively. The above system of equations is the result of the linearization of the complete nonlinear system of equations, obtained in [3, 5].

The problem is solved with zero initial data

$$\mathbf{U}|_{t=0} = \dot{\mathbf{U}}|_{t=0} = 0, \tag{3}$$

$$\mathbf{V}|_{t=0} = \dot{\mathbf{V}}|_{t=0} = 0, \tag{4}$$

and the boundary conditions on a free surface in the plane $x_2 = 0$

$$\sigma_{22} + P|_{x_2=0} = \sigma_{12}|_{x_2=0} = \left.\frac{\rho_l}{\rho_0}P\right|_{x_2=0} = 0, \tag{5}$$

where the pore pressure and the stress tensor are determined by formulas

$$P = (K - \rho\rho_s\,\alpha)\,\mathrm{div}\,\mathbf{U} - \rho\rho_l\,\alpha\,\mathrm{div}\,\mathbf{V}\,, \tag{6}$$

$$\sigma_{ik} = \mu\left(\frac{\partial U_k}{\partial x_i} + \frac{\partial U_i}{\partial x_k}\right) + \tilde{\lambda}\delta_{ik}\mathrm{div}\,\mathbf{U} - \left(1 - \frac{K}{\alpha\rho}\right)\delta_{ik}\,P\,. \tag{7}$$

In formulas (6), (7), δ_{ik} is the Kronecker symbol, $\tilde{\lambda} = \lambda - K^2/(\rho^2\,\alpha)$.

Let us note that when the porosity disappears then with allowance for $\rho^2\alpha_3 \to K_s/\rho_s^f$ [5], where K_s and ρ_s^f, respectively, are the modulus of the triaxial compression and the density of a homogeneous elastic isotropic body, formula (7) goes to the relations of Hooke's law of a homogeneous elastic isotropic body [17].

3. Solution Algorithm

To solve the initial-boundary value problem (1) - (5) we apply the Laguerre integral transform with respect to time:

$$\vec{W}^m(x_1, x_2) = \int\limits_0^\infty \vec{W}(x_1, x_2, t)(ht)^{-\frac{\alpha}{2}} l_m^\alpha(ht) d(ht), \tag{8}$$

with the inversion formulas

$$\vec{W}(x_1, x_2, t) = (ht)^{\frac{\alpha}{2}} \sum_{m=0}^{\infty} \frac{m!}{(m+\alpha)!} \vec{W}^m(x_1, x_2) l_m^\alpha(ht), \tag{9}$$

where $l_m^\alpha(ht)$ is the Laguerre function.

The Laguerre functions $l_m^\alpha(ht)$ are expressed in terms of the classical orthonormal Laguerre polynomials $L_m^\alpha(ht)$ [8]. In this paper, we choose the parameter α to be integer and positive, hence:

$$l_m^\alpha(ht) = (ht)^{\frac{\alpha}{2}} e^{-\frac{ht}{2}} L_m^\alpha(ht).$$

For the first and the second derivatives of the Laguerre polynomials, we have the following formulas:

$$\frac{\partial}{\partial t} L_m^\alpha(ht) = -h \sum_{k=0}^{m-1} L_k^\alpha(ht) \quad , \quad \frac{\partial^2}{\partial t^2} L_m^\alpha(ht) = h^2 \sum_{k=0}^{m-2} (m-k-1) L_k^\alpha(ht).$$

It is easy to see that in order to satisfy the initial conditions of the problem it is sufficient to set the value $\alpha \geq 2$. Moreover, in these formulas we introduce the shift parameter $h > 0$, whose meaning and effectiveness are discussed in detail in [10, 11].

As a result of the transformation conducted, the original problem (1) - (5) reduces to a two-dimensional spatial differential problem in the spectral domain, which is written as:

$$\frac{h^2}{4}\mathbf{U}^m - c_s^2\,\Delta\mathbf{U}^m - a_1\nabla\mathrm{div}\mathbf{U}^m + a_2\nabla\mathrm{div}\mathbf{V}^m = f^m\mathbf{F} - h^2\sum_{j=0}^{m-1}(m-j)\mathbf{U}_j^m, \tag{10}$$

$$\frac{h^2}{4}\mathbf{V}^m - a_4\nabla\mathrm{div}\mathbf{V}^m + a_3\nabla\mathrm{div}\mathbf{U}^m = f^m\mathbf{F} - h^2\sum_{j=0}^{m-1}(m-j)\mathbf{V}_j^m, \tag{11}$$

with the boundary conditions

$$\sigma_{22}^m + P^m\big|_{x_2=0} = \sigma_{12}^m\big|_{x_2=0} = \frac{\rho_l}{\rho_0} P^m\bigg|_{x_2=0} = 0, \tag{12}$$

where the pore pressure and the stress tensor are determined by the following formulas:

$$P^m = (K - \rho\rho_s\,\alpha)\,\mathrm{div}\,\mathbf{U}^m - \rho\rho_l\,\alpha\,\mathrm{div}\,\mathbf{V}^m\,, \tag{13}$$

$$\sigma_{ik}^m = \mu \left(\frac{\partial U_k^m}{\partial x_i} + \frac{\partial U_i^m}{\partial x_k} \right) + \tilde{\lambda} \delta_{ik} \mathrm{div}\, \mathbf{U}^m - \left(1 - \frac{K}{\alpha\rho} \right) \delta_{ik}\, P^m \,. \tag{14}$$

In equations (10) - (14), f^m are the coefficients of the Laguerre expansion of the time function $f(t)$ in the source, and $\mathbf{U}^m$, $\mathbf{V}^m$, σ_{ik}^m, P^m are the coefficients of the Laguerre expansion of the corresponding field components. The superscript m for all the components means the coefficient number in the Laguerre expansion. It is easy to see that the value of m is explicitly present only in the right-hand side of the equations in the form of a recurrence relation for all components of the field.

To solve problem (10) - (14), we use a finite difference approximation of the derivatives with respect to the spatial coordinates with second order of accuracy. For this, in the calculation domain, we introduce a difference grid with a discretization step Δx with respect to both spatial coordinates. We define the required components of the vector of the solution at the nodes of this grid. To approximate equations (10), (11) at the upper boundary we use the boundary conditions (12). For the lateral and the lower boundaries, the boundary conditions of the first or of the second kind for the corresponding components are given.

As a result of the finite difference approximation of the problem, we obtain a system of linear algebraic equations. We represent the required solution vector $\vec{W}$ in the following form:

$$\vec{W}^m = (\vec{V}_0^m, \vec{V}_1^m, ..., \vec{V}_K^m, \vec{V}_{K+1}^m, ..., \vec{V}_{K+N}^m)^T,$$

$$\vec{V}_{i+j}^m = (\, u_1^m(i\Delta x, j\Delta x),\, u_2^m(i\Delta x, j\Delta x),\, v_1^m(i\Delta x, j\Delta x),\, v_2^m(i\Delta x, j\Delta x))^T,$$

where $i = 0, ..., K$ is the number of nodes along the coordinate x_1 and $j = 0, ..., N$ is the number of nodes along the coordinate x_2.

Then, a given system of linear algebraic equations in the vector form can be written down as:

$$(A_\Delta + \frac{h^2}{4} E)\ \vec{W}^m = \vec{F}_\Delta^{m-1}. \tag{15}$$

On the main diagonal of the matrix of system (15), the components appear in the equations of the system as summands having the parameter h as a co-factor. We should note that at the expense of the appropriate choice of the parameter h, there appears a possibility to substantially improve the conditioning of the matrix of the system. Having solved the system of linear algebraic equations (15), we can define the spectral values for all components of the wave field $\vec{W}(m)$. Then, using the Laguerre transform inversion formula (9), we obtain the solution of the original problem (1) - (5).

In the Laguerre analytical transform formula (9) for the determination of the values of functions by their expansion coefficients, a sum with an infinite limit is used. In the numerical implementation, the necessary condition is to determine the required number of terms in a series to be summed for constructing a solution with a given accuracy. The number of the Laguerre harmonics required for the definition of functions by formula (9), depends on a given signal in the source $f(t)$, the choice of the parameter h, and the values of the time interval of the simulated wave field. The way of how one can determine the required number of harmonics and choose the optimal value of the parameter h, is considered in detail in [8-11].

4. Numerical results

The calculation results of the wave field for different media models are presented in Figures 1 - 3. The first model is a medium consisting of three homogeneous layers: the upper layer is an elastic medium; the lower left layer is a porous medium; the lower right layer is an elastic medium. The physical characteristics of the layers were given by the following:

(1) the upper elastic layer $\rho = 1200$ kg/m ^{3}, $c_p = 1400$ m/s, $c_s = 1000$ m/s;
(2) the lower right porous layer $\rho_s = 1500$ kg/m ^{3}, $\rho_l = 1000$ kg/m ^{3}, $c_{p_1} = 1900$ m/s, $c_{p_2} = 400$ m/sec, $c_s =$ 1300 m/s, $d = 0.1$;
(3) the lower left elastic layer $\rho = 1500$ kg/m ^{3}, $c_p = 1900$ m/s, $c_s =$ 1300 m/s.

The thickness of the upper elastic layer is 600 m. The vertical interface between the lower porous layer and the elastic one passes along the line $x =$ 700 m. The wave field was simulated from the expansion center point source with the coordinates $x_0 = 700$ m, $z_0 = 500$ m, located in the upper elastic layer.

The time signal in the source was set in the form of the Puzyrev pulse:

$$f(t) = exp\left(-\frac{2\pi\, f_o(t-t_0)^2}{\gamma^2}\right) sin(2\pi\ f_0(t-t_0)), \tag{16}$$

where $\gamma = 4$, $f_0 = 30$ Hz, $t_0 = 0.05$ s.

The results of numerical calculations of the wave field for the given model are shown in Figure 1. This figure represents a snapshot of the wave field for the vertical displacement component $U_z(x,z)$ at a fixed instant of time for $T = 0.4$ seconds. It can be seen from the figure that in the lower left porous layer there is a secondary longitudinal wave $c_{p_2} = 400$ m/s, while only one longitudinal wave and one transverse wave propagate in the right elastic layer .

As the second model, a three-layer medium with a thin layer in the middle was set. Two cases were considered. In the first case, a wave field for a thin porous layer was simulated, in the second case a thin elastic layer of the same thickness was specified. Physical characteristics of the medium with a porous layer were set by the following:

(1) the upper and the lower elastic layers $\rho = 1200$ kg/m ^{3}, $c_p = 1500$ m/s, $c_s =$ 1000 m/s;
(2) the average porous layer $\rho_s = 1500$ kg/m^3, $\rho_l = 1000$ kg/m^3, $c_{p_1} = 2000$ m/s, $c_{p_2} = 400$ m/s, $c_s = 1300$ m/s, $d = 0.1$.

The thickness of the upper layer is 700 m. The thickness of the middle layer is 10 m. In the case of a thin elastic layer, $c_p = c_{p_1}$, $\rho = \rho_s$ and $c_s = 1300$ m/s. The wave field was simulated from the point source of the center-of-expansion type with the coordinates $x_0 = 650$ m, $z_0 = 650$ m, located in the upper elastic layer. The time signal in the source was set in the form of the Puzyrev pulse by formula (16).

The results of numerical calculations of the wave field for a given medium model are shown in Figure 2. This figure presents snapshots of the wave field for the vertical displacement component $U_z(x,z)$ at a fixed instant of time $T = 0.4$ seconds. Left - for the thin porous layer. Right - for the thin elastic layer. It can be seen from the figure that in the case of a thin porous layer there are waves generated by multiple reflection of the secondary (slow) longitudinal wave in a thin porous layer, whose thickness is approximately one spatial length of this wave.

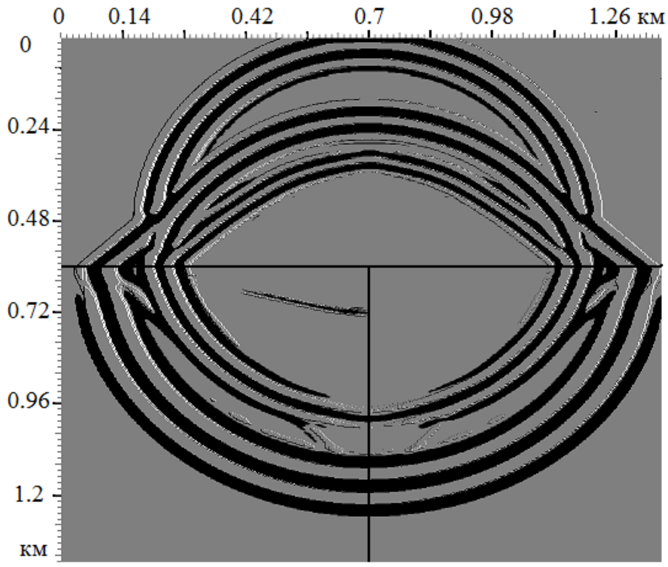

FIG. 1. A snapshot of the wave field for the vertical displacement component $U_z(x, z)$ at the time instant $T = 0.4$ seconds. The layer interfaces are shown as a solid line.

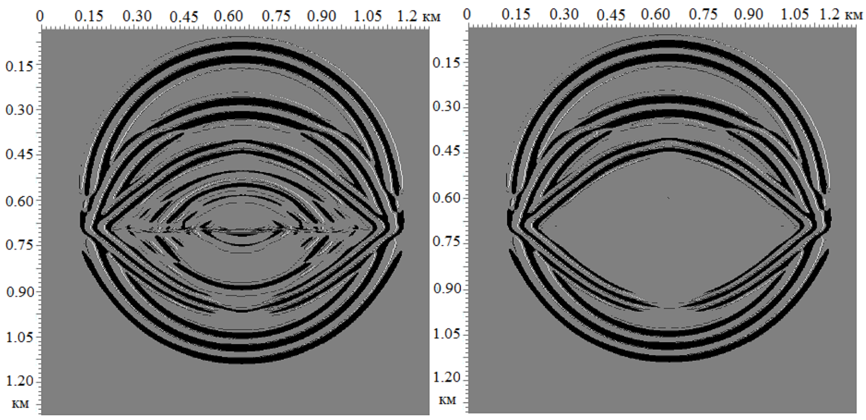

FIG. 2. A snapshot of the wave field for the vertical displacement component $U_z(x, z)$ at the time instant $T = 0.4$ seconds. Left - with a thin porous layer, right - with an elastic layer.

In the next case, the medium model was specified as the one consisting of three homogeneous layers: the upper elastic layer, the middle porous layer, the lower elastic layer. Physical characteristics of the medium were given by the following:

(1) the upper elastic layer $\rho = 1500$ kg/m ^{3}, $c_p = 2000$ m/s, $c_s =$ 1300 m/s;
(2) the average porous layer $\rho_s = 1500$ kg/m^3, $\rho_l = 1000$ kg/m^3, $c_{p_1} = 1600$ m/s, $c_{p_2} = 400$ m/s, $c_s =$ 1100 m/s, the porosity coefficient $d = 0.1$;
(3) the lower elastic half-space $\rho = 2000$ kg/m ^{3}, $c_p = 2000$ m/s, $c_s =$ 1300 m/s.

The thickness of the upper layer is 1.7 km. The thickness of the middle layer is 2.6 km. The wave field was simulated from a point source of the center type extension with the coordinates $x_0 = 2.5$ km, $z_0 = 3.5$ km, located in a porous layer. The time signal in the source was given in the form of the Puzyrev pulse by formula (16). The carrier frequency of the signal in the source is $f_0 = 10$ Hz.

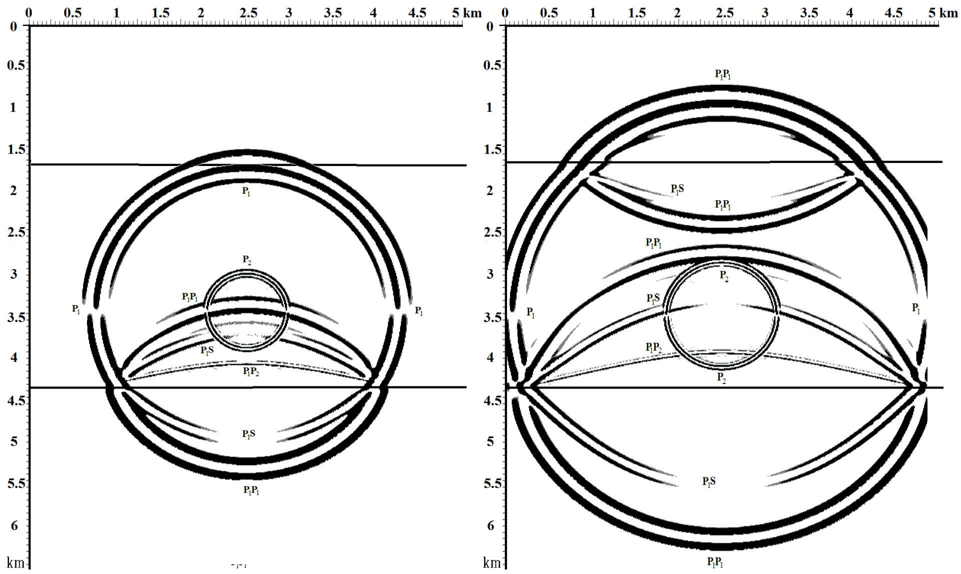

FIG. 3. A snapshot of the wave field for the bias component $U_z(x, z)$. Left - at the time instant $T = 1.2$ seconds, right - at the time instant $T = 1.6$ seconds.

The results of numerical calculations of the wave field for the given medium model are shown in Figure 3. This figure presents snapshots of the wave field for the vertical component of the displacement velocity $u_z(x, z)$ at fixed instants of time. Left - at the time instant $T = 1.2$ seconds, right - at the time instant $T = 1.6$ seconds. It can be seen from the figure that the given source generates in a porous medium two types of longitudinal waves $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, which propagate with the velocities c_{p_1} and c_{p_2}, respectively.

5. Conclusion

The algorithm proposed is an analog of the well-known spectral methods for solving dynamic problems. However, unlike the classical transformations of Fourier and Laplace, the application of the Laguerre transform brings about a system of equations in which the harmonic separation parameter is present only in the right-hand side in the recurrent form. As a result, in the reduced problem the matrix of the system of linear algebraic equations has a good conditionality, which makes it possible to use efficient methods for solving such systems. The analysis of the test calculations reveals the stability of the algorithm presented even for the medium models with drastically contrast interfaces between layers or the medium models containing thin layers comparable with spatial wavelength.

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