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A.D. TAIMANOV AND MODEL THEORY IN KAZAKHSTAN

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ABSTRACT. The article gives an overview of the development of model theory in Kazakhstan over the past 60 years from the standpoint of the tasks that stood at that time. We consider directions that naturally arose from the definition of truth in a model and under the influence of Doctor of Physical and Mathematical Sciences, Academician A. Taimanov, initiator of the growth of model theory in Kazakhstan.

Keywords: A.D. Taimanov, history of model theory, Kazakhstan.

1. INTRODUCTION

This is an expanded version of the first author's attempt to write a recollection of Asan Dabsovich Taimanov. The basics of this article were presented by him at the conference "Actual Problems of Mathematics and Mathematical Modelling" devoted to the 50th anniversary of the Institute of Mathematics and Mechanics, 2015, the conference "Taimanov Readings – 2017", dedicated to the 100th anniversary of A.D. Taimanov [29] (Baizhanov-Kalmenov, 2017), and the 16th Asian Logic Conference, 2019. After a while, the author realized that it is impossible to write about Asan Dabsovich without mentioning his scientific activity, because science is the way of his life. Without a constant search for a solution of a problem that Asan Dabsovich was thinking about at the moment, without attempts to understand the nature of an object he studied, and to give a philosophical explanation, it is impossible to imagine Asan Dabsovich. Therefore, when you talk about the life of Asan Dabsovich, it is necessary to give a context of mathematical problems that worried him at this time. Proceeding from the principle of Asan Dabsovich, who

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told the first author in his postgraduate years, that for a better understanding of the problem it is necessary to study the origins of this problem, to know the history of the issue, to know the range of related tasks, and then to single out the main task in this topic, the author tried to single out the main tasks of this time and to trace the activities of Asan Taimanov and his students at one or another period of time through the prism of these main tasks. The best scientific interests of Asan Dabsovich express a list of his publications and a list of theses of his PhD (candidate of sciences) students' dissertations in this period of time.

In this article, the authors will not deal with the scientific activities of Asan Dabsovich Taimanov in the period before getting to know him as a specialist in model theory. Therefore, for the supplement at the end of the article there will be a short essay on his activities, and his results will be listed, including the results in other areas of mathematics.

As it became clear over time, Asan Dabsovich Taimanov determined for many years the high level of development of Model Theory in Kazakhstan, which was confirmed by the results, publications and the level of international scientific cooperation and reviews of leading world experts. Thus, this attempt of the first author to write about the activities of Asan Dabsovich Taimanov grew into the history of Model Theory in Kazakhstan, the main role in which was played by Academician of the Academy of Sciences of the Kazakh Soviet Socialist Republic Asan Taimanov.

1.1. Basic problems. Since the middle of the 20th century one of the most dynamically developing fields of mathematical logic, along with the set theory, theory of algorithms and proof theory has been model theory. The development of any mathematical field is defined by main problems in the area and stages of their solution. We will examine the history of model theory in Kazakhstan through the prism of the main defining problems, hypotheses and questions associated with attempts to solve these problems. We state the main objectives and formulate important theorems and methods obtained by world's famous mathematicians, and the results of the Kazakhstan mathematicians, in line with the underlying development of these directions. We will indicate the time when, by our opinion, these directions have evolved most intensively (mainstream). Unfortunately, some intensive developments that did not involve Kazakh mathematicians will remain outside of the scope of our overview.

A structure \mathfrak{M} of the first order predicate logic of a language L defines the *elementary theory* of this structure, $T = Th(\mathfrak{M})$, namely, the set of sentences of the language L true in this structure. In this case \mathfrak{M} is said to be a *model* of the theory T . Two structures \mathfrak{M}_1 and \mathfrak{M}_2 of the same language are called *elementarily equivalent* if $Th(\mathfrak{M}_1) = Th(\mathfrak{M}_2)$.

A study of elementary theories arose because of emergence of this concept in five main problems that define the directions of research:

- Decidability (1930-1960);
- Axiomatizability of classes of (algebraic) structures (1950 – first half of 1980), Axiomatizability of classes of expanded structures (1980 -present);
- Number of non-isomorphic models of a complete theory (1950 – present), (uncountable case, second half of 1960 – second half of the 1980's);
- Expansions of models and theories (1980 – present);

- Application of methods and results of model theory for solving problems in other areas of mathematics (1940 – present).

Each of those directions, following its internal development, creation of new methods and approaches to solve its problems, can be divided into natural sub-directions. Notice that research on each of these directions often uses the methods and approaches obtained in other directions; that says our partition is formal and there is mutual influence of these directions.

1.2. Content of the paper. To the five essential directions of research in model theory we add two sections concerning international connections of logicians of Kazakhstan and essential dates of life of A.D. Taimanov.

1. Decidability;
2. Axiomatizable classes of (algebraic) structures;
3. Number of non-isomorphic models of a complete theory;
4. Expansions of models and theories;
5. Application of methods and results of model theory for solving problems in other areas of mathematics;
6. International connections;
7. Biography of A.D. Taimanov.

2. DECIDABILITY

If we enumerate a set of all formulas of a countable language L by the standard enumeration (called a Gödel numbering), then a theory T of the language L can be associated with the set of numbers of all formulas of T . A theory T is *decidable* if the set of its numbers is decidable (a recursive set), that is if there is an algorithm to determine whether or not an arbitrary natural number is a number of some formula from T .

The main problem here is to determine whether elementary theories of classical (algebraic) structures are decidable, and in the case of decidability to describe their algorithms.

2.1. Decidability of elementary theories of the classical (algebraic) structures. [K. Gödel, A. Tarski, A.I. Mal'tsev, Ax-Kochen-Ershov, Z. Sela, O. Kharlampovich, A. Myasnikov]. The most famous accomplishments of Kurt Gödel formulated and proved by him are the incompleteness theorems, which were published in 1931 [79]. The first theorem states that every recursively axiomatizable theory in a fairly rich language, which is sufficient to define the natural numbers with addition and multiplication (arithmetic) operations, is either incomplete or contradictory. The second theorem states that if a formal arithmetic is consistent, then it has a non-deducible formula, which substantially asserts consistency of the arithmetic. A consequence of these theorems is that every theory which satisfies the conditions of Gödel's theorem, including arithmetic, is undecidable. Proven by Gödel theorems have broad implications for both mathematics and philosophy (in particular for ontology and philosophy of science). The second of the famous mathematical problems that David Hilbert presented in 1900 in Paris at the II International Congress of mathematicians asks: "Are the axioms of arithmetic contradictory or not?" Kurt Gödel proved [79] (1931) that consistency of the axioms of arithmetic cannot be proved on the basis of the axioms of arithmetic (if only the arithmetic is not actually contradictory). Alfred Tarski announced in [321, 322, 323, 324] (1949)

and proved decidability of elementary theories of the field of complex numbers, and the field of real numbers [325] (Tarski-Mostowski-Robinson, 1951), undecidability of the theory of lattices, projective geometry, and the theory of algebras with a closure [326] (1953). A.I. Mal'tsev confirmed undecidability of the elementary theory of a class of finite groups [184] (1961), free decidable groups, the elementary theory of the classical linear groups [183] (1961), and that the class of locally free algebras has apparently undecidable theory [185] (1962). Replying to a question of A. Tarski, Ax-Kochen-Ershov proved decidability of elementary theory of the field of p-adic numbers [8] (Ax-Kochen, 1965), [75] (Ershov, 1965). The period 1950–1960 is characterized by a large number of works, in the Soviet Union and abroad, focusing on decidability-non-decidability of specific elementary theories.

Three students of academician of the Kazakh SSR A.D. Taimanov defended their PhD (candidate of sciences) theses precisely in algorithmic problems of model theory.

J.A. Almagambetov, "To algorithmic problems of model theory", 1965;

N.G. Khisamiev, "Questions of elementary theory of lattice-ordered algebraic structures", 1968 (co-supervisor A.I. Kokorin);

T.Sh. Shayakhmetov, "On undecidability of elementary theory of some algebraic structures", 1970.

In this direction at the end of 1960's two difficult Tarski problems remained unresolved:

- **Is $Th(\langle R, +, *, <, e^x \rangle)$ decidable?** That is, is the elementary theory of the field of real numbers expanded by a unary exponent function decidable?

Alex Wilkie [346] proved that this theory is model complete and ω -minimal, the question of decidability is open.

- **Is $Th(F_n)$ decidable for $n \geq 2$?**

The solution to this problem is divided into subtasks: 1) Will two free groups with two and three generators be elementarily equivalent? 2) To investigate decidability of equations in free groups (semigroups), and then move to decidability of the elementary theory.

These tasks immediately became popular among Soviet logicians and remained in the spotlight throughout the second half of the last century. In particular, Yu.I. Khmelevsky [105, 106, 107] (1971, 1972), Khmelevsky-Taimanov [319] (1980), A.D. Taimanov [317] (1984), G.S. Makanin [178, 179, 180] (1977, 1982, 1984), and A.A. Razborov [280] (1984) studied possibility of solving equations in free semigroups and groups. Zlil Sela, using Makanin-Razborov's algorithm for solving equations in a free group, in a series of 7 works [284]–[290] (2001–2006), in which he developed 'algebraic geometry' in free groups, solved the Tarski's problem.

Around 1945, Alfred Tarski asked whether free groups with two or more generators have the same first order theory, and whether this theory is decidable. Z. Sela [290] (2006) answered the first question by showing that any two non-Abelian free groups have the same first order theory, and Kharlampovich and Myasnikov [97] (2006) answered both questions, showing that this theory is decidable.

2.2. Elementary classification. [R. Fraïssé, A. Ehrenfeucht, A.D. Taimanov]. Roland Fraïssé [78] (1954) first proposed the partial isomorphisms method to determine whether two structures of one language have the same elementary theory.

Fraïssé's ideas had interpreted as a modified version of the method of partial isomorphisms in the form of a game [68] (A. Ehrenfeucht, 1961). This method of determining elementary equivalence of two structures was independently rediscovered by A. Taimanov in the form of spread-course multiple mappings [315] (Taimanov, 1962) using of ideas, language and approaches from topology [306, 307, 308, 310] (Taimanov, 1952–1960). This (non-combinatorial) approach to examine elementary equivalence of two structures was popular in that decade in Soviet Union. The method is helpful in matters of decidability of an elementary theory, and in counting the number of non-isomorphic models. The history of this method is described in [82] (Yu. Gurevich, 1979), [71] (2009), [55, p. 462] (2009), [88, p. 128] (Hodges, 1997) [174] (2019)

A.T Nurtazin introduced the notion of a graph with finite chains and proved that the classes of all graphs with finite chains and all finite graphs have the same elementary theory. The elementary theory of every graph of finite length is decidable [222] (2007).

2.3. Model completeness, elimination of quantifiers. [Tarski-Robinson-Mostowski, Ershov-Lavrov-Taimanov-Taitslin]. The main method of studying decidability of elementary theories is the quantifier elimination method. A theory *admits elimination of quantifiers* if for every formula of a given signature there exists an equivalent quantifier-free formula. Since 1927 Tarski with his students used the method of quantifier elimination to study a wide range of different structures. Among the important early examples are the field of real numbers and the set of natural numbers with the symbols 0, 1, and addition (Presburger arithmetic). A theory is model complete if every embedding of its models is elementary. Every theory admitting quantifier elimination is model complete.

Basic methods for proving undecidability were presented in a book of three authors A. Tarski, R. Robinson and A. Mostowski [326] (1953). In general terms, the essence of the method of quantifier elimination is as follows: 1) in indication of an algorithm of conversion of formulas to some canonical form, that is in construction of an algorithm that transforms a formula of a signature of theory T into an equivalent quantifier-free formula; 2) specifying an algorithm that checks whether a formula reduced to a canonical form belongs to the theory T . Thereby an algorithm judging whether an arbitrary formula of a signature of the theory T belongs to T is specified. In connection with the rapid development of this trend and emergence of a large number of new results, there was a need to review and update the list of decidable theories as well as of theories which are undecidable. Yu.L. Ershov, I.A. Lavrov, A.D.Taimanov and M.A. Taitslin wrote such an overview [77] (1965). Essentially this work summed up scientific research in the direction “decidability of elementary theories”, in the appendix a table of the results obtained at the time of writing (the number of theories covered in the article: decidable – 39, undecidable – 82) is presented.

The number of publications on decidability/undecidability of specific algebraic structures has dropped dramatically by the 70's years of the last century. Having a complex structure of definable sets, for the study of which new approaches, new ideas based on the accumulated facts, and persistence were needed, as history of solving Tarski problems has shown, theories remained unexplored.

Model completeness of theories continues to remain in the center of attention of model theory specialists. Abraham Robinson, who developed non-standard analysis

and the theory of model completeness, was a pioneer of this kind of work [281] (1956), [283]. An other early work in this direction was the Tarski's description procedure on elementary Euclidean geometry [325] (1951). This is an effective procedure to determine which (first-order) claims about the structure $\langle R, + \rangle$ are correct. A byproduct of his work was to identifying those subsets of R^n , which are first order definable, using only “+” and “ \times ”, as sets definable by polynomial equations and inequalities. A key result is that definable subsets of the real line itself are finite unions of intervals and points. This feature later (in the 1980s) had became the definition of an o-minimal structure on R [66] (van den Dries, 1998), [270] (Pillay-Steinhorn, 1984). A number of facts on the model pairs of theories was obtained. K.Zh. Kudaibergenov studied the classical notion of a model companion. The general results on the existence and non-existence of a model companion for theories of a structure with a distinguished automorphism were obtained, as well as corresponding results for some specific structures [124] (Kudaibergenov-Macpherson, 2006). An answer was given on the Tsuboi Kikyo's question about existence of a model companion for a theory of C-minimal models with a distinguished automorphism of a special kind [133] (Kudaibergenov, 2013).

2.4. Constructive and strongly constructive models, computable numberings. [A.I. Mal'tsev, Yu.L. Ershov] (1960–present). In 1961, A.I. Mal'tsev wrote the “Constructive Algebras” [183], in which he gave the definition of a constructive model, as a countable model is equipped with a countable numbering so that all the functions and predicates (relations) were recursively enumerable, he introduced the definition of equivalent numberings, and obtained a number of results on constructive models of decidable elementary theories. The article opened a new research direction focused on elementary theories of algebraic structures and the theory of constructive models. Those areas located at the junction of algebra, mathematical logic and theory of algorithms, have found numerous applications. This trend is mainly developed in Siberia and Kazakhstan.

In the late 60s of the last century Yu.L. Ershov introduced the concept of a strongly constructive model [76] (1980). The main problem in this area, in our opinion, is the problem of the existence of a maximal decidable theory, that is a decidable theory that no extension of this theory in the expanded language is not decidable. In terms of strongly constructive models and expansion by new predicate: is there a strongly constructive model, that can not be expand by new recursive relation remaining in the class of strongly constructive models?

In 1974 N.G. Khisamiev proved that all countable models of an \aleph_1 -categorical decidable theory are strongly constructive [98]. In this year an American mathematician L. Harrington published the same result [85]. In Kazakhstan, this trend is driven by Nazif Garifullinovich Khisamiev. Most of the graduates of the department of algebra and mathematical logic of the Kazakh National University (KazNU) of 1970–1980 began their research by studying the properties of structural and strongly constructive models, their diploma and PhD (candidate of sciences dissertations) focused on this topic. For example, A.T. Nurtazin in his PhD thesis obtained important results on computable classes and autostability which determined the nature of further research in this direction [221] (1974). A.T. Nurtazin found the autostability and strong constructivizability criterion for prime models, and proved that a non-autostable strongly constructive model has a countable number of non-autoequivalent strong constructivizations. He has also proved that the class of all

constructivizable torsion-free groups is computable, while the class of all constructive torsion-free groups is not computable.

M.G. Peretyat'kin constructed examples of decidable theories with special properties, answering questions of Yu.L. Ershov and A. Lachlan, [247, 248] (1971, 1973). In particular, he built an example of a decidable theory which has a single strongly constructive model, and this model does not have its own elementary extensions. M.G. Peretyat'kin in [249] (1978) and, independently from him, S.S. Goncharov, answering on M. Morley's question, found a criterion of strong constructivizability of homogeneous models. K.Zh. Kudaibergenov constructed a complete decidable theory, that has exactly two strongly constructible models [109] (1979); this contrasts with famous Vaught's theorem that a countable complete theory can not have exactly two countable models. K.Zh. Kudaibergenov proved that for every natural number n there exists an ω_1 -categorical theory that has exactly n constructivizable models [110] (1980). In [111] (1986) he built a strongly constructivizable homogeneous model that is not effectively homogeneous in any constructivization.

An extensive series of works of N.G. Khisamiev is devoted to the theory of constructive Abelian groups by start with V.P. Dobritsa and A.T. Nurtazin [64] (1978). In [101] (Khisamiev, 1990) was solved the main problem of characterization of constructivizability and strong constructivizability of Abelian p -group of finite ulmean types. The principal problem of existence of a constructive model of a given theory of Abelian groups was solved. In [102] (Khisamiev, 1998) relations of classes of constructivizable and strongly constructivizable Abelian groups were found. In [99] (Khisamiev, 1986) an arithmetic hierarchy of Abelian groups was built. From this it follows that there is a certain computable enumerably definable non-constructivizable Abelian torsion-free group. This is the answer to the question of American mathematician G. Baumslag.

The existence of a major class of computable enumerations of the class of all constructive Abelian torsion-free groups the ranks of which do not exceed a predetermined number and are non-zero. An elementary classification of structurally-ordered Abelian groups was obtained. On its base a new elegant reproof of the known Weinberg's theorem on free structurally ordered Abelian groups was obtained. A fundamental result on undecidability of the elementary theory of a free lattice was obtained. In [100] (Khisamiev, 1987) the problem of British mathematician A. Macintyre on constructivizability of an ordered field of primitive recursive real numbers was negatively solved. In [103, 104] (2004) N.G. Khisamiev and V.A. Roman'kov solved the problems of existence of a (strong) constructivization of linear groups. Those results, published in the journal "Algebra and Logic", were awarded the prize of the magazine for the year 2004. A criterion of constructivizability of a two-step torsion-free nilpotent group was found. Constructivizability of a computably-enumerable nilpotent torsion-free group, whose commutator dimension is finite, was proved. A necessary and sufficient condition of constructivizability of a nilpotent torsion-free group, whose commutator dimension is finite, was obtained. In [220] (Nurizinov-Tyulybergenov-Khisamiev, 2014) an effective description of constructivizable nilpotent torsion-free groups of finite dimensions was given.

D.A. Tussupov studied algorithmic complexity of the Scott family of formulas for different classes of algebraic structures [327, 328, 329, 330] (2006, 2007). In particular, he built an autostable two-step nilpotent group with a single constructivization up to autoequivalences that has no Scott family of finite formulas [329]

(2007). V.P. Dobritsa essentially developed approaches and methods for computable classes of constructive models and autostability established in the early works of A.T. Nurtazin and S.S. Goncharov [221] (Nurtazin, 1974), [81] (Goncharov-Nurtazin, 1973), [80] (Goncharov, 1975), studied computability of computable indexations of computable classes of constructive models with a finite number of constructive models [60, 61, 62, 63] (1986, 1989, 1994).

This area is closely related with research on algorithmic questions of algebra, for instance, with the complexity of algorithms solving whether an arbitrary chosen element of a computable group is in a fixed term of the lower central series of the group (that is in the *central* of the group). It turned out that the Turing complexity of the problem of belonging to the given central can be completely independent from complexity of this problem for different centrals, even with an assumption that the group is nilpotent, that is it has only finite number of non-trivial centrals [171] (Latkin, 1987). N.G. Khisamiev established [99] (1986) that every abelian torsion-free group which has a *positive representation* is computable. I.V. Latkin showed that this result can not be extended to the class of nilpotent groups without torsion of the stage two [172] (1996). Nevertheless, for some classes of unary modules over a commutative ring the analogue of Khisamiev's theorem remains true [173] (2002). It was also shown that the tensor product of computable Abelian torsion-free groups is computable, and that the requirement for absence of elements of a finite order is significant.

In this direction the following PhD and doctor of sciences dissertations of scientists from Kazakhstan were defended:

PhD theses:

M.G. Peretyat'kin, "Constructive models", 1974 (supervisor Yu.L. Ershov).

A.T. Nurtazin, "Computable classes and algebraic criteria of autostability", 1974 (supervisor A.D. Taimanov).

V.P. Dobritsa "Computable classes of algebras and constructivizations of abelian groups", 1976 (supervisor N.G. Khisamiev);

B.N. Drobotun, "Numberings of special models", 1977 (supervisors A.D. Taimanov, S.S. Goncharov);

K.Zh. Kudaibergenov, "Constructive models of complete theories", 1983 (supervisor Yu.L. Ershov);

S.S. Zaurbekov, "Estimates of algorithmic complexity of some semantic sentence classes", 1990 (supervisor M.G. Peretyat'kin);

D.A. Tussupov, "Algorithmic complexity of homogeneous and saturated models", 1991 (supervisor S.S. Goncharov).

B.S. Kalenova, "Some questions of theory of constructive abelian groups", 1997 (supervisor N.G. Khisamiev);

S.Zh. Karatabanova, "Low estimates of algorithmic complexity and structure of semantic sentences", 1999 (supervisor M.G. Peretyat'kin);

I.V. Latkin, "Problems of natural subgroups in constructive groups", 2000 (supervisor S.S. Goncharov);

M.K. Nurizinov, "On computable nilpotent groups", 2016 (supervisors N.G. Khisamiev, A. Sorbi);

R.K. Tyulyubergenev, "Constructive nilpotent groups", 2017 (supervisors N.G. Khisamiev, A. Sorbi);

A.A. Konyrkhanova, "Computable decidable groups", 2017 (supervisors N.G. Khisamiev and V. Roman'kov).

Doctor of science dissertations:

N.G. Khisamiev, "Constructive abelian groups", 1990;

V.P. Dobritsa, "Computable classes of constructive models", 1991 (supervisors S.S. Goncharov, N.G. Khisamiev);

D.A. Tussupov, "The problems of definability and algorithmic complexity of relationships over algebraic structures", 2007 (supervisors S.S. Goncharov, S.A. Ba-daev).

3. AXIOMATIZABLE CLASSES OF (ALGEBRAIC) STRUCTURES

Let K be a class of structures of a fixed language, and $Th(K)$ be the set of all sentences of the language true on all structures of the class K . For a given theory T $Mod(T)$ denotes the set of all models of the theory T .

Main problem: when $K = Mod(Th(K))$?

Connection of the class of structures with a set of formulas true in all structures of this class (incomplete elementary theories) is defined by the following objectives: 1) to establish the minimal number of formulas, from which all the rest of the formulas logically true in all structures of this class, are logically implied by – axioms of the class, in extremis, all the true formulas can be taken as the axioms; 2) to determine the properties of the set of axioms: a special kind of axioms (universal \forall -formulas, existential \exists -formulas, $\forall\exists$ -formulas, $\exists\forall$ -formulas, identities, quasi-identities, Horn formulas, etc.), finiteness/infiniteness of the set of axioms; 3) to describe axiomatizable classes (formulas) stable with respect to direct, boolean, filtered products and ultraproducts; 4) to check whether the class of structures satisfy the joint embedding property (JEP) and the amalgamation property (AP), and finding such special classes of models.

This area can be divided into the following subdirections:

a) Direct products, filtered products and ultraproducts [Keisler, Shelah, Taimanov, Omarov].

This sub-directions had two main tasks. First is G. Keisler's hypothesis that two models are elementarily equivalent if and only if under a suitable ultrafilter their ultra-degrees are isomorphic [94, 96] (1961, 1967). G. Keisler proved the theorem using the continuum hypothesis. The proof of this theorem without use of the continuum hypothesis was given by S. Shelah [292] (1971). The second main task is to describe formulas stable with respect to various types of products. J. Łoś proved that all formulas are stable under ultraproducts [176] (1955).

In the paper of A.D. Taimanov [309] (1960) there is a characteristic of formulas reducible to a Horn form (conditional class), and there is an example of a multiplicatively closed formula that is not reducible to a Horn form. The kinds of multiplicatively closed formulas are indicated.

In the works of A.D. Taimanov [316] (1966) and J.A. Almagambetov [3] (1965), non-recursiveness of classes of filtered products in the case when the signature contains at least one non-unary predicate, and recursiveness if it does not contain multi-ary predicates have been proved. Continuing those works Omarov Amangeldy Iskakovich determined formulas, stable (filterable) with respect to filtered products under a filter containing an atomless element stable (filter) with respect to the filtered pro-reference of the filter containing an atomless element (P-formulas from

[242] (Palyutin, 1980)) [235, 236] (1991), described (multiplicative) formulas, filterable under the Cartesian product [238] (1993). Those works ended a cycle of works by A.I. Omarov [226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237] (1967-1991), dedicated to a variety of issues on filtered products, reduced degrees, and ultraproducts, in which A.I. Omarov answered a number of questions posed by foreign mathematicians L. Pacholski and B. Węglorz on equationally compact algebras and G. Keisler and S. Burris on B-separable algebras [95] (Keisler, 1965), [56] (Burris, 1975). In particular, A.I. Omarov proved non existence of a B-separable abelian group; assuming the Continuum Hypothesis he proved that k -saturation of the Boolean degree implies k -saturation of the Boolean algebra [233]. Algebras are called B-separable, if isomorphism of Boolean degrees implies isomorphism of the Boolean algebras (Johnson, Tarski). Together with the Łoś's result [176] (1955) and Taimanov' result [309] (1960), the last works of A.I. Omarov on P-formulas [236] (1991) and multiplicative formulas [238] (1993) constitute a complete finished picture.

b) Special types of axioms. Varieties and quasivarieties.

In 1961 A.D. Taimanov, while continuing the works of Łoś [176] (1955) and Chang [58] (1959), gave the characteristics of axiomatizable classes of models and classes of models described by axioms of $\forall\exists\forall\exists$ -formulas and $\exists\forall\exists\forall$ -formulas type. He proved theorems that are amplifications of well-known theorems of Tarski-Łoś, Łoś-Sushko, Chang, and others. In particular, the announced characteristics of axiomatizable classes from works of Łoś [176] (1955) and Mycielski [219] (1957), found their proof in [311] (Taimanov, 1961). A.D. Taimanov [312] (1961) investigated the conditions for reducibility of formulas to formulas of a special form, through examination of equivalence of two systems of axioms, using a third system of axioms; as a consequence he gets a strengthening of the A. Robinson's theorem on sigma-stability and reducibility to existential formulas with free variables [282] (1959).

Zh.A. Omarov [240] (1988) explored varieties of different algebras. He studied the concept of a characterizable class of structures. He proved existence of characterizable varieties of lattices, some of which in the lattice of all varieties of lattices is not a characterizable variety. Zh.A. Omarov found two lattices, co-intersection of which in the lattice of quasivariety of lattices is a variety, as well as constructed an example of characterizable locally finite variety of lattices [241] (2012).

A.T. Nurtazin in a series of works investigated existentially closed models and obtained the following results: the criterion to be existentially closed model for arbitrary universally axiomatizable theory [223](2016); a criterion for an arbitrary theory to be an elementary theory of its existentially closed models [224] (2018); applying the method of forcing to prove that any $\exists\forall\exists$ -sentence true in some forcing structure is satisfied in any existentially closed model [225](2018)

c) Finite axiomatizability [Kleene, Hanf, Vaught, Taimanov, Peretyat'kin, Zilber].

In 1961 A.D. Taimanov gave a description of a finite axiomatizability for axiomatizable classes meeting certain conditions [313, 314] (1961). In 1965, in his famous work, Michael Morley raised several questions on categorical theories, including the question about finite axiomatizability of categorical theories. In 1980, M.G. Peretyat'kin constructed an example of a finitely axiomatizable categorical in uncountable cardinalities theory and, on the basis of his solution of the M. Morley's question on existence of finitely axiomatizable uncountably categorical theory [251,

254] (1980, 1986), he strengthens the Hanf's theory [83, 84] (1965, 1975) on similarity of properties of finitely axiomatizable and recursively axiomatizable theories [252, 253] (1982). This theme became central to his scientific work. M.G. Peretyat'kin continues finding model-theoretic properties that connect finitely axiomatizable and recursively axiomatizable complete theories [255, 256, 257, 260, 261, 262, 263, 264] (1989-1997). One of these works was included into a list of works in Russian language, recommended by the American Mathematical Society for translation into English [265] (Peretyat'kin, 1999).

Notice that M.G. Peretyat'kin was the first who constructed an example of a complete finitely axiomatizable uncountably categorical theory with rank of $x = x$ is equal to 4 [251] (1980). Later, A.D. Taimanov gave a new simpler example of such a theory [318] (1989) of $r(x = x) = 3$. M.G. Peretyat'kin was the first who constructed an example of a complete finitely axiomatizable uncountably categorical theory with rank of $x = x$ is equal to 2 [256] (1991). There is no answer for the question on existence of a complete uncountably categorical theory with rank of $x = x$ is equal to 1.

As a matter of fact, M. Morley formulated two problems of existence of a finitely axiomatizable theory: one for uncountably categorical but not omega-categorical theory, and the other for everywhere categorical theory. The second problem was solved by B.I. Zilber [349, 350] (1980, 1981), who proved nonexistence of a finitely axiomatizable everywhere categorical theory. The technique of the proof offered by B.I. Zilber turned out to be in demand in model theory, and found its development for other classes of theories [59] (Cherlin-Harrington-Lachlan, 1985), as well as served as the basis for the so-called geometric theory of stability.

d) Similarity of theories, mutual interpretability of models.

Each theory can be associated with the Boolean algebra of its definable sets (the Boolean algebra of Lindenbaum-Tarski). We can say that theories are similar in their properties, if their Boolean algebras coincide. As mentioned above, M.G. Peretyat'kin, continuing Hanf's idea of recursive isomorphism of Lindenbaum algebras of a recursively axiomatizable and finitely axiomatizable theory, explored issues of similarity of finitely axiomatizable and infinitely axiomatizable theories [256, 257, 263, 264] (1991, 1997). M.G. Peretyat'kin defined the notion of a model-theoretic property and develops this concept [258, 259], (1991) [266, 267, 268] (2013-2015). In particular, he asked the question: "Is o-minimality a model-theoretic property?" In [137] (2015) K.Zh. Kudaibergenov proved that o-minimality is not a model-theoretic property. In addition, he proved that the property of having a prime model is not preserved with respect to mutual interpretability of theories [137] (2015).

B.I. Zilber has used the concept of rational equivalence of two structures, when there is a bijection between the structures, and all definable subsets are mapped into definable subsets of the second structure. He studied strongly minimal quasiurbanik structures, that is, structures whose definable closure coincides with algebraic closure, and proved that a strongly minimal quasiurbanik structure is rationally equivalent to one of the following three structures: 1) a group of bijections on an infinite set; 2) a vector space over a division ring with a subspace of selected items; 3) an affine space over a division ring with a subspace of selected transfers [351] (1989).

T.G. Mustafin introduced two kinds of similarity notions, semantic and syntactic [211, 212] (1990, 1993). To determine the semantic similarity, first the semantic

characteristics of the theory is given, as a triple, which includes the class of elementary substructures of some large saturated model, the group of all automorphisms and the class of all universes and substructures. Two theories are semantically similar, if there is a special mapping of one semantic triple of one theory into a semantical triple of the other theory. Syntactical similarity is defined as equality of Boolean algebras of n -definable sets for each positive integer n . The concepts of “proximity” of theories and the concept of quasi-similar theories were introduced, and the characterization of theories quasi-similar to unary predicates was given [211] (Mustafin, 1993). The concept of a polygon was introduced by L.A. Skornyakov [300](1969), and P. Berhause [51] (1967) (as an S-act). T.G. Mustafin proved that for every theory there is a corresponding polygon theory which is “close” to the given one [208] (1998). T.G. Mustafin prepared a great work on Jonsson classes, in which he tried to transfer techniques and approaches developed for the study of complete theories, in particular the stability theory onto the study of incomplete theories (axiomatizable classes of models closed under Jonsson conditions). Unfortunately, this work was published four years after after his early departure from life [213] (1998). His apprentice A.R. Yeshkeyev continues studying properties of Jonsson theories.

A.R. Yeshkeyev and O.I. Ulbrikht obtained criteria for cosemanticness of both modules and abelian groups [353, 355]. A.R. Yeshkeyev and B. Poizat proved a criterion for k -extensibility of a model of a positive Jonsson theory [279]. A.R. Yeshkeyev, M.T. Kassymetova and N.K. Shamatayeva obtained both a criterion for ω_1 -categoricity of the $\#$ -companion of a perfect Jonsson fragment and a criterion for positive model completeness of the $\#$ -companion of a perfect Jonsson fragment [354]. Observe that three PhD students defended their theses under supervision of A.R. Yeshkeyev in 2019.

In the class of models of some theory in a countable first-order language, M.I. Bekenov considered relationships between isomorphism, elementary embeddability, and elementary equivalence. A model \mathfrak{A} is *elementarily embedded* into a model \mathfrak{B} if there exists an isomorphism between \mathfrak{A} and an elementary submodel of \mathfrak{B} .

Let $\omega \leq \lambda \leq \mu$, \mathfrak{A} and \mathfrak{B} models of T in a language \mathcal{L} , $|A| = |B| = \mu$. The models \mathfrak{A} and \mathfrak{B} are λ -similar, if for any $\mathfrak{A}' \prec \mathfrak{A}$ and $|A'| \leq \lambda$ it follows that $\mathfrak{A}' \prec \mathfrak{B}$ and for any $\mathfrak{B}' \prec \mathfrak{B}$ and $|B'| \leq \lambda$ it follows that $\mathfrak{B}' \prec \mathfrak{A}$.

Assume that T is a theory and $\omega \leq \lambda \leq \mu$. M.I. Bekenov states that $B_T(\lambda, \mu) = 1$ is the spectral function if it computes the number of classes of models of this theory of cardinality μ relative to λ -similarity. In particular, he proved [44] (2018) that if all models of a theory T are infinite, the union of any chain of models of T is a model of T and $B_T(\lambda, \mu) = 1$ for some λ and μ such that $\omega \leq \lambda \leq \mu$, then T is model complete.

e) Special models of axiomatizable classes (prime, homogeneous, saturated) and properties of algebraic structures (automorphisms groups).

Part of the works of K.Zh. Kudaibergenov was devoted to study of homogeneous models. He solved Keisler and Morley’s problem on the number of homogeneous models in various cardinalities [113] (1988). He disproved the hypothesis of Shelah about cardinality of absolutely homogeneous models and in some way related to it Keisler and Morley’s hypothesis [112] (1987). He also investigated homogeneous models with a condition weaker than stability, the condition of stability of a diagram [121] (2002). By K.Zh. Kudaibergenov questions on homogeneity for

specific algebraic structures were studied. He as well investigated homogeneous modules [123] (2004) and linear orders [126] (2008). By him a countable strictly 2-homogeneous distributive lattice [131] (2012) was constructed, the question of existence of which was raised by Macpherson and Droste. In general, the concept of homogeneity has been studied from various sides. It was proved that the notion of homogeneity is not absolute in the sense of set theory [118] (Kudaibergenov, 2000). K.Zh. Kudaibergenov investigated the question of conservation and non-conservation of homogeneity under a specific model expansion, a conditions under which homogeneity of a model implies homogeneity of an expanded model were obtained. He also showed that, in general, homogeneity of a model does not entail homogeneity of the expanded model [127] (2009).

In the work [11] (2006) K.A. Meirembekov, A.T. Nurtazin and E.R. Baisalov investigated rings, which are definably minimal, it was proved that a definably minimal associative ring either Jordan ring, alternative ring, or a ring of characteristic zero is a field. E.R. Baisalov studied linear minimal rings and algebras as well as Lie algebras [12, 13] (2012, 2013), [4] (Al'dzhuie-Baisalov, 2016).

K.Zh. Kudaibergenov solved the Hodges's problem on possibility of transferring one of the Fraïssé theorems to the case of an uncountable class of finitely generated structures of a given language: he proved that in the general case such a transfer is impossible, and in the case of a finite language an analog of the Fraïssé theorem holds [139] (2017). In [224] (2018) A.T. Nurtazin studied forcing formulas in Fraïssé structures and classes.

f) Automorphisms. Generic automorphisms of models were studied. It was proved that every fixed field of a generic automorphism of a separably closed field is regularly closed, and its absolute Galois group was calculated [119] (2001). This generalizes the corresponding Macintyre's results on algebraically closed fields. In [122] (Kudaibergenov, 2003) generic automorphisms of models were studied; they were used in proving existence of dense free subgroups of automorphisms groups of homogeneous models [132], what supplements the results of Melles and Shelah obtained for saturated models. J.T. Baldwin [39] (1973) while studying automorphisms of uncountably categorical theories raised the question on automorphisms of Morley's tower. K. Zhetpisov answered this question [352] (1990). W. Hodges with his group in London on a seminar in 1988 examined relationship between an automorphism groups and coordinatizability over a unary predicate. Hypothesis about coincidence of these concepts were expressed. Independently and in different ways B.S. Baizhanov [17, 19] (1989, 1990) and E. Hrushovski built counterexamples to those hypotheses.

K.Zh. Kudaibergenov studied the small index property. There are two definitions of this important notion; W. Hodges and D. Macpherson raised the question of their equivalence. K.Zh. Kudaibergenov proved that from the point of view of the ZFC set theory these definitions are equivalent [136] (2014).

K.Zh. Kudaibergenov developed a general approach that made it possible to uniformly prove the small index property and solve the question of (strong) cofinality of an automorphism group for some classes of models [138] (2016).

In this direction the following and PhD and doctor of sciences dissertations of scientists from Kazakhstan were defended:

PhD theses:

A.I. Omarov, “Some applications of filtered products in model theory”, 1967 (supervisor A.D. Taimanov);

T.G. Mustafin, “Some questions of axiomatizable classes of models”, 1971 (supervisor A.D. Taimanov);

R.T. Kel’tenova, “Equational compactness of some algebras”, 1975 (supervisors T.I. Amanov and A.I. Omarov);

K.A. Nauryzbaev “Equational compactness of complete continuous distributive lattices”, 1987 (supervisor A.I. Omarov);

Zh.A. Omarov, “Characterization of certain classes of lattices”, 1993. (supervisor V.A. Gorbunov);

K. Zhetpisov, “The number of automorphisms of models of superstable theories”, 1992 (supervisor T.G. Mustafin);

A.R. Yeshkeyev, “Johnson theories”, 1995 (supervisor T.G. Mustafin);

B.K. Dauletbayev, “Groups of automorphisms of boolean algebras”, 2000 (supervisor A.S. Morozov);

P. Dossanbai, “Definable fragments of arithmetic structures”, 2006 (supervisors A.I. Omarov and E.R. Baisalov);

A.M. Kungozhin, “Some properties of universally axiomatizable classes”, 2012 (supervisors N.T. Danaev, A.T. Nurtazin and B. Poizat);

N.K. Shamataeva, “The structure of convex existential prime Jonsson theories and their classes of models”, 2019 (supervisors A.R. Yeshkeyev and B. Poizat);

M.T. Kassymetova, “Model-theoretic properties of companions of Jonsson sets fragments”, 2019 (supervisors A.R. Yeshkeyev and B. Poizat);

O.I. Ulbrikht, “Classification of models of Jonsson theories with respect to cosemantic equivalence”, 2019 (supervisors A.R. Yeshkeyev and B. Poizat).

Doctor of science dissertations:

M.G. Peretyat’kin, “Finitely axiomatizable theories”, 1984;

A.I. Omarov, “P-formulas and Boolean constructions in model theory and universal algebra”, 1992.

A.R. Yeshkeyev, “Structure of perfect positive Johnson theories”, 2010 (supervisor S.S. Goncharov);

4. NUMBER OF NON-ISOMORPHIC MODELS

Two models of a fixed language are *isomorphic* if there exists a bijective mapping from one model onto another which preserves basic relations and operations. Obviously, if two models are isomorphic, then their universes have same cardinalities.

J. Łoś in 1954 [175] conjectured if a complete theory is categorical in some uncountable cardinality, then it is categorical in all other uncountable cardinalities. M. Morley [197] in 1965 confirmed the Łoś’s hypothesis and proved homogeneity of all models of categorical theories, while changing the quality of research in model theory, systematically introducing methods of working with types (locally consistent sets of formulas), through introducing ranks of the types and formulas based on study of category of topological spaces of n -types and elementary embeddings. This article, as well as Baldwin-Lachlan’s article [42] (1971), played an important role in development of model theory throughout the next two decades. M. Morley formulated a list of unsolved problems on uncountably categorical theories, which included, besides the above-mentioned question on finite axiomatizability,

the question of finiteness of Morley rank, a question on the number of countable non-isomorphic models, suggesting that it may not be finite. J.T. Baldwin [38] (1973) and independently, B.I. Zilber [348] (1974), proved finiteness of Morley rank for uncountably categorical theories. T.G. Mustafin studied countable models of uncountably categorical theories with restriction on models [200] (1968). T.G. Mustafin and A.D. Taimanov, with an additional condition on the Morley tower (an increasing sequence of elementarily embedded models) given, proved non-finiteness of the number of countable models [218] (1970). The final solution to the Morley's problem on the number of countable models was given in the work by J.T. Baldwin and A. Lachlan [42] (1971), in which they proved that the number of countable models of an uncountably categorical theory can be either 1 or countable. In addition, they reproved M. Morley's theorem, meanwhile establishing that every model of such a theory is characterized by a dimension of a strongly minimal formula. This work defined the nature of research in model theory, in particular for questions related to counting the number of non-isomorphic models, the idea of a dimension began to play a decisive role.

A *spectrum* of a complete theory is a function that assigns to a cardinal λ the number of models of cardinality λ of the given theory up to an isomorphism, $I(T, \lambda)$.

Main problem: To prove that for every complete theory the spectrum function is non-decreasing for uncountable cardinals.

Saharon Shelah in a series of papers [291, 293, 294, 295] (1971, 1972, 1974) proved that for a class of non-stable theories, and stable but not non-superstable theories, such a function on uncountable cardinals takes the maximum value. While doing so, he developed the stability theory, now it had become classics in model theory [296] (1978). In addition, it became clear that for the class of totally transcendental theories and the class of superstable but not totally transcendental theories the spectrum functions will be different, and it is necessary to conduct the research of properties of models of these theories by means of rank functions.

4.1. Spectrum and rank functions.

4.1.1. *Spectrum of totally transcendental theories.* A theory in which each type has an ordinal Morley rank is called *totally transcendental*. Every countable ω_1 -categorical theory is totally transcendental, the class of countable totally transcendental theories coincides with the class of omega-stable theories [197] (Morley, 1965). S. Shelah proved that an omega-stable theory has a saturated model for every cardinality $\lambda \geq \omega$. For a totally transcendental theory two prime over some set models are isomorphic over this set [293] (1972). A. Lachlan investigated totally transcendental theory of rank 2. He gave a complete description of all possible spectrum functions of rank 2 and degree 1 [168] (1976). B.S. Baizhanov extended the full description of spectrum functions for rank 2 and arbitrary degree n [14] (1980), meanwhile he specified the list of spectrum functions for the degree 1. A. Lachlan in 1978 made an important step in the study of spectral functions, proving that in class of totally transcendental theories, there is no constant functions except for uncountably categorical, and the most important, that every function is non-decreasing.

Theorem [169] (Lachlan, 1978). If T is a totally transcendental theory, then for the spectrum S_T one of the following possibilities holds:

- 1) $S_T(\omega) \in \{1, \omega\}$ and $S_T(\omega_\alpha) = 1$ for all ordinals $\alpha \geq 1$;

- 2) $S_T(\omega_\alpha) = |\alpha + 1|^\omega$ for all ordinals α and $S_T(\omega_\alpha) = |\alpha|$ for $\alpha \geq \omega$;
- 3) $S_T(\omega_\alpha) = |\alpha + 1|^\omega$ for all $\alpha \geq 1$;
- 4) $S_T(\omega_\alpha) \geq \omega^{|\alpha|}$ for all ordinals α .

Of course, the case 4) in this classification has great uncertainty. A. Lachlan conjectured that in this case spectrums of totally transcendental theory are limited to the following range:

- a) $S_T(\omega_\alpha) = \omega^{|\alpha|}$, $\alpha \geq 1$;
- b) $S_T(\omega_\alpha) = 2^{\omega_\alpha}$, $\alpha \geq 1$;
- c) $S_T(\omega_\alpha) = \max(2^\omega, \omega^{|\alpha|})$, $\alpha \geq 1$.

B. Baizhanov extended this list [15] (1980), he constructed for every ordinal $\gamma < \omega_1$ totally transcendental theories with the following spectrums:

- d) $S_T(\omega_\alpha) = \min(2^{\omega_\alpha}, \beta(|\alpha + 1|, \gamma))$, $\alpha \geq 1$ (where the cardinal $\beta(\chi, \alpha)$ is defined by induction and is the standard definition in axiomatic set theory);
- e) $S_T(\omega_\alpha) = \min(2^{\omega_\alpha}, \beta(|\alpha + 1|^\omega, \gamma))$, $\alpha \geq 1$.

About this B.S. Baizhanov's extension it was told in the review article by E.A. Palyutin [243] (1982). The works of A. Lachlan [168] (1976) and B.S. Baizhanov [16] (1980) identified a condition that provides maximality of the number of non-isomorphic models in all uncountable cardinalities (by A. Lachlan for rank 2, by B.S. Baizhanov it was generalized to the class of omega-stable theories), based on dimensions of types associated with various copies of one formula, defined by different constants, connected by a non-trivial relation in the realization of a type (connected type). In the next decade the results of A. Lachlan and B.S. Baizhanov were strengthened, absorbed and blocked by numerous at that time works dedicated to spectrum of omega-stable and superstable theories. For omega-stable theories, the condition of existence of a connected type re-opened in other terms (later named by S. Shelah, the dimensional order property, B.S. Baizhanov [18] (1989)), together with the condition of an infinite depth constituted a necessary and sufficient condition for maximality of the spectrum of omega-stable theories.

4.1.2. *Spectrum of superstable theories.* Finally the spectrum problem was solved by Shelah in the second half of the 1980's for the class of superstable theories, and hence for the class of all complete countable theories [297] (1986), and Hart-Hrushovski-Laskowski carefully considered all non-obvious places in the proof of Shelah, closing all gaps in the proof [86] (2000). Note that the list of spectral functions for superstable theories is different from the list for omega-stable theories with adding e'), where instead of ω in the exponent in the definition of beta function e) 2^ω is used.

4.1.3. *Stable theories and rank functions.* In the 1970s – 1980s, the central object of study of model theory were stable theories and different properties characterizing the models and formulas in them: rank functions, properties of formulas and types, one-cardinal, two-cardinal formulas, spectral functions for the class of homogeneous models of a complete theory, manifolds, quasi-manifolds, universals, and various classes of structures. We will not dwell on all these issues, obtained by foreign scientists, just we note an article of Daniel Lascar and Bruno Poizat [170] (1979) with excellent representation of Shelah's theory of forking by fundamental order on types and the book of John Baldwin on fundament of Shelah's stability theory [40](1988) with complete bibliography.

Let us consider the works of Kazakh scientists. In 1977 T.G. Mustafin, developing an analogue of Marsh's theory from the famous work of J.T. Baldwin and A. Lachlan [42] (1971), described spectral functions for the so-called quasitranscendental theories with a strong base [201] (1977). It is interesting to note that later in 1978, in the book by S. Shelah [296], the notions of a regular type, orthogonal regular base and non-multidimensional superstable theory generalize, respectively, of the notions of a strong type, strong base and quasitranscendental theory with a strong base. K.J. Kudaibergenov comprehensively studied homogeneous models of stable theories [115] (1993), including homogeneous models of one-dimensional theories [116] (1995), locally modular theories of finite rank [120] (2002), and weakly minimal theories [117] (1999). He describes the range of homogeneous models of totally transcendental non-multidimensional theories [114] (1991).

In 1971 J.T. Baldwin and A. Lachlan proved that an omega-stable theory with a non-two-cardinal strongly minimal formula is omega-categorical [42] (1971). Later, A. Lachlan had shown omega-stability of an arbitrary theory with non-two-cardinal strongly minimal formula [167] (1975). In 1975 as well, M.M. Erimbetov proved a stronger statement: if a theory has an omega-stable non-two-cardinal formula, then it is omega-stable [72]. In 1985 M.M. Erimbetov proved stability of a theory, having a non-two-cardinal stable formula [74] (1985). It should be noted that Erimbetov's theorem of the year 1975 was included into all textbooks on the model theory, as the Erimbetov's formula.

T.G. Mustafin strengthened the results as follows: if there exists a non-two-cardinal formula stable in some cardinal, then in the same cardinal would exist a stable theory [203, 204] (1980). T.G. Mustafin studied rank functions, introduced axiomatization of rank functions [202] (1980), found the conditions which guarantee normality of rank functions [207] (1985), and with T.A. Nurmagambetov they established connection with forking of Shelah types [215, 216] (1982, 1983). M.I. Bekenov and T.G. Mustafin explored splitting of types in stable theories [45, 46] (1979, 1981). T.G. Mustafin proved theorems on normalizing formulas for stable theories [206] (1981). V.S. Bogomolv, T.G. Mustafin and B. Poizat studied stable polygons, [52] (Bogomolv-Mustafin, 1989), [208, 210] (Mustafin, 1988, 1990), [217] (Mustafin-Poizat, 1995). One of these works was included into a list of works in Russian language, recommended by the American Mathematical Society for translation into English [214] (Mustafin, 1999). In general, Tulendy Garifovich Mustafin stood out by his ability to define new profound concepts [209, 211, 208] (1990, 1988), which attract attention and in future become an object of study [244, 245, 246] (Palyutin, 2013, 2014), [301] (Stepanova, 2014).

In 1980s – 1986s K.A. Meirembekov studied spectral functions of Abelian groups. Here it should be mentioned that the works with a description of Abelian groups with bounded spectrum, that is the range of groups which spectrum do not exceed two to the power of continuum [192] (1980). With help of this work it turned out to be possible to describe all possible spectra of Abelian groups [193] (1981). It turned out, in general terms, that if the spectrum of an Abelian group is not maximal, then it does not exceed α to the power of continuum. It was proved that Abelian groups do not have the finite cover property [194] (1982). In the work of K.A. Meirembekov [195] (1986) these concepts are generalized onto groups that do not have infinite conjugacy classes. In [196] (1993) K.A. Meirembekov and K.M. Shegirov described spectra of stable geometric lattices.

Hyttinen and Lessmann introduced a rank for a superstable diagram and noted that finding a rank for a stable diagram is more problematic. In [141] (Kudaibergenov, 2019), such rank functions are constructed that if any D-type has an ordinal-valued rank, then the diagram D is stable; and if the diagram D is stable, then there are a lot of non-algebraic D-types having ordinal-valued rank, which means density in some natural topology.

4.2. Number of countable models.

4.2.1. *The Vaught's Conjecture.* The number of countable non-isomorphic models of a countable theory can be either finite, countable, have cardinality of a continuum, or have an intermediate cardinality between a countable set and the continuum [198] (Morley, 1970) ($I(T, \omega) \in \omega \cup \{\omega, \omega_1, 2^\omega\}$). Vaught proved [333] (1961), that this number can not be equal to 2. A theory is called *small* if the number of its n -types is not maximal for every finite n . If a theory is not small, the number of its countable models is maximal, that is, 2^ω . For the class of omega-stable theories the conjecture was confirmed by S. Shelah, L. Harrington and M. Makkai in [298] (1984). Laura Mayer, using D. Marker's theory of orthogonality of 1-types in o-minimal theories [186] (1986), confirmed the Vaught conjecture for the class of o-minimal theories [191] (1988). B.Sh. Kulpeshov and S.V. Sudoplatov confirmed the Vaught's conjecture for quite o-minimal theories [161] (2017). A. Alibek, B.S. Baizhanov, B.Sh. Kulpeshov, and T.S. Zambarnaya proved the Vaught's conjecture for weakly o-minimal theories of convexity rank 1 [1] (2018) and gave a description of their countable spectrum.

At the moment, there is no answer on the Vaught conjecture, but model theory experts continue to work on it, among them B.S. Baizhanov, S.V. Sudoplatov and V.V. Verbovskiy [31] (2012), S.V. Sudoplatov [305] (2014). A. Alibek, B.S. Baizhanov, and T.S. Zambarnaya proved maximality of the number of countable models of theories having a quasi-successor formula [2] (2014). B.Sh. Kulpeshov and S.V. Sudoplatov confirmed almost ω -categoricity of Ehrenfeucht quite o-minimal binary theories [162] (2017). B.S. Baizhanov, J.T. Baldwin and T.S. Zambarnaya introduced various kinds of trivial types in small theories and proved that a theory of (an expansion of) linear order which has an extremely trivial type has 2^ω countable non-isomorphic models [34] (2018). In [157] (2019) B.Sh. Kulpeshov gave a criterion for the countable spectrum to be maximal in small binary quite o-minimal theories of finite convexity rank.

In model theory, there is a method of transfer from a theory T to a theory $T \cup p(\bar{c})$, where $p(\bar{x})$ is a complete type. Such an extension is called an unessential extension because for any model of T realized type p the set of definable sets is the same. This transfer preserves basic properties such as stability, model completeness, and uncountable spectrum. R. Woodrow [347] (1978) constructed examples of theories with four countable models such that an unessential expansion has a countable set of nonisomorphic models. M.G. Peretyatkin [250] (1980) answering the question of R. Woodrow built an example of a theory with three models and an unessential expansion having countable number of countable models. B. Omarov for the first time proved that unessential expansion can decrease number of countable models of initial theory to expanded theory from \aleph_0 to finite number and from 2^{\aleph_0} to \aleph_0 and finite number [239] (1983).

The first author of the article, as a PhD student talking with his supervisor A. Taimanov, asked that “if there is no continuum hypothesis, then there is no Vaught’s conjecture, and therefore this does not require research”. To which A. Taimanov replied that there are two reasons for continuing the research. The first reason is model-theoretical, indicating the number of nonisomorphic models of a complete theory, we indicate why they are not isomorphic, and therefore explain the nature of models and formulas, what leads to classification of complete theories. The second, set-theoretical reason, is that the number of countable models is a concept that arose in a very natural way, and thus, confirmation of the Vaught’s conjecture would be a weighty argument in favor of the continuum hypothesis.

On the other hand, S. Shelah comments [296] (1990) on Vaught’s Conjecture “Some people think this is the most important question in model theory as its solution will give us an understanding of countable models which is the most important kind of models. We disagree with all those three statements.”

4.2.2. *The Lachlan problem.* After A. Lachlan proved that every superstable theory can not have a finite number of non-isomorphic models except 1 [166] (1973), he formulated the problem that existence of a stable theory with finitely many non-isomorphic countable models. T.G. Mustafin proved that if a stable theory has a non-principal superstable type, then the number of countable non-isomorphic models can not be finite [205] (1981). S.V. Sudoplatov constructed a stable theory that has finitely many non-isomorphic countable models [304] (2009). He also obtained a classification of countable models of complete theories with respect to two basic characteristics: Rudin-Keisler preorders and distribution functions for numbers of limit models [305] (2014, 2018).

4.2.3. *Isolating formulas.* B.Sh. Kulpeshov, D.Yu. Emelyanov and S.V. Sudoplatov obtained a description of algebras of distributions for binary isolating formulas over an 1-type of \aleph_0 -categorical weakly o-minimal theories [69] (2017), they as well obtained a criterion for generalized commutability of an algebra of distributions for binary isolating formulas over a pair of 1-types in \aleph_0 -categorical weakly o-minimal theories [69] (2017), and a description of algebras of distributions of binary isolating formulas over an 1-type for quite o-minimal theories with non-maximal number of countable models [70] (2019). B.Sh. Kulpeshov and S.V. Sudoplatov obtained a description of hypergraphs of models of a theory in terms of freedom and independence in general case and for some natural classes of theories [163] (2018), and a characterization for relative separability of hypergraphs of models of a theory both in a general case and for almost \aleph_0 -categorical quite o-minimal theories [164] (2018). In [9] (2018) K.A. Baikalova, D.Yu. Emelyanov, B.Sh. Kulpeshov, E.A. Palyutin and S.V. Sudoplatov obtained a description of algebras of distributions for binary isolating formulas of theories of abelian groups and some of their ordered expansions.

The following PhD and doctor of science dissertations had been defended in this area:

PhD theses:

M.M. Erimbetov, “On some questions of countable complete theories related to the concept of rank”, 1975 (supervisor A.D. Taimanov);

B.S. Baizhanov, “Spectral questions of totally transcendental theories of finite rank”, 1981 (supervisor A.D. Taimanov);

B.I. Omarov “Constant expansions and rank functions of complete theories”, 1983 (supervisors *A.D. Taimanov, M.G. Peretyat'kin*);

M.I. Bekenov “Properties of superstable theories”, 1984 (supervisor *A.D. Taimanov*);

T.A. Nurmagambetov, “Almost ω -stable theories”, 1984 (supervisor *A.D. Taimanov*);

K.A. Meirembekov, “Spectra of elementary theories of groups close to Abelian”, 1985 (supervisor *M.G. Peretyat'kin*);

E.R. Baisalov, “Countable models of superstable theories” 1991 (supervisor *A.D. Taimanov*);

K. Shegirov, “Stable lattices”, 1992 (supervisor *E.A. Palyutin*);

A.A. Vikentyev, “Theories with a cover and definable subsets”, 1992 (supervisors *A.D. Taimanov and E.A. Palyutin*);

T.S. Zambarnaya, “Countable models of small dependent theories”, 2019 (supervisors *B.S. Baizhanov and J.T. Baldwin*).

Doctor of science dissertations:

T.G. Mustafin, “Stable theories”, 1990;

K.Zh. Kudaibergenov, “Homogeneous models”, 1990;

5. EXPANSIONS OF MODELS AND THEORIES

We say that a model $\langle M, \Sigma^+ \rangle$ is an **essential expansion** of a model $\langle M, \Sigma \rangle$, if Σ^+ contains Σ , and there is an n -ary formula $\varphi(x)$ of the signature Σ^+ such that the set $\varphi(M^n)$ is not definable with parameters from the model $\langle M, \Sigma \rangle$.

Main problem: to find conditions on new relations, under which the desired properties of the initial model are preserved.

Among such properties are the following: decidability of an elementary theory of an expanded model (for example, decidability of the theory of the field of real numbers, expanded with a unary exponential function), model completeness, stability, superstability, omega-stability, strong minimality, weak and strong o-minimality, finite cover property, absence of a formula with the independence property, and so on. As a new predicate, non-definable in the original model, elementary submodels, selected automorphisms, indiscernible subsets, and just a subsets without any restrictions (the general case). As a result of studies carried out in 60s – 90s years of the last century the class of complete theories, depending on the behavior of definable sets definable and systems of definable sets, had been divided into the following classes of complete theories:

I_1 — the class of strongly minimal theories;

I_2 — the class of o-stable theories;

I_3 — the class of superstable theories;

I_4 — the class of stable theories;

I_5 — the class of o-minimal theories;

I_6 — the class of weakly o-minimal theories;

I_7 — the class of dependent theories;

Note that, $I_1 \subset I_2 \subset I_3$ and $I_5 \subset I_6 \subset I_7$. Then the expansion problem can be specified (narrowed) as follows:

If $\langle M, \Sigma \rangle \in I_j$, then which conditions on the introduced relations are necessary and/or sufficient in order to have $\langle M, \Sigma^+ \rangle \in I_k$, where $I_j \subseteq I_k$?

I_1 — strongly minimal theories.

A theory is called *strongly minimal* if definable set of every formula either itself, or its negation is finite in itself [42] (1971). A. Macintyre studied expansions of an algebraically closed field by unary function that is an automorphism. Within the framework of studying spectrum of complete theories and the question of final axiomatizability, B. Zilber began to study the classification of strongly minimal theories. He proposed a hypothesis about geometry of strongly minimal theories, that they can only be of three types: trivial, locally modular or expansions of an algebraically closed field. In 1988 E. Hrushovski constructed a counterexample to this conjecture — an example of a strongly minimal non-locally-modular theory, in which a group is not interpreted, and which can not be interpreted into a field. V.V. Verbovskiy proved that an elementary theory of this example does not admit elimination of imaginary elements [336, 337] (2002, 2006). E. Hrushovski, responding to H. Cherlin's question about existence of a strongly minimal theory, maximal in the sense of impossibility of its own strongly minimal expansion, proved that in one strongly minimal theory it is possible to combine two arbitrary strongly minimal theory [89, 90] (1992, 1993). B.S. Baizhanov and J.T. Baldwin proved that any expansion of a strongly minimal theory with a unary predicate preserves stability (superstability if and only if the geometry of a strongly minimal formula is trivial [26] (2004). On the other hand, J.T. Baldwin and K. Holland proved that it is possible to expand an algebraically closed field by a unary predicate so that an expanded structure would be omega-stable of rank κ for arbitrary natural κ [43] (2004). R.D. Aref'ev built an unsaturated generic structure [5] (1995) R.D. Aref'ev, J.T. Baldwin and M. Mazzucco showed there are countably many δ -invariant (strong) amalgamation classes of finite graphs which are closed under subgraph and describe the countable generic models for these classes [7] (1999). B.Sh. Kulpeshov being a student under B.S. Baizhanov's supervision studied an article by B.I. Zilber [351] (1989) published in Almaty collection of papers, and on its base he wrote one of his first works [143] (1995), where a characterization of completeness of the theory of elementary pairs for strongly minimal structures with the condition of coincidence of definable and algebraic closures was obtained.

I_2 - omega-stable theories. J.T. Baldwin and K. Holland proved that a two-colored omega-stable field of finite Morley rank, constructed by B. Poizat [277, 278] (1999, 2001), has a non-model-complete theory. They found sufficient conditions for an ω_1 -categorical expansion of a strongly minimal model to have a model complete theory [43] (2004). The basis of the construction for building of the Hrushovski's example was the concept of dimension and pre-dimension for finite structures. While constructing omega-stable theories dimension was defined as pre-dimension of a finite extension of the given finite structure. During transition to building stable theories, there were difficulties with defining dimension. Basing on idea of completion of a topological space, V.V. Verbovskiy invented a method of defining pre-dimension on one class of infinite structures, which allowed to transfer the technique of studying generic omega-stable structures onto studying generic stable structures [345] (Verbovskiy-Yoneda, 2003).

I_3 — superstable theories [Bouscaren-Poizat, Baizhanov-Baldwin-Shelah].

Elisabeth Bouscaren considered expansion of a model of superstable theory, emphasizing an elementary submodel by a new non-definable unary predicate (a pair of models). A theorem that a theory of a pair of models is stable (superstable) if and only if the initial superstable theory does not have the dimension order property

was proved by her [53] (1989). E. Bouscaren and B. Poizat proved that this result can not be transferred to stable theories [54] (1988), constructing an example of a stable non-superstable theory with the dimension order property, whose theory of pairs of models is complete and stable. An important step in the proof of the Bouscaren's theorem [53] (1989) was the theorem that equality of types of two tuples over a small model in the original language implies equality of types of those tuples in the expanded language. This property is called being *benign*. B.S. Baizhanov, J. Baldwin and S. Shelah proved that equality of strong types of two tuples in the initial language over any set of a superstable model implies equality of types of those tuples in the expanded language [27] (2005). This property is called being *weakly benign*. Thus, it was proved that all the sets of superstable theories are weakly benign.

I_4 - stable theories [Poizat, Bouscaren, Baizhanov, Baldwin-Benedict, Casanovas-Ziegler].

B. Poizat studied pairs of models of a stable theory, such that the large model is saturated over a small submodel. Such a pair of models he called lovely. He proved that the theory of lovely pairs is complete if and only if the original theory does not have the finite cover property [276] (1983). J. Baldwin and M. Benedict proved stability of an expansion of model of a stable theory by a predicate, which distinguishes a non-definable set, over which the model is saturated [41] (2000). E. Casanovas and M. Ziegler strengthened the results from [41], removing the condition of indiscernibility, but adding non finite cover property with respect to this set [57] (2001). B.S. Baizhanov and J. Baldwin strengthened the results of B. Poizat and E. Bouscaren on the one hand, and the results of Baldwin-Benedikt-Casanovas-Ziegler on the other, by proving that expansion of model of a stable theory by a weakly benign set has a stable theory iff the restriction of an expanded model on this set has a stable theory (analogous result is also true for the class of ω -stable and superstable theories). In addition, B.S. Baizhanov and J.T. Baldwin answered the Casanovas-Ziegler's question on characterization of the property when the model \mathfrak{M} has the finite cover property with respect to a set A [26] (2004).

I_5 - o-minimal theories.

The mid 1980s was the beginning of active study of linearly ordered theories basing on the notion of o-minimality. Alex Wilkie proved that an expansion by a unary exponential function the field of real numbers, whose elementary theory admits quantifiers elimination is decidable and o-minimal, has a model complete and o-minimal theory [346] (Wilkie, 1996). Any o-minimal structure on \mathbb{R} generates a family of spaces (definable sets), which is quite stable under different topological / geometrical operations and has local triviality, stratification and properties of uniform finiteness [67] (van den Dries-Macintyre-Marker, 1997). A. Pillay, Ch. Steinhorn and J. Knight proved that if a linearly ordered structure is o-minimal, then any its elementary equivalent structure is also o-minimal. In addition, they described properties of definable functions [271, 108, 272] (1986, 1988). K.Zh. Kudaibergenov studied the concept of o-minimality. The results of Marker on small extensions of models of o-minimal theories were strengthened by him in [125] (2007). By B.S. Baizhanov it was proved that an o-minimal expansion of a model of a dense o-minimal theory T which admits quantifier elimination will be essential if and only if in some elementary extension the class of unary partial definable with parameters functions does not coincide with the class of unary partial functions, definable with

parameters in the initial language in this extension [25] (2007). B.S. Baizhanov, J.T. Baldwin and V.V. Verbovskiy proved that for any linearly ordered group there is an o-minimal theory, such that this group definably acts on a set of realizations of some 1-type of the o-minimal theory, and conversely, every group of definable bijections on the set of realizations of an o-minimal model is a linearly ordered group [28] (2007).

E. Hrushovski and A. Pillay in [93] (1994) introduced the notion of geometric theories that is a common generalization of the classes of strongly minimal and dense o-minimal theories because it has by the definition the exchange property for the algebraic closure and the property of elimination of the quantifier “there are infinitely many”. Based on the notion of lovely pair of models of a simple theory (a simplicity theoretic version of B. Poizat’s belle paire) developed and studied in [47] (Ben-Yaacov-Pillay-Vassiliev, 2003) and [331, 332] (Vassiliev, 2003, 2005), as well as [273, 274] (Pillay-Vassiliev, 2004, 2005), A. Berenstein and E. Vassiliev considered expansions of geometric theories by a unary predicate with density and extension properties and studied connection between properties of the original theory with properties of such expansions [48, 49, 50] (2010–2019).

I_6 — **weakly o-minimal theories** [Macpherson–Marker–Steinhorn, Baizhanov, Verbovskiy, Kulpeshov, Aref’ev].

In June 1994 in Almaty was hosted another Kazakh-French Colloquium on model theory, which was attended by many prominent scientists from distant foreign countries and Russia. An american scientist Charles Steinhorn (who is currently one of the heads of the Association for Symbolic Logic) gave a lecture on o-minimality at the Colloquium. This was the beginning of the collaboration of C. Steinhorn with Kazakh mathematicians.

Professor Charles Steinhorn sent by mail a packet of copies of articles on o-minimality [271, 272] (Pillay-Steinhorn, 1986, 1988), [108] (Knight-Pillay-Steinhorn, 1986), [186] (Marker, 1986), [191] (Mayer, 1988), [190] (Marker-Steinhorn, 1994), [269] (Pillay, 1994) and a draft of a work on weak o-minimality [177] (Macpherson-Marker-Steinhorn, 2000). These works were discussed in detail at seminars of Kazakh logicians and determined the direction of research of some kazakh logicians for next decades. Some problems raised in the draft (1994) of the last of the above works have been resolved by Kazakh scientists in [20, 21] (Baizhanov, 1995, 2001), [6] (Aref’iev, 1997), [334, 335] (Verbovskiy, 1997, 2001).

External definability. Approach of Macpherson-Marker-Steinhorn. In the article [177] (2000, preprint 1994) D. Macpherson, D. Marker and Ch. Steinhorn proved weak o-minimality of the expansion of an o-minimal structure by unary convex predicate, such that the predicate is traversed by a uniquely realizable cut (1-type). Following D. Marker [186], a uniquely realizable 1-type $p \in S_1(M)$ over a model is that prime model over model and one realization of this 1-type p contains just this element from the set of realization of the type. A uniquely realizable 1-type has the next property: there is no definable function acting on the set of realizations of this 1-type p . Macpherson-Marker-Steinhorn considered at the same time two structures $\mathfrak{M}^+ = \langle M; \Sigma \cup \{U^1\} \rangle$ and $\mathfrak{N} = \langle N; \Sigma \rangle$, where \mathfrak{N} is a model of an o-minimal theory of the signature Σ and a saturated elementary extension of \mathfrak{M} . They defined a new unary convex predicate U by using an element $\alpha \in N \setminus M$ from the set of realizations of an irrational 1-type $p \in S_1(M)$ such that for every $a \in M$ the following holds:

$$\mathfrak{M}^+ \models U(a) \iff \mathfrak{N} \models a < \alpha.$$

By induction on construction of formulas $\phi(\bar{y})$ of the signature $\Sigma^+ = \Sigma \cup \{U^1\}$ there is a formula $K_\phi(\bar{y}, \alpha)$ of the signature Σ such that for any $\bar{a} \in M$ the following holds:

$$(1) \quad \mathfrak{M}^+ \models \phi(\bar{a}) \iff \mathfrak{N} \models K_\phi(\bar{a}, \alpha).$$

The crucial point in this construction was the case $\phi(\bar{y}) = \exists x \psi(x, \bar{y})$. They proposed

$$K_{\exists x \psi(x, \bar{y})}(\bar{y}, \alpha) := \exists z_1 \exists z_2 \exists x (z_1 < \alpha < z_2 \wedge \forall z (z_1 < z < z_2 \rightarrow K_{\psi(x, \bar{y})}(x, \bar{y}, z)).$$

Since the 1-type $p \in S_1(M)$ is uniquely realizable, two convex to right and to left from α M - α -1-formulas have solutions out of $p(\mathfrak{N})$. Thus for any $\bar{a} \in M$, if $\mathfrak{N} \models K_{\exists x \psi(x, \bar{y})}(\bar{a}, \alpha)$, then for some $b_1, b_2 \in M$,

$$\mathfrak{N} \models \exists x (b_1 < \alpha < b_2 \wedge \forall z (b_1 < z < b_2 \rightarrow K_{\psi(x, \bar{y})}(x, \bar{a}, z)).$$

This means that in an elementary submodel of \mathfrak{N} the part of the last formula holds on $\mathfrak{M} \models \exists x \forall z (b_1 < z < b_2 \rightarrow K_{\psi(x, \bar{y})}(x, \bar{a}, z))$. Then there is an element $c \in M$ such that $\mathfrak{M} \models \forall z (b_1 < z < b_2 \rightarrow K_{\psi(x, \bar{y})}(c, \bar{a}, z))$. So, $K_{\psi(x, \bar{y})}(c, \bar{a}, z) \in p$.

Thus, any Σ^+ - M -1-formula $\phi(x, \bar{a})$ has the set of its realizations, $\phi(\mathfrak{M}^+, \bar{a}) = K_\phi(\mathfrak{N}, \bar{a}) \cap M$, being a finite union of convex sets because $K_\phi(\mathfrak{N}, \bar{a})$ is a finite union of intervals and points. The elementary theory of \mathfrak{M}^+ is weakly o-minimal since the number of convex sets is bounded and consequently does not depend on parameters.

Non uniquely realizable one-type. For the case when $p \in S_1(M)$ is a non uniquely realizable type, B.S. Baizhanov proposed [20] (1995) to take the constants for $K_{\exists x \psi(x, \bar{y})}$ for the formulas of (1) from an indiscernible sequence $I = \langle \alpha_n \rangle_{n < \omega}$, $\alpha_n \in p(\mathfrak{N})$, with finite realisability in M of the type $tp(\alpha_n \mid M\bar{\alpha}_{n-1})$ and elaborated the approach in prolongation of the idea of Macpherson-Marker-Steinhorn to consider expanded model by constants from elementary saturated extension \mathfrak{N} .

The idea to use an indiscernible sequence consists from three observations B1-B3 and one fact B4.

B1 If $K_{\psi(x, \bar{y})}(\mathfrak{N}, \bar{a}, \bar{\alpha}_n) \cap M = \emptyset$, then there is a finite number irrational cuts (1-types over M) such that for any such 1-type $r \in S_1(M)$, $K_{\psi(x, \bar{y})}(\mathfrak{N}, \bar{a}, \bar{\alpha}_n)$ is a subset of quasi-neighborhood of $\bar{\alpha}_n$ in r

$$QV_r(\bar{\alpha}_n) := \{\beta \in r(\mathfrak{N}) \mid \text{there exists an } M\bar{\alpha}_n\text{-1-formula } \Theta(x, \bar{\alpha}_n), \text{ such that } \beta \in \Theta(\mathfrak{N}, \bar{\alpha}_n) \subset r(\mathfrak{N})\}.$$

B2. On the one hand, if for some $c \in M$, $\mathfrak{N} \models K_{\psi(x, \bar{y})}(c, \bar{a}, \bar{\alpha}_n)$, then for any $\bar{\gamma} = \langle \alpha_{i_0}, \dots, \alpha_{i_n} \rangle$ ($n < i_0 < \dots < i_n$), $\mathfrak{N} \models K_{\psi(x, \bar{y})}(c, \bar{a}, \bar{\gamma})$, because $\bar{\alpha}_n$ and $\bar{\gamma}$ have the same type over M .

B3. On the other hand, to find a sequence I such that for any $r \in S_1(M)$, for any $\bar{\gamma} = \langle \alpha_{i_0}, \dots, \alpha_{i_n} \rangle$ ($n < i_0 < \dots < i_n$), $QV_r(\bar{\alpha}_n) \cap QV_r(\bar{\gamma}) = \emptyset$ in the form $QV_r(\bar{\alpha}_n) < QV_r(\bar{\gamma})$ or $QV_r(\bar{\alpha}_n) > QV_r(\bar{\gamma})$.

B4. [186] (Marker 1986). Let $q, r \in S_1(A)$, and let type $q(x) \cup r(y)$ be non complete (q is non weakly orthogonal to r , Shelah, 1978). Then there is an A -definable monotonic bijection $g : q(\mathfrak{N}) \rightarrow r(\mathfrak{N})$ and consequently, q is irrational if and only if r is irrational; q is uniquely realizable if and only if r is uniquely realizable.

Let \mathfrak{M} be a model of an arbitrary complete theory T of the signature Σ and $J \subset S(M)$. We say that \mathfrak{M}_J^+ is expansion of \mathfrak{M} by the set of types $J \subset S(M)$, if

$\mathfrak{M}_J^+ := \langle M; \Sigma_J^+ \rangle$, where $\Sigma_J^+ := \{R_{(\psi,p)}(\bar{y}) \mid \psi \in \Sigma, p \in J\}$ and for any $p \in J$, for any $\bar{a} \in M$, $\mathfrak{M}_J^+ \models R_{(\psi,p)}(\bar{a})$ iff $\psi(\bar{x}, \bar{a}) \in p$.

We say that \mathfrak{M}_J^+ admits uniform representation of Σ_J^+ -formulas by Σ -formulas, if for any formula $\phi(\bar{y})$ of Σ_J^+ there exists Σ -formula $K_\phi(\bar{y}, \bar{z})$, and there exists $\bar{\alpha} \in N \setminus M$ such that for any $\bar{a} \in M$ the following holds:

$$\mathfrak{M}_J^+ \models \phi(\bar{a}) \iff \mathfrak{N} \models K_\phi(\bar{a}, \bar{\alpha}).$$

In fact, Macpherson-Marker-Steinhorn [177] (preprint 1994) considered the expansion of the ordered field of algebraic numbers $\mathfrak{M} = \langle R_{alg}; =, <, +, *, 0, 1 \rangle$ by an orthogonal family of irrational one-types $J \subset S_1(R_{alg})$. In this structure any irrational one-type is uniquely realizable and, consequently, orthogonality of the family of one-types J means algebraic independence of the set $\{\beta \in R \mid tp(\beta \mid R_{alg}) \in J\}$. Thus it follows from their consideration that for an arbitrary o-minimal model \mathfrak{M} , for any orthogonal set of irrational uniquely realizable one-types $J \subset S_1(M)$, \mathfrak{M}_J^+ admits uniform representation of Σ_J^+ -formulas by Σ -formulas and, consequently has an weakly o-minimal theory. B.S. Baizhanov [20] (1995) extended J to set of an arbitrary orthogonal set of irrational one-types $J \subset S_1(M)$. Ye. Baisalov and B. Poizat [10] (1998, preprint 1996) in the paper on “beautiful” pairs of models of o-minimal theories proved elimination of quantifier $\exists x \in M$. This is equivalent to $J = S(M)$ in theorems of Macpherson-Marker-Steinhorn and Baizhanov, and consequently, strengthening. It is difficult to say whether the approach from [10] is alternative to the approach from [20] (Baizhanov, 1995), since they used the same principles B1-B4 from [20].

B.S. Baizhanov in 1996 obtained a classification of 1-types over a subset of a model of weakly o-minimal theory (6 kinds of 1-types, and 2 kinds of non-orthogonality of types, which define equivalence relations and preserve the 6 kinds of 1-types), and solved the problem of expanding a model of weakly-o-minimal theory by a family of unary convex predicates in the preprint "Classifications of 1-types in weakly o-minimal theories and its applications" and submitted to the Journal of Symbolic Logic, the revised version was published in [21](2001). In this paper B.S Baizhanov proved that the expansion by all externally definable subsets admits quantifier elimination and has weakly o-minimal theory.

In his paper, S. Shelah [299] (2009, preprint 2004) considered a model of NIP theory and proved that the expansion by all externally definable subsets admits quantifier elimination and thereby is NIP. The key problem here is eliminating the quantifier “there exists in the submodel”. In his proof in the way of contradiction Shelah used an indiscernible sequence $\langle \bar{b}_n \mid n < \omega \rangle$ in order to show that if eliminating quantifier “there exists x in the submodel” $\varphi(x, \bar{a})$ fails, then $\varphi(\alpha, \bar{b}_n)$ holds if and only if n is even, for some α , which implies the independence property, for a contradiction.

Frank Wagner and Viktor Verbovskiy [338] (2008, preprint 2005) found a somewhat simplified account of Shelah’s proof, namely by using noting of a finitely realizable type. A. Pillay [275] (2007, preprint 2006) gave two re-proofs of Shelah’s theorem, the first going through quantifier-free heirs of quantifier-free types and the second through quantifier-free coheirs of quantifier-free types.

R.D. Aref’ev proved the monotonicity property for weakly o-minimal structures [6] (1997). V.V. Verbovskiy constructed an example of a weakly o-minimal structure, which does not have a weakly o-minimal theory [334] (1997). V.V. Verbovskiy

constructed an example of a weakly o-minimal ordered group, which does not have a weakly o-minimal theory [335] (2001). B.Sh. Kulpeshov obtained both a characterization of weak o-minimality of a linearly ordered structure in terms of convexity of the set of realizations of 1-types and a complete description of weakly o-minimal linear orders [144] (1998); he introduced the notion of convexity rank of unary formula [144] (1998), which appeared to be useful in studying \aleph_0 -categorical structures: a criterion for binarity of \aleph_0 -categorical weakly o-minimal structures in terms of convexity rank and binarity of types was found [148, 150] (2007, 2011); he described \aleph_0 -categorical weakly o-minimal structures of convexity rank 1 [145] and proved their binarity [146] (2006); as well he described both \aleph_0 -categorical quite o-minimal structures [151, 153] (2011, 2013) and \aleph_0 -categorical weakly o-minimal theories of finite convexity rank [156] (2016). B.S. Baizhanov and B.Sh. Kulpeshov gave a characterization of behavior of 2-formulas (so-called p -preserving convex to the right (left) formulas) in weakly o-minimal theories [30] (2006).

B.Sh. Kulpeshov and S.S. Baizhanov in a series of works [35, 36, 37] (2018, 2019) studied preservation properties (\aleph_0 -categoricity, weak o-minimality, convexity rank) at expanding \aleph_0 -categorical weakly o-minimal structures by a convex unary predicate, an equivalence relation or an arbitrary binary predicate.

K.Zh. Kudaibergenov introduced and studied the notions of a weakly quasi-o-minimal model and theory [128] (2010). He introduced and studied various extensions of o-minimality into partial orders [130, 135] (2012, 2013). K.Zh. Kudaibergenov continued generalization in different directions of the classical concept of o-minimality. He introduced and studied the concepts of multi-R-minimality, right o-minimality, and their variants [140] (2018).

B. Herwig, H.D. Macpherson, G. Martin, A.T. Nurtazin, and J.K. Truss studied \aleph_0 -categorical weakly o-minimal structures and proved that any \aleph_0 -categorical weakly o-minimal 3-indiscernible structure is n -indiscernible for every natural n . They constructed examples of \aleph_0 -categorical weakly o-minimal 2-indiscernible structures that are not 3-indiscernible [87] (2000).

B.Sh. Kulpeshov and H.D. Macpherson [142] (2005) introduced the notion of weak circular minimality on circularly ordered structures, obtained a description of \aleph_0 -categorical weakly circularly minimal structures with a primitive automorphism group up to binarity; they proved high homogeneity of any 6-homogeneous \aleph_0 -categorical weakly circularly minimal structure. A.B. Altayeva and B.Sh. Kulpeshov obtained a complete description of countably categorical non-1-transitive weakly circularly minimal n -convex (where $n > 1$) almost binary theories of convexity rank 1 [158] (2016). Other results in the area of weakly circularly minimal structures had been obtained by B.Sh. Kulpeshov in a series of papers [147, 149, 154, 155] (2006-2016).

B.Sh. Kulpeshov, N.D. Markhabatov and S.V. Sudoplatov obtained some interesting results on combinations of structures, families of theories, ranks for families of theories, and pseudofinite structures in [159, 160, 187, 188, 189]. In particular, B.Sh. Kulpeshov and S.V. Sudoplatov [160] obtained both a criterion for Ehrenfeuchtness of a P -combination of countably many copies of an \aleph_0 -categorical linear order (a structure expanded by countably many unary predicates P_i where each of them distinguishes the universe of a copy) and a description of its countable spectrum, namely, the theory of such a P -combination has 2^ω countable models or

exactly $(k+2)^m(k^2+3k+2)^s$ countable models for some non-negative integers k, m and s with $k \geq 1$ and $m^2 + s^2 \neq 0$.

The main characteristic of a stable theory is definability of any type over any set [296] (Shelah, 1978), that is, an elementary theory is stable if and only if for every type p over B and for each formula $H(x, \bar{y})$ there exists a controlling formula $d(H)(\bar{y}, \bar{a})$ such that for every tuple \bar{b} of elements of the set B , the formula $H(x, \bar{b})$ belongs to the type p type if and only if $d(H)(\bar{b}, \bar{a})$ holds. An elementary extension \mathfrak{N} of a model \mathfrak{M} is called to be *n-conservative* if the type over M of any n -tuple of elements of N is definable. L. van den Dries proved that every type over a field of real numbers is definable, that is every elementary extension is conservative [65](1984). D. Marker and Ch. Steinhorn strengthened this result to the class of o-minimal theories, proving that if a couple of models of an o-minimal theory is 1-conservative, then it is n -conservative [190] (1994), A. Pillay re-proved this result and noticed that the theory of conservative pairs of models of o-minimal theory is axiomatizable [269] (1994). B.S. Baizhanov showed that this result does not apply to a wider class, the class of weakly o-minimal theories, he constructed a 1-conservative, non-2-conservative pair of models of a weakly minimal theory [22, 23] (2005, 2006). B.S. Baizhanov proved that for every (except for an o-minimal expansion of a theory of discrete linear order with maximal and minimal elements) model of a weakly o-minimal theory there exists a conservative saturated elementary extension and established the condition for axiomatizability of conservative pairs models of weakly o-minimal theories [24] (2007).

I_7 — dependent theories.

Since the end of 90s, and the beginning of 2000s there a problem of classification of dependent theories appeared. Proper subclasses of the class of dependent theories were obtained, such as o-minimal, weakly o-minimal and quasi-o-minimal classes of complete theories. On the basis of the approach developed in the theory of stability, B.S. Baizhanov and V.V. Verbovskiy defined the class of ordered stable (o-stable) theories, and besides the analogue of strongly minimal theories are o-minimal theories, weakly o-minimal theories will be omega-stable, and quasi-o-minimal will be o-superstable. It was proved that the theory of a pure linear order is o-superstable, and that the class of o-stable theories is a subclass of the class of dependent theories [32] (2011). In [33] (2015) B. Baizhanov and V. Verbovskiy investigated definability of one-types for o-stable theories. V.V. Verbovskiy investigated o-stable ordered groups and fields. He proved commutativity of an o-stable ordered group. In addition, he described definable subsets, and constructed a number of non-trivial examples of o-stable ordered groups [340] (2012). In frame of classification of dependent theories S. Shelah introduced the concept of a dp-minimal theory. V.V. Verbovskiy proved that dp-minimal theories with linear order are o-stable [339] (2010). Later he introduced the concept of a stable up to delta theory and proved that dependent theories are stable up to some subset of formulas without the independence property [341] (2013).

V.V. Verbovskiy introduced the notion of relative stability and proved a characterization of NIP theories via stability up to Δ in [341] (2013). A theory is *stable up to Δ* if any Δ -type over a model has few extensions up to complete types. V.V. Verbovskiy continued to study relatively stable theories and definability of types in [344] and proved that for a theory T that is stable up to Δ it holds that any one-type over a model of T is definable if and only if its Δ -part is

definable. In [342] (2018) V. Verbovskiy continued to study o -stable ordered groups and proved that any ordered group of Morley o -rank 1 with boundedly many definable convex subgroups is weakly o -minimal and constructed an example of an ordered group of Morley o -rank 1 and Morley o -degree at most 4. In the articles [165] (Kulpeshov-Verbovskiy, 2015) and [343] (Verbovskiy, 2018) circularly ordered groups were studied. B. Kulpeshov and V. Verbovskiy proved that weakly circularly minimal groups are abelian, and later V. Verbovskiy extended this result up to the class of circularly ordered o -stable groups.

K.Zh. Kudaibergenov studied the independence property for first-order theories. The strong form of Shelah's hypothesis of existence of infinite indiscernible sequences in models of big cardinalities of theories without the independence property was refuted [129] (2011). He constructed a theory with independence property, atomic formulas of which does not have the independence property [134] (2013), what disproves the Adler's claim.

The following PhD and doctor of sciences dissertations were defended in this area:

PhD theses:

B.Sh. Kulpeshov, "Quasiurbanik minimal structures", 1997 (supervisor B.S. Baizhanov);

V.V. Verbovskiy, "Properties of functions definable in structures with the condition of minimality on the family of definable sets", 2002 (supervisor B.S. Baizhanov).

Doctor of science dissertations:

B.S. Baizhanov, "Expansion of models of weakly o -minimal and stable theories", 2008 (supervisor E.A. Palyutin);

B.Sh. Kulpeshov, "Binarity and countable categoricity for variants of o -minimality: weak o -minimality and weak circular minimality", 2009 (supervisor B.S. Baizhanov);

V.V. Verbovskiy, "Methods of stability theory in the study of ordered structures", 2010 (supervisor B.S. Baizhanov).

6. APPLICATION OF THE METHODS AND RESULTS MODEL THEORY TO SOLVE PROBLEMS IN OTHER AREAS OF MATHEMATICS

We note three results, when by methods of model theory were firstly established facts in other areas of mathematics, which at the moment were not known.

Anatoly Ivanovich Mal'tsev proved the following theorem (Compactness Theorem for first-order predicate logic). For a set of first order formulas T , if every finite subset of T has a model, then there exists a model, on which T holds [181, 182] (Mal'tsev, 1936, 1941). A.I. Mal'tsev applied this to theorem to the group theory.

Saharon Shelah by proving that for every set of totally transcendental theories there is a prime up to an isomorphism model [293] (1972), obtained a proof of existence of a differential closure for the fields of characteristic 0, since the theory of differentially closed fields is totally transcendental. This fact was not known in algebra.

Udi Hrushovski using methods of the stability theory, published the first proof of the Mordell-Lang conjecture from algebraic geometry [91] (1996). By use the

methods of model theory of difference fields Manin-Mumford conjecture was proved [92] (E. Hrushovski, 2001)

From 1979 to 1984, Professor Mikhail Abramovich Taitlin worked in Kazakhstan (Kazakh State University). He organized a city science seminar on model theory. Based on the results of the workshop, edited by Mikhail Taitlin, 3 collections of scientific papers on model theory, dynamic and program logic were released. Under the direction of Taitlin during this period, three candidate of physical and mathematical sciences (PhD) dissertations were prepared. The scientific school in Alma-Ata, created by Taitlin, was engaged in research in the field of dynamic logics and the theory of finite models. At that time, a number of important results were published by Erimbetov [73] (1981), Taitlin, Stolboushkin and Musikaev [199] (Musikaev-Taitlin, 1990), [302, 303] (Stolboushkin-Taitlin, 1983), [320] (Taitlin, 1983). Later, M.A. Taitlin and his student A.P. Stolboushkin solved a number of relevant open problems in the field of dynamic logics, and, in particular, proved inevitability of non-determinism and recursion in program logic. He and his student I.Kh. Musikaev solved the problem of the influence of finite memory on the expressive power of software logics.

In this direction the following PhD dissertations of scientists from Kazakhstan were defended:

S.S. Magazov, "On countable fragments of infinite logic", 1982 (supervisor M.A. Taitlin);

A.P. Stolboushkin, "Comparison of expressive power of some dynamic logics", 1984 (supervisor M.A. Taitlin);

T.Kh. Musikaev, "Comparison of expressive power of some dynamic logics", 1986 (supervisor M.A. Taitlin).

7. INTERNATIONAL CONNECTIONS. PARTICIPATION OF KAZAKH MATHEMATICIANS IN THE MAINSTREAM OF MODEL THEORY

Cooperation with Russia, France, USA and UK.

Russia. Of course, without regular contact with a wide variety of colleagues, the scientist becomes a "provincial" and his studies do not always become known to a wide range of professionals.

The researcher do not work for himself, but for colleagues and, very importantly, needs an assessment and recognition of his results. During the Soviet period Kazakhstan's scientists did not had such a problem.

Regularly Soviet Union conferences on mathematical logic were held, the most important results were discussed and views were exchanged. The Kazakh school of model theory was a part of the Soviet school of mathematical logic and a part of the Siberian school of algebra and logic. When new results were obtained, they were always transported to Siberia for reports. Almost all Kazakhstan's doctors in mathematical logic (except B.Sh. Kulpeshov and V.V. Verbovskiy) spent a long time in Akademgorodok, went to Yu.L. Ershov, Yu.I. Merzlyakov, E.A. Palyutin, and V.N. Remeslennikov's courses, participated in the seminars "Algebra and Logic", "Model theory", "Theory of numberings", "Group Theory", and "Ring theory". Yu.L. Ershov was a supervisor of candidate dissertations of doctors from Kazakhstan — M.G. Peretyat'kin, K.Zh. Kudaibergenov and S.A. Badaev; E.A. Palyutin was a supervisor of the doctoral dissertation of B.S. Baizhanov; S.S. Goncharov was a

supervisor of doctoral dissertations of V.P. Dobritsa, D.A. Tussupov and A.R. Yeshkeev. Thank them for this!

France. Cooperation with France is impossible to imagine without Bruno Poizat. Professor B. Poizat is a honorary professor of two Kazakh universities. Five scientists from Kazakhstan defended their PhD theses under Bruno Poizat's supervision.

In frames of the project of SCST USSR – CNRS France (1989–1992, coordinators B. Poizat and B.S. Baizhanov) two joint Soviet-French Colloquiums on Model Theory (1990 and 1992) were held. Further the Kazakh-French Colloquium on Model Theory in Almaty (1994); the Fourth French-Turan Colloquium on Model Theory in Marseille (1997); the Fifth Kazakh-French Symposium on Model Theory in Karaganda (2000); and the French-Kazakh Conference on Model Theory and Algebra in Astana (2005).

As a part of the C.N.R.S. (France) P.I.C.S. 541 avel le Kazakhstan program, many Kazakh experts on model theory held from two weeks to two months on an internship at the Lyon-1 University. In cooperation with professor F. Wagner from the Lyon-1 University V.V. Verbovskiy won the grant INTAS YSF 04-83-3042 for young scientists. Many logicians from Kazakhstan and Russia went on long-term scientific trips to Lyon-1 and Paris-7 universities, among them R.D. Aref'ev, E.R. Baisalov, K.Zh. Kudaibergenov, T.G. Mustafin, T.A. Nurmagambetov, E.A. Palyutin, A.N. Ryaskin, S.V. Sudoplatov, and K. Zhetpisov.

We thank all French logicians, especially Professor Poizat, for friendship and hospitality!

United States. Cooperation with the United States began with a report by Charles Steinhorn at French-Kazakh Colloquium in Almaty in 1994, when he presented a number of modern articles on o-minimality and weak o-minimality. Slightly later an active collaboration of the group under B.S. Baizhanov's leadership began with John T. Baldwin, professor of the University of Illinois at Chicago, which resulted in two CRDF grants: in 1999-2001, award number KM2-2246, "Problems in Logic: the Theory of Models and Relation Databases", and in 2004-2006, award number KZM12620-AL-04, "Model theory: Problems of Expansions of Stable and Ordered Structures"; joint works were published. J.T. Baldwin was a foreign scientific advisor of PhD T. Zambarnaya.

We thank Professor Baldwin and Professor Steinhorn for their cooperation and support!

United Kingdom. Kazakh logicians have collaborated and are currently collaborating with British professors such as W. Hodges, D. Macpherson and J. Truss.

Wilfried Hodges was the organizer of an international project on Model Theory with the countries of the former Soviet Union (EU-Kazakhstan-Russia), the project won a grant from the European Union INTAS-93-3547 "Combinatorial problems of model theory" (1993-1994). Coordinators - W. Hodges (EU), E.A. Palyutin (Russia), B.S. Baizhanov (Kazakhstan). Within the framework of the project, scientific trips took place and a small, but important and timely financial support was provided to participants from Kazakhstan and Russia during the most difficult period of the formation of statehood and the collapse of the economy, in particular in Kazakhstan. We are very grateful for the support to all Western mathematicians of this project, especially Professor Hodges!

A.T. Nurtazin by the INTAS grant in 1996 was on a month-long research trip in Leeds (the result of the trip is a paper with D. Macpherson and J. Truss

published in APAL in 2000); B.S. Baizhanov by the grant of the Royal Society of Great Britain had a month-long research trip to London in 2000; By the grant of Kazakh government B.Sh. Kulpeshov was on a three-month-long internship in Leeds in 2003, which resulted in a paper with D. Macpherson, published in MLQ in 2005; K.Zh. Kudaibergenov was a visitor of D. Macpherson as well and had a joint paper with him. Professor D. Macpherson is a foreign supervisor of PhD student S.S. Baizhanov.

In conclusion, we note that Kazakh mathematics in each period of time, in a varying degree, participated in the search for solutions to the major challenges standing at that time before mathematical logic specialists. For instance, Kazakh mathematicians were solving problems that were raised by leading in this field in the world mathematics. We list the major mathematicians whose questions were answered by Kazakh mathematicians-logicians: G. Keisler (A.I. Omarov [233] (1986); K.Zh. Kudaibergenov [112, 113] (1987, 1988)), M. Morley (M.G. Pere-tyat'kin [249, 251] (1978, 1980), K.Zh. Kudaibergenov [112, 113] (1987, 1988)), S. Shelah (K.Zh. Kudaibergenov [112, 129] (1987, 2011)), A. Lachlan (M.G. Pere-tyat'kin [248] (1973)); B.S. Baizhanov [15](1980)), J.T. Baldwin (T.G. Mustafin, K. Zhetpisov [352] (1990), R.D. Aref'ev [7] (1999)), W. Hodges (B.S. Baizhanov [17] (1989); K. Zh. Kudaibergenov [139] (2017)), W. Hodges – D. Macpherson (K.Zh. Kudaibergenov [135] (2014)) A. Macintyre (N.G. Khisamiev [100] (1987)), G. Cherlin – D. Macpherson – D. Marker – Ch. Steinhorn (B.S. Baizhanov [20, 21] (1995, 2001)), D. Macpherson – D. Marker – Ch. Steinhorn (R.D. Aref'ev [6] (1997); V.V. Verbovskiy [334, 335] (1997, 2001)), D. Macpherson (K.Zh. Kudaibergenov [131] (2012)), G. Baumslag (N.G. Khisamiev [100] (1987)), E. Casanovas – M. Ziegler (B.S. Baizhanov, J.T. Baldwin [26] (2004)), etc.

The recognition of Kazakhstan's school of mathematical logic is an invitation to the international conference as plenary speakers of the following scientists: M.G. Pere-tyat'kin spoke on the section "Mathematical logic" of the International Congress of Mathematicians in 1986 (Berkeley, USA), with an hour-long report "Finitely axiomatizable theories"; T.G. Mustafin presented an hour-long report "On similarities of complete theories" at the plenary session of the European Logic Colloquium in 1990 (Helsinki, Finland); B.S. Baizhanov made an hour-long report "Expansion of models of stable and weakly o-minimal theories" at the plenary session of the Asian Logic Colloquium in 2009 (Singapore); B.Sh. Kulpeshov presented an hour-long report "The countable spectrum of weakly o-minimal theories of finite convexity rank" at the plenary session of the 16th Asian Logic Conference in 2019 (Nur-Sultan, Kazakhstan). The fact that Kazakhstan's mathematicians-logicians were often on the direction of the "main attack", in our opinion, there is a great merit of Asan Dabsovich Taimanov. Asan Dabsovich carefully followed the novelties, trying to catch fundamentally important articles and monographs, suggesting to study such articles on the seminars and offered to translation foreign monographs, and himself was and editor of translation. In the period from the end of 1960s to the middle of 1980's there had been times when the same new articles were studied on seminars on model theory in Novosibirsk and Alma-Ata. Explicitly or implicitly Asan Dabsovich's focus on main problems was manifested, he was always in search of solutions to the major challenges, "infecting" his apprentices and surrounding society with his passion and tasks. In every time period and in every area of mathematics revolutionary articles and monographs appear. In these publications

milestone problems are solved, new systems of notions are formed, providing and supporting new methods and approaches, and the mainstream research direction for the next decade is determined. It was hard to overlook such works being close to Asan Dabsovich, what cannot be said about the next period. Among such works in model theory were the following: A.I. Mal'tsev "Constructive algebras" [183] (1961), an article by Yu.L. Ershov, I.A. Lavrov, A.D. Taimanov and M.A. Taitslin, "Elementary theories" [77] (1965); an article by M. Morley, "Categoricity in power" [197] (1965); an article by J.T. Baldwin and A. Lachlan, "On strongly minimal sets", [42] (1971); A. Lachlan's articles [166, 169] (1973, 1978); articles by S. Shelah [291, 295] (1971, 1974); monographs.

The following monographs (edited by A.D. Taimanov) were translated and published by the publishing houses "Mir" and "Nauka":

A. Robinson, "Introduction to model theory and metamathematics of algebra", 376 p., 1967;

K. Kuratowski, A. Mostowski, "Set theory", 416 p., 1970;

S. Feferman, "Numeric Systems: Basis of algebra and analysis", 440 p., 1971;

G. Sacks, "Theory of saturated models", 190 p., 1976;

H.J. Keisler, C.C. Chang, "Model theory", 614 p., 1977 (with Yu.L. Ershov);

Handbook of Mathematical Logic, Part 1, "Model theory", 391 p., 1983 (with Yu.L. Ershov and E.A. Palyutin).

All translations were made on time and accurately. Books of G. Sacks and H. Keisler and C. Chang were translated and published in 4 years after appearance of the English version, in the midst of the search for solutions of spectrum of complete theories and the development of the stability theory. Unfortunately, important books on the model theory (B. Poizat, E. Bouscaren), translated into Russian in the subsequent period, came too late, and without such quality. But it is not as critical, due to availability of any scientific information nowadays, due to possibility of foreign travel, and emergence of Internet.

8. BIBLIOGRAPHY OF A.D. TAIMANOV

8.1. Key dates of life and activity of A.D. Taimanov.

Asan Dabsovich Taimanov was born at 25 October (7 November) 1917 in Urda district of West Kazakhstan region.

1927–1932. Orphanage in Urda village, school of kolkhoz youth, pedagogical institute named after A.S. Pushkin.

1933–1936. Student of the Ural State Pedagogical Institute named after A.S. Pushkin.

1936–1938. Assistant of the Department of Mathematics of the Ural State Pedagogical Institute named after A. S. Pushkin; student of the correspondence department of the Faculty of Mathematics and Mechanics of the Lomonosov Moscow State University.

1938–1941. Postgraduate student of the Moscow Pedagogical Institute named after V.I. Lenin.

1941–1945. Service in the Soviet Army.

1945. He was awarded medals "For the Capture of Königsberg" and "For the Victory over Germany in the Great Patriotic War 1941–1945".

1945–1947. Postgraduate student of the Moscow Pedagogical Institute named after V.I. Lenin.

1947. Defense of a thesis for the scientific degree of Candidate of Physical and Mathematical Sciences on the theme “On quasicomponents of disconnected sets”.

1947–1954. Head of the Department of Mathematics of the Kyzylorda Pedagogical Institute named after N.V. Gogol.

1951. By the decision of the Higher Attestation Commission approved in the rank of a docent in the “Mathematics” department.

1954–1956. Docent of the Shuya Pedagogical Institute.

1956–1960. Docent of the Ivanovo Textile Institute named after M. V. Frunze.

1960. By the decision of the Presidium of the USSR Academy of Sciences approved in the rank of a Senior Research Fellow on the “Abstract algebra” specialty.

1960–1968. Senior Research Fellow of the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences.

1961. Defense of a thesis for the scientific degree of Doctor of Physical and Mathematical Sciences on theme “Some questions on mappings spreading”.

1961–1962. Docent of the Department of Algebra of the Novosibirsk State University.

1962. Was elected as a full member of the Academy of Sciences of the Kazakh SSR. By the decision of the Higher Attestation Commission approved in the rank of Professor of the “Geometry and Topology” Department.

1962 – 1968. Head of the Department of Geometry of the Novosibirsk State University, member of joint academic senate on physical-mathematical and technical sciences of the Siberian Branch of the USSR Academy of Sciences, member of academic senate of Novosibirsk State University, of the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences.

1963. Seconded to Poland for familiarization with the work of research centers in Warsaw, Torun and Wroclaw, and for establishing of scientific contacts.

1965. Joined the ranks of CPSU.

1966. Participated in the International Congress of Mathematicians in Moscow.

1967. Was awarded the Order of the Red Banner of Labour for achievements in developments in science.

1968. Elected as a member of the Presidium of the Scientific and Methodological Council of the Ministry of Education of the USSR.

1968–1970. Academician-Secretary of the Department of Physics and Mathematics of the Academy of Sciences of the Kazakh SSR, Head of the Department of Mathematical Logic of the Kazakh State University named after S. M. Kirov, Director of the Institute of Mathematics and Mechanics of the Academy of Sciences of the Kazakh SSR.

1970. Was awarded the Medal “In Commemoration of the 100th Anniversary of the Birth of Vladimir Ilyich Lenin”. Senior Research Fellow of the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences. Seconded to France to participate in work of the International Congress of Mathematicians in Nice.

1972. Seconded to the Federal Republic of Germany to participate in work of the model theory conference in Oberwolfach.

1973. Seconded to Poland for lecturing in the International Mathematical Centre named after Hahn-Banach.

1974. Seconded to the German Democratic Republic for lecturing in the university named after Arndt.

1976. Seconded to the Federal Republic of Germany to report on the model conference in Oberwolfach.

1978. Was awarded the honorary sign of the USSR Ministry of Education, "Excellence in enlightenment of USSR".

1979. Seconded to the Federal Republic of Germany to participate with a report in the work of the VI International Congress of Logic, Methodology and Philosophy.

1983. Was awarded the Medal for Battle Merit.

1984. Seconded to the German Democratic Republic for lecturing in the University of Greifswald named after Arndt.

1985. Was awarded the Order of the Patriotic War of 1st Class.

1986. Was awarded the Order of the Red Banner of Labour.

8.2. A brief sketch of scientific, educational and social activities of A.D. Taimanov.

One of the leading mathematicians of the USSR, receiver of fundamental results in a number of mathematical disciplines, holder of two orders: the Labor Red Banner and the Order of the Patriotic War of 1 degree, the founder of the Kazakh school of mathematical logic, academician of the Academy of Sciences of the Kazakh SSR Asan Dabsovich Taimanov was born on October 25, 1917 in a family of poor Kazakh herdsman in Urda region of West Kazakhstan (now the Ural) region.

Coeval of Soviet power, A.D. Taimanov grew and matured together with her. His fate is a prime example of the heights which talented goal-oriented man in conditions of Soviet reality can reach.

Since 1927 A.D. Taimanov was brought up in a children's home in the villages Urda and Smolihin (now Bishkek), where he studied in a peasant youth school (then the collective farm youth school) converted to pedagogical school. In 1933 he enrolled at the Ural State Pedagogical Institute named after Pushkin, after which was enrolled as an assistant of the department of mathematics, and at the same time passing an examination on the correspondence department of Mechanics and Mathematics Faculty of Moscow State University named after M.V. Lomonosov.

In 1938, after the third year Taimanov entered the graduate school of Moscow State Pedagogical Institute named after Lenin, where he was greatly influenced by an outstanding scientist and a major teacher, corresponding member of the Academy of Sciences of the USSR A.Y. Khinchin. During the period of study in graduate school Taimanov participated in various scientific seminars, which were held under the leadership of such prominent scientists as A.A. Lyapunov, Keldysh, P.S. Aleksandrov, V.V. Stepanov, M.V. Bebutov, Gantmaher and others.

The Second World War interrupted the hard and interesting work. By joining the people's militia in July 1941, A.D. Taimanov participated in the battles of Moscow, for Belarus and Lithuania, and ended the war in Prussia. In 1945, demobilized from the army, A.D. Taimanov continued his studies at Moscow State Pedagogical Institute named after Lenin and graduated under the guidance P.S. Novikova, L.V. Keldysh, and Lyapunov resumed classes on the descriptive theory of sets and set-theoretic topology. In 1945, demobilized from the army, A.D. Taimanov continued his studies at Moscow State Pedagogical Institute graduate. Lenin and under the guidance P.S. Novikova, LV Keldysh, and Lyapunov resumed classes on the descriptive theory of sets and set-theoretic topology. He obtained a number of fundamental results in set-theoretic topology, which formed the basis for his

doctoral dissertation. Academician Aleksandrov called thesis of A.D. Taimanov outstanding.

During these years in Kazakhstan, which was experiencing an acute shortage of highly qualified personnel, network of new institutions of higher education created. Asan Dabsovich, get a start in life from Kazakh Komsomol of 40th years, from 1947 to 1954 working as a teacher in the Kyzyl-Orda Pedagogical Institute named after Gogol. Here he organized a permanent scientific seminar on mathematics for students and teachers, introduced the practice of mathematical competitions and evenings. In 1951 he organized city and regional mathematics Olympiads the first in the republic.

During this period, A.D. Taimanov conducted deep series of studies in the descriptive theory of sets and set-theoretic topology. Final results on the Hausdorff problem of preserving classes of B -sets under open mappings. Taimanov theorem among other corollaries have emerged as well-known theorem of P.S. Alexandrov and L.V. Keldysh.

In his article “On closed mappings” fundamental advance was made by a solution of known problem of Alexandrov–Weinstein on the preservation of classes of B -sets under closed mappings. Slightly enhanced in 1976 by the French mathematician Saint-Raymond and further generalized by the Moldavian mathematician Choban theorem named Taimanov–Saint–Raymond–Choban is considered a classic in descriptive set theory.

During the same years A.D. Taimanov derived another classical theorem relating to the set-theoretic topology. Studying the possibility of extending to the closure of a continuous mapping of a part of a completely regular space into another completely regular space, he introduced the notion of a uniformly continuous mapping and showed that these mappings possess the possibility of such an extension. From this result as a corollary the many well-known theorem of domestic and foreign mathematicians deduced: Y.M. Smirnova, E. Cech, Eilenberg, Steenrod, Vulikh, and others.

Becoming interested in the theory of functions, A.D. Taimanov in the article “On a problem of N.N. Luzin” constructed a continuous function of a complex variable, whose monogenicity sets at the points of countable everywhere dense sets coincide with the complex plane. This solved the well-known problem of N.N. Luzin and found a new class of continuous functions, a theory that was further developed in the school by Yu.Yu. Trokhimchuk and E.P. Dolzhenko.

In 1954 A.D. Taimanov was invited to Shuya Pedagogical Institute. Where, together with D.A. Raikov he organized a seminar on functional analysis and theory of functions. In 1956, he, being a delegate to the Soviet Union Mathematical Congress, became acquainted with the report of Academician A.I. Mal'tsev on model theory, a new and topical direction of mathematical logic. This acquaintance identified further scientific activity of Asan Taimanov. Working on the problem of elementary equivalence of models, he opened a kind of inductive process, allowing to analyze the connection between the internal structure of models and entry of defining axioms. In this process induction goes on changes of quantifiers, i.e. by projective classes construction of formulas. Such an application of the ideas of descriptive set theory to logic led to the discovery of the elementary equivalence criteria of two models, called by A.I. Mal'tsev as “method of throwing”. With this method, A.D. Taimanov managed to solve a number of mathematical problems

and get characteristics of definability and Diophantine predicates in axiomatizable classes of models, as well as to prove decidability of some elementary theories.

Found regardless of Fraïssé the method of “back-and-forth” has now become a classic in mathematical logic. In the scientific literature it is known as the method of Fraïssé-Taimanov-Ehrenfeucht. From 1960 to 1968 A.D. Taimanov worked in the Institute of Mathematics of SB AS USSR and the Novosibirsk State University, where he paid great attention to the training of scientific personnel. Here he organized seminar on the study of scientific papers on the theory of models, one of results was a review article “Elementary Theory”, written by Yu. Ershov, I.A. Lavrov and M. Taitslin and A.D. Taimanov. Subsequently, the article has been translated into English and remained the simplest manual for many years for beginners to learn the theory of models. A.D. Taimanov is one of the founders of the Soviet school on the theory of models, which currently holds a leading position in the world.

In the 1960s academician Mal'tsev first introduced in the program of NSU course in mathematical logic. Further methodological improvement of the course carried out by A.D. Taimanov.

In 1960 A.D. Taimanov with academician A.I. Mal'tsev developed a program for the training of scientific personnel in mathematics for Kazakhstan.

In 1962 A.D. Taimanov was elected as a full member of the Academy of Sciences of the Kazakh SSR, and by the decision of the Higher Attestation Commission was approved in the rank of Professor of the “Geometry and Topology” Department. At that time he devoted lot of attention to training of qualified mathematics personnel for Kazakhstan. In June 1963, by the suggestion of K.I. Satpayev with participation of G.I. Marchuk, M.M. Lavrentiev, O.A. Zhautykov, and A.D. Taimanov, a decision on personnel training in applied areas of mathematics was made.

In 1968, by invitation of the Academy of Sciences of the Kazakh SSR, A.D. Taimanov moves to Alma-Ata, where he was elected as an Academician-Secretary of the Department of Physics and Mathematics of the Academy of Sciences of the Kazakh SSR, and was appointed to be Director of the Institute of Mathematics and Mechanics. He gives a lot of energy and effort for development of mathematical science in Kazakhstan. On his initiative, National Physics and Mathematics School, Department of Algebra and Mathematical Logic in KazNU, laboratory of algebra and mathematical logic, and a number of applied mathematics laboratories in the Institute of Mathematics and Mechanics were opened.

In 1970, continuing to supervise the algebra and mathematical logic laboratory in the Institute of Mathematics of the Academy of Science of KazSSR and the reopened Department of Algebra and Mathematical Logic of the Karaganda State University, he returned to the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of the USSR.

In Novosibirsk, A.D. Taimanov continued to be actively engaged in scientific and pedagogical activity. To this period refers his researches on topological algebras. He supervises post-graduate students from Kazakhstan. As the motto of this his work the following proverb can be considered: “student is not a container you have to fill but a torch you have to light up”.

Together with scientific, A.D. Taimanov conducts a social work: he was elected as a member of the Presidium of the Scientific and Methodological Council of the Ministry of Education of the USSR and of a number of research councils.

Asan Dabsovich Taimanov represented the Soviet Union on the international congresses on mathematics in Nice, logic, methodology and philosophy of science in Hannover, participated in a number of international model theory conferences, was a member of the organizing committee of almost all Soviet Union conferences on mathematical logic. He gives a lot of effort to promoting the Soviet mathematical science abroad. In the period between 1973 and 1984 he traveled three times to give lectures at foreign universities.

For his merits to Soviet science, scientific-pedagogical and scientific-organizational activity, A.D. Taimanov was awarded the Orders of the Red Banner of Labour and the honorary sign of the USSR Ministry of Education, “Excellence in enlightenment of USSR”.

In November 1990, the 10th Soviet Union Conference on Mathematical Logic dedicated to the memory of A.D. Taimanov was held in Alma-Ata. Kazakhstan regularly holds conferences in memory of Academician A.D. Taimanov: in Almaty, Kyzylorda, Uralsk. On August 22-25, 2017, at the Institute of Mathematics and Mathematical Modeling of the Ministry of Education and Science of Kazakhstan, Almaty, the International conference “Actual problems of pure and applied mathematics”, dedicated to the 100th anniversary of the birth of academician Taimanov Asan Dabsovich was held. On October 25, 2007, in the city of Uralsk, the International Conference in memory of A.D. Taimanov was held. In Uralsk, a street and a school are named after A.D. Taimanov. A memorial plaque was erected in Almaty on a house along Tulebaev Street, in the Kyzylorda State University — a bust of A.D. Taimanov.

8.3. Literature about life and works of A.D. Taimanov.

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2. *Asan Dabsovich Taimanov*, USSR Academy of Sciences, Siberian Branch of the Institute of Mathematics, Novosibirsk, 1979 (in Russian)
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7. *Mathematics in the USSR: 1958 – 1967*, In two volumes, Executive editors S.V. Fomin, G.E. Shilov, Moscow: Mir, Volume 2: Bibliography, Issue 2, 1970, 1580 pages (in Russian)
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9. R. Engelking, *Introduction to general topology*, Translation from English (in Russian)
10. S.S. Goncharov, Yu.L. Ershov, M.M. Lavrent’ev, L.L. Maksimova, T.G. Mustafin, S.P. Novikov, E.A. Palyutin, M.G. Peretyat’kin, Yu.G. Reshetnyak, D.M. Smirnov, *Asan Dabsovich Taimanov (obituary)*, Russian Mathematical Surveys, **45:5** (1990), 213–215.

8.4. Chronological index of works of A.D. Taimanov.

1949

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1950

2. *On rigid bases of δs -operations*, Izvestiya Akademii nauk SSSR: Seriya matematicheskaja (Proceedings of the USSR Academy of Sciences: Series Mathematical), **14**:5 (1950), 443–448 (in Russian)

1952

3. *On quasicomponents of disconnected sets. II*, Matematicheskii Sbornik (Sbornik: Mathematics), **30**:3 (1952), 465–482 (in Russian)

4. *On extension of continuous mappings of topological spaces*, Matematicheskii Sbornik (Sbornik: Mathematics), **31**:2 (1952), 459–463 (in Russian)

1953

5. *On quasicomponents of non-closed sets*, Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys), **8**:2 (1953), 162–163 (in Russian)

6. *On multiple separability of closed sets*, Izvestiya Akademii nauk SSSR: Seriya matematicheskaja (Proceedings of the USSR Academy of Sciences: Series Mathematical), **17**:1 (1953), 51–62 (in Russian)

7. *On one N.N. Luzin's problem*, Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys), **8**:5 (1953), 169–171 (in Russian)

1955

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9. *On closed mappings. I*, Matematicheskii Sbornik (Sbornik: Mathematics), **36**:2 (1955), 349–352 (in Russian)

10. *On open bases of Borel sets*, Matematicheskii Sbornik (Sbornik: Mathematics), **37**:2 (1955), 293–300 (in Russian)

11. *On universal sets*, Matematicheskii Sbornik (Sbornik: Mathematics), **37**:1 (1955), 117–120 (in Russian)

1956

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1958

13. *On a class of models, closed with respect to direct product*, Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys), **13**:3 (1958), 231–232 (in Russian)

14. *Extension of mappings of compacta*, Izvestija Vysshih Uchebnyh Zavedenij. Matematika (Proceedings of Higher Education Institution. Mathematics), **3** (1958), 198–202 (in Russian)

1959

15. *On closed images of B-sets*, 2nd Soviet Union topological conference: Book of Abstracts, Tbilisi, 1959, 44–45 (in Russian)

16. *On a class of models, closed with respect to direct product*, Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys), **14**:5 (1959), 202–203 (in Russian)

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1960

18. *Closed multi-valued images of B -sets*, Trudy Tbilisskogo matematicheskogo instituta (Proceedings of the Tbilisi Mathematical Institute), **27** (1960), 53–56 (in Russian)

19. *On closed mappings. II*, Matematicheskii Sbornik (Sbornik: Mathematics), **52**:1 (1960), 579–588 (in Russian)

20. *On closed mappings*, Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys), **15**:5 (1960), 187–190 (in Russian)

21. *On closed mappings*, American Mathematical Society Translations: Series 2, **30** (1963), 291–294.

22. *On a class of models, closed with respect to direct product*, Izvestiya Akademii nauk SSSR: Seriya matematicheskaja (Proceedings of the USSR Academy of Sciences: Series Mathematical), **24**:4 (1960), 493–510 (in Russian)

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1962

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34. *Characterization of axiomatizable classes of models*, Algebra and Logic, **1:4** (1962), 5–31 (in Russian)

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1963

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1964

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1972

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PhD (candidate) theses performed under supervision of A.D. Taimanov

1. J.A. Almagambetov, “To algorithmic problems of model theory”, Novosibirsk, 1965.

2. B.S. Baizhanov, “Spectral questions, of totally transcendent theories of finite rank”, Novosibirsk, 1981.

3. E.R. Baisalov, “Countable models of superstable theories”, Omsk, 1991;

4. M.I. Bekenov, “Properties of superstable theories”, Novosibirsk, 1984.

5. B.N. Drobotun, “Numberings of special models”, Novosibirsk, 1977 (co-supervisor S.S. Goncharov).

6. M.M. Erimbetov, “On some questions of countable complete theories related to the concept of rank”, Novosibirsk, 1975.

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