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THE CONFERENCE "DYNAMICS IN SIBERIA", NOVOSIBIRSK, FEBRUARY 24 – 29, 2020

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Abstract. In this article abstracts of talks of the Conference "Dynamics in Siberia" held in Sobolev Institute of Mathematics, February 24 – 29, 2020 are presented.

The conference "Dynamics in Siberia" was dedicated to 70th birthday of Valery Vasil'evich Kozlov. The conference was held in House of Scientists of SB RAS and in Sobolev Institute of Mathematics SB RAS (Novosibirsk) from February 24 to 29, 2020. Members of the program committee were as follows: A.V. Borisov, I.A. Dynnikov, A.A. Glutsyuk, A.E. Mironov, I.A. Taimanov and A.Yu. Vesnin.

More than 50 experts on dynamical systems, mathematical physics, geometry and topology participated in the conference. The conference program consisted of plenary talks and short talks. The talks were made by well–known experts from Dubna, Moscow, Nizhny Novgorod, Novosibirsk, St. Petersburg, Troitsk, Vladivostok, Ufa, and also by well–known mathematicians from Benin, France, Israel, Italy, Mongolia, Poland, Ukraine and US. About 15 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

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Program (Plenary talks)

February 24

09:40 – 10:20	V. Kozlov (*Moscow*). Квадратичные законы сохранения уравнений математической физики.
10:25 – 11:05	A. Sorrentino (*Rome, Italy*). Inverse problems and rigidity questions in Billiard Dynamics.
11:25 – 12:05	V. Dragovic (*Richardson, TX*). Integrable Billiards in the Minkowski plane and space.
12:10 – 12:50	A. Chupakhin (*Novosibirsk*). On the energy of a hydroelastic system.

February 25

09:00 – 09:40	D. Treschev (*Moscow*). On inclusion of a diffeomorphism into a flow.
09:45 – 10:25	S. Maksymenko (*Kiev, Ukraine*). Homotopy types of groups of foliated diffeomorphisms for Morse–Bott foliations.
10:45 – 11:25	A. Slizewska (*Bialystok, Poland*). A family of integrable perturbed Kepler systems.
11:30 – 12:00	L. Shalom (*el Aviv University, Israel*). Non–ordinary Gutkin billiards.

February 26

9:00 – 09:40	S. Bolotin (*Moscow*). Динамика быстро–медленных гамильтоновых систем около гомоклинического множества.
09:45 – 10:25	M. Hounkonnou (*Cotonou, Benin*). Geometry and probability on the noncommutative 2–torus in a magnetic field.
10:55 – 11:35	O. Pochinka (*Nizhny Novgorod*). On paths connecting polar diffeomorphisms.
11:40 – 12:20	A. Borisov (*Izhevsk*). Об одной неголономной системе, близкой к шару Чаплыгина.
12:25 – 13:05	A. Tsiganov (*St. Petersburg*). On rigid body dynamics in a magnetic field.

February 27

09:00 – 09:40	S. Kabanikhin (*Novosibirsk*). Inverse problems for mathematical models in epidemiology.
09:45 – 10:25	N. Kuznetsov (*St. Petersburg*). Theory of hidden oscillations and stability of control systems.
10:45 – 11:25	S. Dobrokhotov (*Moscow*). Новые интегральные представления канонического оператора Маслова и эффективные асимптотики в виде специальных функций.
11:30 – 12:10	S. Tikhomirov (*St. Petersburg*). Various parabolic equations with hysteresis.

February 28

9:00 – 09:40	V. Nazaikinskii (*Moscow*). Uniformization and semiclassical asymptotics for equations with Bessel–type degeneration on the boundary.
09:45 – 10:25	Yu. Kordyukov (*Ufa*). Quasiclassical approximation for magnetic monopoles.
10:45 – 11:25	V. Vedyushkina (*Moscow*). Liouville foliation of integrable billiards on cell complexes.
11:30 – 12:10	V. Grines (*Nizhny Novgorod*). Геодезические ламинации и хаотическая динамика на поверхностях.
12:15 – 12:55	A. Plakhov (*Portugal*). New results in Newton's aerodynamic problem for convex bodies.

February 29

09:30 – 10:10	A. Shafarevich (*Moscow*). Lagrangian Tori and Quantization Conditions Corresponding to Spectral Series of the Laplace Operator on a Surface of Revolution with Conical Points.
10:15 – 10:55	A. Gaifullin (*Moscow*). Redistribution of the combinatorial curvature and local combinatorial formula for the first Pontryagin class.
11:15 – 11:55	V. Timorin (*Moscow*). Combinatorial models for spaces of dendritic polynomials.
12:00 – 12:40	Yu. Trakhinin (*Novosibirsk*). Structural stability of shock waves and current–vortex sheets in the solar tachocline.

Plenary talks

Short talks

Plenary talks

On the energy of a hydroelastic system

M. Mamattukov, A. Khe, D. Parshin, A. Chupakhin (Novosibirsk)

The energy approach to the study of a hydroelastic system consisting of an elastic blood vessel, viscous fluid flow and an aneurysm has been developed to evaluate the various energy components of the system:viscous flow dissipation energy,stretching and bending energies of the aneurysm wall.To calculate the total energy of the system we developed a computing complex.The performance of the complex has been tested on model geometric configurations and configurations corresponding to blood vessels with cerebral aneurysms of real patients and reconstructed by angiographic images.The calculated values of the Willmore functional caracterizing the shell bending energy are consistent with analytical data.

Redistribution of the combinatorial curvature and local combinatorial formula for the first Pontryagin class

A. Gaifullin (Moscow)

The definition of the Pontryagin classes of a manifold substantially uses the smooth structure on it. However, by a classical result due to Rokhlin and Schwarz (1957), and independently Thom (1958), rational Pontryagin classes are invariant under PL homeomorphisms. This result raised a problem on explicit computation of the rational Pontryagin classes of a manifold from a triangulation of it. In the context of a smooth manifold with smooth triangulation this problem was solved for the first Pontryagin class in a famous work of Gabrielov, Gelfand, and Losik (1975). Nevertheless, their approach gave no answer in a purely combinatorial situation, i.e., for a triangulated manifold without given smoothing.

In 2004, the author suggested another approach based on the usage of bistellar moves, and constructed purely combinatorial local formulae for the first rational Pontryagin class of a triangulated manifold. More precisely, this result gave explicit description of all local combinatorial formulae for the first rational Pontryagin class, but no choice of a particular local formula was made. Recently, Gorodkov and the author have constructed effectively a particular local combinatorial formula for the first rational Pontryagin class. The key ingredient is the study of the redistribution of the combinatorial Gaussian curvature of a triangulated 2-sphere under bistellar moves.

The talk is based on a joint work with Denis Gorodkov.

Геодезические ламинации и хаотическая динамика на поверхностях

В. Гринес (Нижний Новгород)

Доклад посвящен описанию взаимосвязей между свойствами геодезических ламинаций и динамикой потоков, слоений и диффеоморфизмов на ориентируемых замкнутых поверхностях рода большего нуля.

Будет показано, как нетривиальные геодезические ламинации естественным образом появляются при описании потоков и слоений, обладающих свойством транзитивности, и позволяют классифицировать такие объекты. Соответствующие результаты были получены в серии работ С.Х. Арансона, В.З. Гринеса и Е.В. Жужомы с использованием идей А. Вейля и Д.В. Аносова, касающихся изучения асимптотических свойств незамкнутых кривых без самопересечений посредством исследования их асимптотического поведения на универсальной накрывающей поверхности, являющейся евклидовой плоскостью для тора и плоскостью Лобачевского для поверхностей отрицательной эйлеровой характеристики (см. [1], [2], для знакомства с основными понятиями теоремами и ссылками).

Кроме того, будет показано, что наличие хаотической динамики у гомеоморфизма из заданного гомотопического класса связано с существованием на поверхности пары трансерсальных геодезических ламинаций, каждая из которых состоит из рекуррентных незамкнутых геодезических. Наличие такой пары трансверсальных ламинаций является ключом к топологической классификации нетривиальных базисных множеств, в частности, одномерных аттракторов и репеллеров, полученной в серии работ Р.В. Плыкина, В.З. Гринеса и А. Ю. Жирова (см. [3], [4] для знакомства с основными понятиями теоремами и ссылками).

Будет также представлен недавний результат автора доклада и Е.Д. Куренкова (к глубокому сожалению, трагически погибшему 5 августа 2019 года) о классификации совершенных аттракторов на поверхностях отрицательной кривизны посредством псевдоаносовских гомеоморфизмов с отмеченным множеством седловых периодических точек [5], [6].

Благодарность. Доклад подготовлен при финансовой поддержке международной лаборатории динамических систем и приложений НИУ ВШЭ и гранта Министерства науки и высшего образования РФ (соглашение № 075–15–2019–1931).

Литература

[1] С. Х. Арансон, В. З. Гринес. *Топологическая классификация потоков на замкнутых двумерных многообразиях* УМН, 1986, Т. 41, № 1(247), С. 149–169.

[2] V. Grines, E. Zhuzhoma, *Around Anosov–Weil Theory*, Contemporary Mathematics Vol. 692: Modern Theory of Dynamical Systems: A Tribute to Dmitry Victorovich Anosov. Providence, Rhode Island : American Mathematical Society, 2017. pp. 123–154.

[3] С. Х. Арансон, В. З. Гринес, *Топологическая классификация каскадов на замкнутых двумерных многообразиях* УМН, 1990, Т. 45, №1(271), С. 3–32.

[4] V. Grines, T. Medvedev, O. Pochinka, *Dynamical Systems on 2– and 3–Manifolds*, Switzerland. Springer International Publishing, 2016.

[5] В. З. Гринес, Е. Д. Куренков, *Классификация одномерных аттракторов диффеоморфизмов поверхностей посредством псевдоаносовских гомеоморфизмов*, Доклады Академии Наук. Математика, 2019, Т. 485, № 2б С. 135–138.

[6] В. З. Гринес, Е. Д. Куренков, *Диффеоморфизмы двумерных многообразий с одномерными просторно расположенными базисными множествами*, Известия академии наук. Серия математическая. (принято к печати в 2020 году).

Geometry and probability on the noncommutative 2–torus in a magnetic field

M. Hounkonnou (Cotonou, Benin)

This talk addresses the geometric and probabilistic properties of a noncomutative 2– torus in a magnetic field. We study the volume invariance, integrated scalar curvature and volume form by using the operator method of perturbation by inner derivation of the magnetic Laplacian operator in the noncommutative 2–torus.

Then, we analyze the magnetic stochastic process describing the motion of a particle subject to a uniform magnetic field on this manifold, and discuss the related main properties.

Quasiclassical approximation for magnetic monopoles

Yu. Kordyukov (Ufa)

We construct a quasiclassical approximation to describe the eigenvalues of the magnetic Laplacian on a compact Riemannian manifold in the case when the magnetic field form is not exact. For this, we apply the multi–dimensional WKB method in the form of Maslov canonical operator. In this case, the canonical operator takes values in sections of a nontrivial line bundle. As an example, we consider the Dirac magnetic monopole on the two–dimensional sphere.

This is joint work with Iskander A. Taimanov.

Theory of hidden oscillations and stability of control systems

N. Kuznetsov (Nizhny Novgorod)

The development of the theory of absolute stability, the theory of bifurcations, the theory of chaos, and new computing technologies made it possible to take a fresh look at a number of well-known theoretical and practical problems in the analysis of multidimensional control systems, which led to the emergence of the theory of hidden oscillations which represents the genesis of the modern era of Andronov's theory of oscillations. For the engineering dynamical models the importance of identifying hidden oscillations is related with the classical problems of determining the exact boundaries of global stability and identifying classes of models for which the necessary and sufficient conditions for global stability coincide. This lecture is devoted to well–known theoretical and engineering problems in which hidden oscillations (their absence or presence and location) play an important role.

Homotopy Types of Groups of Foliated Diffeomorphisms for Morse–Bott Foliations

S. Maksymenko, O. Khohliyk (Ukraine)

Let $f : M \to \mathbb{R}$ be a Morse-Bott function on a smooth closed manifold M. Thus the set C of critical points of f is a disjoint union of finitely many submanifolds $C_1, \dots, C_k$ such that f is "non–degenerate on C in transversal directions". Denote by $\mathcal{F}$ be a *singular* foliation on M consisting of $C_1, \dots, C_k$ and connected components of the sets $f^{-1}(y) \setminus C$.

Let also $\mathcal{D}(\mathcal{F})$ be the group of diffeomorphisms of M, leaving invariant each leaf of $\mathcal{F}$, $\mathcal{D}(\mathcal{F}, C)$ be its subgroups consisting of diffeomorphisms fixed on C, and $\mathcal{D}(C)$ be the group of diffeomorphisms of C. Notice that there is a natural restriction to C homomorphism

$$r : \mathcal{D}(\mathcal{F}) \to \mathcal{D}(C), \qquad r(h) = h|_C,$$

with kernel $\mathcal{D}(\mathcal{F}, C)$.

Theorem 1. [3] *The map r is a locally trivial fibration over its image $r\big(\mathcal{D}(\mathcal{F})\big)$.*

This statement is a foliated analogue of well–known results by J.Cerf [1], R.Palais [5], and E. Lima [4].

In particular, we have exact sequence of homotopy groups of that fibration:

$$\cdots \to \pi_k \mathcal{D}_{\mathrm{id}}(\mathcal{F}, C) \to \pi_k \mathcal{D}_{\mathrm{id}}(\mathcal{F}) \to \pi_k \mathcal{D}_{\mathrm{id}}(C) \to \pi_{k-1} \mathcal{D}_{\mathrm{id}}(\mathcal{F}, C) \to \cdots$$
$$\cdots \to \pi_1 \mathcal{D}_{\mathrm{id}}(C) \to \pi_0 \mathcal{D}(\mathcal{F}, C) \to \pi_0 \mathcal{D}(\mathcal{F}) \to \pi_0 \mathcal{D}(C),$$

where subscript id means the corresponding identity path component.

Thus a knowledge about homotopy types of $\mathcal{D}(\mathcal{F}, C)$ and $\mathcal{D}(C)$ would give some information about the homotopy type of $\mathcal{D}(\mathcal{F})$.

In the present talk, we will show how to further simplify the group $\mathcal{D}(\mathcal{F}, C)$.

Let $N \subset M$ be a tubular neighborhood of C, $p : N \to C$ be a vector bundle projection, and U be an open neighborhood of C consisting of such that $C \subset U \subset \overline{U} \subset N$. Denote by $\mathcal{D}^{lin}(\mathcal{F}, C)$ the subgroup of $\mathcal{D}(\mathcal{F}, C)$ consisting of diffeomorphisms h such that $h(p^{-1}(x) \cap U) \subset p^{-1}(x)$ and the restriction $h : p^{-1}(x) \cap U \to p^{-1}(x)$ is a linear map.

In other words, each $h \in \mathcal{D}(\mathcal{F}, C)$ is linear on fibres of p near C.

Theorem 2. *The inclusion $\mathcal{D}^{lin}(\mathcal{F}, C) \subset \mathcal{D}(\mathcal{F}, C)$ is a homotopy equivalence.*

This theorem is a *foliated and parametric* variant of another well-known result that each diffeomorphism of $\mathbb{R}^n$ fixing the origin it isotopic to its linear part, e.g. ([2], Chapter 4, Theorem 5.3)

We will discuss application of Theorem 2 to computation the homotopy type of $\mathcal{D}(\mathcal{F})$.

References

[1] J. Cerf, *Topologie de certains espaces de plongements*, Bull. Soc. Math. France, 1961, 89, pp. 227–380.

[2] M. Hirsch, Differential Topology, 1976

[3] O. Khohliyk, S. Maksymenko, *Diffeomorphisms preserving Morse-Bott functions*, to appear in Indagationes Mathematicae, arXiv:1808.03582, doi:10.1016/j.indag.2019.12.004.

[4] E. L. Lima, *On the local triviality of the restriction map for embeddings*, Comm. Math. Helv., 1964, Vol. 38, pp. 163–164.

[5] R. S. Palais, *On the local triviality of the restriction map for embeddings*, Comm. Math. Helv., 1960, Vol. 34. pp. 306–312.

Uniformization and semiclassical asymptotics for equations with Bessel–type degeneration on the boundary

V. Nazaikinskii (Moscow)

Let X be a compact C^∞ manifold with smooth boundary ∂X, and let $D(x)$ be a smooth function on X such that $D(x) > 0$ in $X^0 = X, \partial X$, $D(x) = 0$ on ∂X, and $\nabla D(x) \neq 0$ on ∂X,. Consider an operator L on X of the form $L = -\langle \nabla, D(x)A(x)\nabla \rangle$ in local coordinates, where the matrix $A =^T A = (A_{jk})_{j,k=1}^n \in C^\infty(X)$ is real and positive definite up to the boundary. We consider the Cauchy problem for the wave equation $u_{tt} + Lu = 0$ degenerating on the boundary as well as the eigenvalue problem $Lu = \omega^2 u$ for the operator L. In applications (e.g., to tsunami waves generated by a localized source, waves trapped by the coast, or seiches), these problems contain a small (or large) parameter, and so it is natural to use the semiclassical approximation. However, the standard scheme of Maslov's canonical operator does not apply here because of the degeneration, and therefore, the author and his colleagues have earlier constructed asymptotic solutions of these problems by introducing a nonstandard phase space Φ and a modified canonical operator.

Here we suggest a completely different approach to the construction of semiclassical asymptotics in the above-mentioned problems based on an idea resembling that of Leray's uniformization for differential equations on complex–analytic manifolds. Namely, we construct a closed manifold M with an action of the group S^1 and a smooth projection $\pi : M \to X \simeq M/S^1$. This projection permits one to lift the problems in question to M, thus obtaining problems whose asymptotic solutions can be written out by standard methods. The solutions of the original problems are just the fiberwise constant solutions of these new problems. The nonstandard phase space Φ arises in this approach as the simplest version of the Marsden–Weinstein symplectic reduction of T^*M by the action of S^1. The surprisingly simple implementation of this approach provides a complete analysis of asymptotic solutions of the original problems and simple efficient formulas for these solutions.

The talk is based on joint work with S.Yu. Dobrokhotov.

Thanks. The research was supported by the Russian Science Foundation under grant no. 16–11–10282.

New results in Newton's aerodynamic problem for convex bodies

A. Plakhov (Portugal)

We prove two conjectures in Newton's aerodynamic problem stated in 1995 and 1993, respectively:
(a) the slope of the graph of an optimal function at its upper part equals 1;
(b) an optimal function equals zero at the boundary of its domain.
The proof of conjecture (a) utilizes the notion of surface area measure of a convex body.

О путях, соединяющих полярные диффеоморфизмы

О.В. Починка (Нижний Новгород)

Полярным диффеоморфизмом на n–многообразии называется диффеоморфизм Морса–Смейла, имеющий в точности один сток и один источник. Из теории Морса известно, что такие диффеоморфизмы существуют на любых многообразиях. В настоящем докладе будут изложены результаты, касающиеся построения устойчивых дуг, соединяющих полярные диффеоморфизмы на данном многообразии. Простейшим полярным диффеоморфизмом является диффеоморфизм источник–сток на n–сфере. Будет показано, что для сферы, размерности меньшей 4, такую дугу можно построить без бифуркационных точек [1], [2]. Тогда, как начиная с размерности 7, существуют полярные диффеоморфизмы не соединяющиеся никакой дугой, из–за наличия различных гладких структур на многомерной сфере [2]. Также будет показано, что на поверхности с нетривиальной фундаментальной группой, любые изотопные полярные диффеоморфизмы соединяются устойчивой дугой, однако эта дуга в общем случае содержит биффуркационные точки.

Благодарность. Автор частично поддержан Лабораторией динамических систем и приложений НИУ ВШЭ, Министерства науки и высшего образования РФ, договор № 075–15–2019–1931.

Литература

[1] Х. Бонатти, В.З. Гринес, В.С. Медведев, О.В. Починка, *Бифуркации диффеоморфизмов Морса–Смейла с дико вложенными сепаратрисами*, Труды МИ РАН, 2007, Т. 256, С. 54–69.

[2] O. Pochinka, E. Nozdrinova, *On the existence of a smooth arc without bifurcations joining source–sink diffeomorphisms on the 2–sphere*, Journal of Physics: Conference Series, 2018, Vol. 990. No. 1. P. 1–7.

Lagrangian Tori and Quantization Conditions Corresponding to Spectral Series of the Laplace Operator on a Surface of Revolution with Conical Points

A. Shafarevich (Moscow)

Semiclassical spectral series of the Laplace operator on a two–dimensional surface of revolution with a conical point are described. It is shown that in many cases asymptotic eigenvalues can be calculated from the quantization conditions on special Lagrangian tori, with the Maslov index of such tori being replaced by a real invariant expressed in terms of the cone apex angle.

A family of integrable perturbed Kepler systems

A. Slizewska (Bialystok, Poland)

We consider a family of perturbed 3–dimensional Kepler systems. We show that Hamilton equations of this systems are integrated by quadratures. Their solutions for some subcases are given explicitly in terms of Jacobi elliptic functions.

Based on the paper A. Odzijewicz, A. Slizewska, E. Wawreniuk, A family of integrable perturbed Kepler systems, RUSS J MATH PHYS 26 (2019), no. 3, 368–383.

Inverse problems and rigidity questions in Billiard Dynamics

A.Sorrentino (Rome, Italy)

A mathematical billiard is a system describing the inertial motion of a point mass inside a domain, with elastic reflections at the boundary. The study of the associated dynamics is profoundly intertwined with the geometric properties of the domain (e.g. the shape of the billiard table): while it is evident how the shape determines the dynamics, a more subtle and difficult question is to which extent the knowledge of the dynamics allows one to reconstruct the shape of the domain. This translates into many intriguing unanswered questions and difficult conjectures that have been the focus of very active research over the last decades. In this talk I shall describe several of these questions, with particular emphasis on recent results obtained in collaborations with Guan Huang and Vadim Kaloshin, related to the classification of integrable billiards (also known as Birkhoff conjecture), and to the possibility of inferring dynamical information on the billiard map from its Length Spectrum (i.e., the lengths of its periodic orbits). This talk is based on joint works with Guan Huang and Vadim Kaloshin.

Structural stability of shock waves and current–vortex sheets in the solar tachocline

Yu. Trakhinin (Novosibirsk)

The equations of shallow water magnetohydrodynamics (SMHD) were proposed by Gilman [1] for studying the global dynamics of the solar tachocline which is a thing transition layer between the Sun's radiative interior and the dikerentially rotating outer convective zone. The tachocline is believed to play a crucial role in the dynamo that maintains magnetic activity in the Sun. We study the structural stability of shock waves and current-vortex sheets in SMHD in the sense of the local–in–time existence and uniqueness of discontinuous solutions satisfying corresponding jump conditions. The equations of SMHD form a symmetric hyperbolic system which is formally analogous to the system of 2D compressible elastodynamics for particular nonphysical deformations. Using this analogy and the recent results in [2] for shock waves in 2D compressible elastodynamics, we prove that shock waves in SMHD are structurally stable if and only if the fluid height increases across the shock front. For current–vortex sheets the fluid height is continuous whereas the tangential components of the velocity and the magnetic field may have a jump. Applying a so–called secondary symmetrization of the symmetric system of SMHD equations, we find a condition sufficient for the structural stability of current–vortex sheets.

References

[1] P.A. Gilman, *Magnetohydrodynamic "shallow water"equations for the solar tachocline*, Astrophys. J. Lett., 2000, Vol. 544, L79–L82.

[2] A. Morando, Y. Trakhinin, P. Trebeschi, *Structural stability of shock waves in 2D compressible elastodynamics*, Math. Ann., 2019, https://doi.org/10.1007/s00208-019-01920-6.

On rigid body dynamics in a magnetic field

A. Tsiganov (St. Petersburg)

When a magnetic field is applied to a charged dielectric rigid body, it starts to spin and uniform precession frequency of the magnetic top is not the Larmor frequency but involves in addition terms depending on higher powers of the magnetic field (quadratic Zeeman effect). When a magnetic field is applied to a ferromagnetic rigid body, it starts to spin (Einstein–de Haas effect). When a magnetic field is applied to a superconducting rigid body, it also starts to spin (gyromagnetic effect), and when a normal metal in a magnetic field becomes superconducting and expels the magnetic field (Meissner effect) the body also starts to spin. All these effects play a key role in modern astrophysics, in the physics of molecular and atomic magnets, macro, micro and nano magneto–mechanical systems, spintronics, ultrafast magnetism and so on.

We plan to discuss integrable Hamiltonian and non–Hamiltonian systems of classical mechanics appearing in a few existing phenomenological theories of rigid body dynamics in a magnetic field starting with the work of V.V. Kozlov (1985).

Liouville foliation of integrable billiards on cell complexes

V. Vediuskina (Moscow)

Integrable systems with two degrees of freedom on isoenergy three–dimensional surfaces are classified by invariants, the so–called "marked molecules". Recently, an important class of billiard books has been discovered – billiards on cell complexes glued from planar billiard sheets. In the particular case, when the complex is an orientable manifold, such a billiard is the so–called topological billiard. It turned out that such billiards (topological and books) are Liouville equivalent to many interesting integrable systems in mechanics and symplectic geometry (that is, such equivalent systems have the same closures of almost all integral trajectories). Relying on the results already obtained, A.T. Fomenko formulated a conjecture of 6 points, the first of which has already been proved, and in the rest interesting progress has been obtained. We give the first 3 points.

Conjecture A (atoms). Any bifurcations of two–dimensional Liouville tori in an isoenergy 3–manifold of any integrable non–degenerate system with two degrees of freedom are modeled using integrable billiards.

Conjecture B (coarse molecules). Any coarse molecules defining the set of all integrable systems up to coarse Liouville equivalence are modeled by integrable billiards.

Conjecture C (marked molecules). Many Liouville foliations of non–degenerate integrable systems on isoenergy 3–surfaces are fiberwise homeomorphic (i.e., Liouville equivalent) to the corresponding foliations of some topological billiard.

Any answer to these conjecture is interesting. For example, if it turns out that not all “marked molecules” are realized by billiards, then it is useful to describe the class of realized systems. This will reveal topological obstacles that distinguish between realizable and unrealizable Liouville foliations. It will become clear which non–degenerate integrable systems are Liouville equivalent to integrable billiards, and which are not. The talk will present current results on the proof (or refutation) of these conjectures. So, in particular, conjecture B is proved "almost completely namely, a proof is obtained for molecules consisting of the so–called atoms without asterisks describing bifurcations with orientable separatrix diagrams. It will also be said that integrable geodesic flows on orientable two–dimensional Liouville surfaces are equivalent to suitable topological billiards.

Short talks

Invariants of winding–numbers in dynamics of flux lines and its visualization

P. Akhmet'ev (Troitsk), O. Cépas (France)

Winding loops in models with local constraints have a natural integer dynamics consisting in the evolution of their integer winding numbers. The dynamics in this case, known as Kempe moves, results in disconnected stable and unstable sectors [1]. Using Pontryagin–Thom construction, we show that the stable invariant I_2, is charged by the stable homotopy group of spheres Π_2 and is visualized as right–left configurations of winding loops on the immersed Konstantinov torus [2]. The stable invariant I_3, which is defined for each chiral sector of I_2 by a polynomial of the degree 6, see [1], is also visualized as (higher) right–left configurations for dynamics on characteristic surfaces in the $3D$ homogeneous space $S^3/\mathbf{Q}$, $\mathbf{Q} = \pm 1, \pm \mathbf{i}, \pm \mathbf{j}, \pm \mathbf{k}$. We will show that examples with I_3 are charged by the elements $\pm 4 \pmod{16}$ in 2–component of the stable homotopy group of spheres ${}_2\Pi_7 = \mathbb{Z}/16$.

References

[1] O. Cépas and P. M. Akhmetiev, *Invariants of winding–numbers and steric obstruction in dynamics of flux lines*, SciPost Phys. 7, 032 (2019) doi: 10.21468/SciPostPhys.7.3.032, https://arxiv.org/abs/1907.04701.

[2] P. M. Akhmetiev and O. Cépas, *A visualization of stable invariants of integer dynamics of three–colorings*, Труды XI Приокской научной конференции, посвященной 70–ти летию В.П.Лексина, Коломна ГСГУ 2019 ISBN 978-5-98492-402-3

Forecasting Limiting Dynamics in the Shapovalov Model of a Mid–Size Firm

T. A. Alexeeva (Novosibirsk), N. V. Kuznetsov, Iu. A. Polshchikova

Understanding and predicting of the behavior of complex systems is one of the important tasks of current research in various fields [1]. The events of the last decade have demonstrated the danger of unpredictable development of economic and financial systems, which can lead to systemic failures or the collapse of the global financial–economic system. One of the main tasks in the study of financial and economic processes is forecasting and analysis of their dynamics. Within this task one could pose such important research questions as determining the qualitative properties of the dynamics (stable, unstable, deterministic chaotic, or stochastic process) as well as estimating its quantitative indicators: dimension, entropy, and correlation characteristics [2, 3, 4].

In this paper, we develop analytical methods [5, 6, 7, 8] for the study of deterministic dynamical systems based on Lyapunov stability theory and chaos theory. These methods make it possible not only to obtain analytical stability criteria and estimate limiting behavior (localization of self–excited and hidden attractors, study of multistability), but also to overcome the difficulties related to implementing reliable numerical analysis of quantitative indicators (such as Lyapunov exponents and Lyapunov dimension). We demonstrate the effectiveness of the proposed methods using as example the "mid–size firm"model suggested recently by V.I. Shapovalov [9].

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References

[1] A. Medio, *Chaotic Dynamics: Theory and Applications to Economics*, Cambridge University Press, 1992.

[2] J. Benhabib, R. Day, *Rational choice and erratic behaviour*, Review of Economic Studies XLVIII, 1981, pp. 459–471.

[3] W. Brock, C. L. Sayers, *Is the business cycle characterized by deterministic chaos?*, Journal of Monetary Economics, 1998, Vol. 22, pp. 71–90.

[4] W. Barnett, A. Serletis, *Martingales, nonlinearity, and chaos*, Journal of Economic Dynamics and Control, 2000, Vol. 24, pp. 703–724.

[5] N. Kuznetsov, T. Alexeeva, G. Leonov, *Invariance of Lyapunov exponents and Lyapunov dimension for regular and irregular linearizations*, Nonlinear Dynamics, 2016, Vol. 85 (1), pp. 195–201.

[6] N. Kuznetsov, G. Leonov, T. Mokaev, A. Prasad, M. Shrimali, *Finite–time Lyapunov dimension and hidden attractor of the Rabinovich system*, Nonlinear Dynamics, 2018, Vol. 92(2), pp. 267–285.

[7] N. Kuznetsov, *The Lyapunov dimension and its estimation via the Leonov method*, Physics Letters A, 2016, Vol. 380 (25–26), pp. 2142–2149.

[8] G. Leonov, T. Alexeeva, N. Kuznetsov, *Analytic exact upper bound for the Lyapunov dimension of the Shimizu–Morioka system*, Entropy, 2015, Vol. 17 (7), pp. 5101–5116.

[9] V. Shapovalov, V. Kablov, V. Bashmakov, V. Avakumov, *Synergetics and problems in control theory*, FIZMATLIT, 2004, Ch. Sinergeticheskaya model ustoychivosty sredney firmy (in Russian), pp. 454–464.

On the Connected Components of Fractal Cubes

D. Drozdov (Gorno–Altaisk)

Take a set $\mathcal{D} \subset \{0, 1, \ldots, n-1\}^k$, $2 \leq \#\mathcal{D} < n^k$ and call it a *digit set*. For any digit set there is unique non-empty compact set $F \subset \mathbb{R}^k$ such that $F = \frac{F+\mathcal{D}}{n}$. The set F is called a *fractal k-cube*.
Along each fractal cube we consider its $\mathbb{Z}^k$- periodic extension $H = F + \mathbb{Z}^k$ and its complement $H^c = \mathbb{R}^k \setminus H$.

If $k = 2$ then F is a fractal square. It was proved in [2] for fractal squares, that either
(A) H^c has a bounded component, which is equivalent to: F contains a non-trivial component that is not a line segment; or
(B) H^c has an unbounded component, and F is either totally disconnected or all its non-trivial components are parallel line segments.

We consider fractal cubes in $\mathbb{R}^3$, and prove the following

Theorem 1. *There is a fractal cube $F \subset \mathbb{R}^3$ such that H^c is connected and H is an uncountable union of unbounded components, each being invariant with respect to $\mathbb{Z}^3$-translations.*

Since all components K_α of F are not line segments, the equivalence (A) of [2] does not hold. From the other side, the set H^c is connected and unbounded, but all components of F are not line segments, so (B) does not work too.

Theorem 2. *The set $Q \subset C(\mathbb{R}^3)$ of connected components $\mathcal{K}_\alpha$ of F is a self-similar set generated by two contractions $\tilde{T}_0$ and $\tilde{T}_1$ of the hyperspace $C(\mathbb{R}^3)$. There is a Hölder homeomorphism $\varphi : Q \to C_{1/3}$ of the set Q to the middle-third Cantor set $C_{1/3}$ which induces the isomorphism of self-similar structures on these sets.*

We prove the following estimates for the dimension of the components K_α:

Theorem 3. *For any $\alpha \in \{0,1\}^\infty$*

$$\underline{\dim}_B(\mathcal{K}_\alpha) = \bar{\lambda}_\alpha \log_5 13 + (1 - \bar{\lambda}_\alpha) \log_5 44,$$

$$\overline{\dim}_B(\mathcal{K}_\alpha) = \underline{\lambda}_\alpha \log_5 13 + (1 - \underline{\lambda}_\alpha) \log_5 44.$$

If α is preperiodic, then

$$\dim_H(\mathcal{K}_\alpha) = \dim_B(\mathcal{K}_\alpha) = \lambda_\alpha \log_5 13 + (1 - \lambda_\alpha) \log_5 44.$$

References

[1] L. L. Cristea, B. Steinsky, *Curves of infinite length in 4×4–labyrinth fractals*, Geom. Dedicata, 2009, Vol. 141, 1–17.
[2] K. S. Lau, J. J. Luo and H. Rao, *Topological structure of fractal squares*, Math. Proc. Camb. Philos. Soc., 2013, Vol. 155, pp. 73–86.
[3] J.J. Luo, J.–C. Liu, *On the classification of fractal squares*, Fractals, 2016, Vol. 24, 1650008.

On analytic projective billiards with open sets of triangular orbits

C. Fierobe (Lyon, France)

This talk will present a generalization of billards called projective billards. In such billards, the law of reflexion is not defined by the usual orthogonal symetries with respect to the tangent lines of the billard tables. Instead, the curves defining the billard tables are endowed with a field of lines, giving place to another reflexion law at each point of the borders. Playing on these tables, we can therefore investigate the same questions as for the usual billards, and for example try to answer Ivrii's conjecture: are there billard tables on which one can find a two–dimensionnal set of periodic orbits? Even more, is it possible to classify such tables? I will present a result for triangular periodic orbits, and try to show how analytic geometry can be useful in such theory.

Conformal Invariance of the Zero–Vorticity Lagrangian Path in 2D Turbulence

V. N. Grebenev (Novosibirsk), M. Wac Lawczyk, M. Oberlack

It was clearly validated experimentally in [1] that the zero–vorticity isolines in 2D turbulence belongs to the class of conformal invariant SLE_k (Schram–Löwner evolution) curves with $k = 6$. The diffusion coefficient k classifies the conformally invariant random curves. With this motivation, we performed a Lie group analysis in [2] of the first equation (i.e. for the evolution of the 1–point probability density function (PDF) $f_1(x_{(1)}, \omega_{(1)}, t)$) of the inviscid Lundgren–Monin–Novikov (LMN) equations for 2D vorticity fields. We proved that the conformal group (CG) is broken for the 1–point PDF but the CG is recovered for the equation restricted on the characteristics with zero–vorticity. As for the zero–vorticity isolines, it implicitly leads to their CG invariance. The main focus of the present work is directed to a Lie group analysis of the characteristic equations of the inviscid LMN hierarchy truncated to the first equation. With this, the CG invariance of the characteristics with zero–vorticity is explicitly derived. Actually, this chain describes the motion of Lagrangian fluid particles that are moving within the conditionally averaged velocity fields.We also show the CG invariance of the separation and coincidence properties of the PDFs. Besides the derivation of the CG invariance of the zero–vorticity isolines, we demonstrate that the infinitesimal operator admitted by the characteristic equations forms a Lie algebra which is the Witt algebra, whose central extension represents exactly the Virasoro algebra. The numerical value of the central charge c occurring here could not be calculated exactly without additional impact into the mathematical tools. But from the previous DNS results performed by Bernard et al the value $c = 0$ is given and corresponds to $k = 6$ for the SLE_k.

References

[1] D. Bernard, G. Boffetta, A. Celani and G. Falkovich, *Conformal invariance in two–dimensional turbulence.*, Nature Physics, 2006, Vol. 2(2), pp. 124–128.

[2] V.N. Grebenev, M Wac lawczyk. and M. Oberlack, *Conformal invariance of the Lungren–Monin–Novikov equations for vorticity fields in 2D turbulence*, J. Phys. A: Math. Theor., 2017, Vol. 50(43) pp. 129–188.

Inhomogeneous distribution of characteristics in ideal crystal

M. A. Guzev (Vladivostok)

We consider an ideal crystal system as a uniform harmonic chain of particles. The exact solution for the particle system is presented. To analyze non-stationary thermal effects in an ideal crystal the temperature is calculated as a measure of the averaged kinetic energy of the particles. The corresponding energy averaging is performed over the initial velocities of the particles, provided that they obey the Boltzmann principle. Over a small time interval, the temperature was shown to depend monotonically on the number of particles. This means that the non-uniformity of thermal characteristics distribution, i.e. dependence on the number of particles, occurs in the system without additional assumptions about the structure of the initial conditions on a macroscopic scale. The obtained formula for the distribution of kinetic energy is presented through Bessel functions. The functional dependence on the number of particles was shown to appear in the index of Bessel functions, and the parity of the number of particles affects the temperature distribution. The distribution of the kinetic energy for a large time was asymptotically analyzed as well.

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Обтекание препятствий потоком тяжелой несжимаемой жидкости

С. Жамбаа (Улан-Батор, Монголия)

Численное конформное отображение, с применением метода Куфарева, позволяет получить линии тока при обтекании различного рода препятствий [3]. Этот же способ можно применить и к не менее интересному случаю, когда поток воды глубины h движется по неровному дну со скоростью С. Свободная поверхность жидкости в случае такого потока представляет собой неизвестную кривую линию, вид которой нужно найти при условии, что давление на ней постоянно. В качестве практического приложения можно рассматривать течения воды, например, в реке текущей по камням и перекатам. Мы, конечно, не имеем возможности решить такую задачу точно, так как это трудная математическая проблема о струйном течении под влиянием силы тяжести. Она до сих пор еще не решена, хотя было много попыток ее решить, в том числе и великими математиками, такими например как Стокс, Релей, Леви–Чивита, Струик, Некрасов, Жуковский [2], и др. Изложение многих интересных способов решения можно найти в книге Л.Н. Сретенского [2].

Математическая постановка задачи состоит в определении потенциала скорости (x, y), который удовлетворяет уравнению Лапласа $_{xx} + _{yy} = 0$ в горизонтальной полосе, ограниченной нижним и верхним краями. Нижний край соответствует неровному дну потока, и на нем должно выполняться условие не протекания: $_n = 0$. На верхнем крае полосы течения должно выполняться условие постоянства давления, которое имеет более сложный вид. Чтобы найти приближенное решение мы, как обычно применяем постулаты линейной теории волн [1], согласно которым граничное условие на верхней (свободной) границе полосы имеет вид:

$$\frac{\partial^2}{\partial x^2} + \frac{g}{C^2}\frac{\partial}{\partial y}|_{y=h} = 0.$$

Литература

[1] Л.Е. СретенскийСретенский,*Теория волновых движений жидкости*, М.: Наука, ФМ, — 1977, 816 с.

[2] Н.Е. Жуковский, *Определение движения жидкости при каком-нибудь условии, данном на линии тока*, статья в книге Собрание сочинений Том II, Гидродинамика. М.: НАУКА, ФМ, — 1949, – 760 с.

[3] С. Жамбаа, Т.В. Касаткина, А.М. Бубенчиков, *Об определении констант в интеграле Кристоффеля-Шварца по методу П.П. Куфарева*, Вестник ТГУ, Математика и Механика, УДК 512.541, №5(43), - С.21-27, г.Томск, РФ, –2016г.

On the 3D consistency of Grassmann extended lattice systems

S. Konstantinou–Rizos (Yaroslavl)

In this talk, we formulate a "Grassmann extension" scheme for constructing noncommutative (Grassmann) extensions of systems of PDEs together with their associated Yang–Baxter maps. As illustrative examples, we use the discrete potential KdV equation and a discrete Boussinesq system. We present some novel, noncommutative systems of difference equations of KdV and Boussinesq type.

Action-Angle Duality for a Poisson–Lie Deformation of the Trigonometric BCn Sutherland System

I. Marshall (Moscow)

The property of action–angle duality was first brought to light in a systematic way by Ruijsenaars. The method of Hamiltonian reduction reveals a natural mechanism for how such a phenomenon can arise. I will give a general overview of this and present as a special case the new result, obtained together with Laszlo Feher, referred to in the title of my talk.

Characteristic Lie algebras of hyperbolic systems

D. Millionshchikov (Moscow)

The concept of characteristic Lie algebra $\chi(f)$ of a hyperbolic system of PDE

$$u^i_{xy} = f^i(u^1, \dots, u^n), i = 1, \dots, n, \tag{1}$$

was introduced by Leznov, Smirnov, Shabat and Yamilov [5, 3]. Consider a vector field

$$X(f) = \frac{\partial}{\partial y} = f^\alpha \frac{\partial}{\partial u_1^\alpha} + D(f^\alpha)\frac{\partial}{\partial u_2^\alpha} + D^2(f^\alpha)\frac{\partial}{\partial u_3^\alpha} + \dots + D^{k+1}(f^\alpha)\frac{\partial}{\partial u_k^\alpha} + \dots$$

A Lie algebra $\chi(f)$ generated by $n+1$ vector fields $X(f), \frac{\partial}{\partial u^1}, \dots, \frac{\partial}{\partial u^n}$, is called *characteristic Lie algebra* of the hyperbolic system (1) [3, 5]. An important step in the study of hyperbolic nonlinear Liouville-type systems was made in [2, 3] where exponential hyperbolic systems were considered

$$u^j_{xy} = e^{\rho_j},\ \rho_j = a_{j1}u^1 + \dots + a_{jn}u^n,\ j = 1, \dots, n. \tag{2}$$

It pas proved in [2] that if $A = (a_{ij})$ is a non–degenerate Cartan matrix then the corresponding exponential hyperbolic system (2) is Darboux–integrable and the corresponding characteristic Lie algebra $\chi(f)$ is solvable and finite–dimensional. A Degenerate (generalized) Cartan matrix A leads to a different kind of integrability, integrability in the sense of the inverse scattering problem, and the corresponding characteristic Lie algebra is infinite–dimensional of slow–growth.

In [4] it was shown that for $n = 1$ the natural growth functions of the characteristic Lie algebras $\chi(\sinh u)$ and $\chi(e^u + e^{-2u})$ grow with average speeds $\frac{3}{2}$ and $\frac{4}{3}$ respectively.

Recently Fedor Pokrovsky proved that in the case of $\frac{\lambda_0}{\lambda_1}$ in general position and $C_0, C_1 \neq 0$ the characteristic Lie algebra $\chi(C_0e^{\lambda_0 u} + C_1e^{\lambda_1 u})$ will be isomorphic to the free Lie algebra $\mathcal{L}(2)$ of two generators. The last statement is not trivial if we take into account the result by Kirillov and Kontsevich [1]. They proved that the Lie algebra generated by two smooth vector fields $f(u)\frac{\partial}{\partial u}$ and $g(u)\frac{\partial}{\partial u}$ on the real line $\mathbb{R}$ is always not free.

References

[1] A. A. Kirillov, M. L. Kontsevich, *The growth of the Lie algebra generated by two generic vector fields on the line*, (Russian) Vestnik Moskov. Univ. Ser. I Mat. Mekh., 1983, Vol. 4, pp. 15–20.

[2] A.N. Leznov, *On the complete integrability of a nonlinear system of partial differential equations in two-dimensional space*, Theoret. and Math. Phys., 1980, Vol. 42, No. 3, pp. 225–229.

[3] A.N. Leznov, V.G. Smirnov, A.B. Shabat, *The group of internal symmetries and the conditions of integrability of two–dimensional dynamical systems*, Theoret. and Math. Phys., 1982, Vol. 51, No. 1, pp. 322–330.

[4] D.V. Millionshchikov, *Lie algebras of slow growth and Klein-Gordon equation*, Algebras and Representation Theory, 2018, Vol. 21, No. 5, pp. 1037–1069.

[5] Shabat A.B., Yamilov R.I.: *Exponencialnie systemy typa I i matricy Cartana (in russian)*, Preprint of Bashkir filial of the Soviet Academy of Sciences, Ufa (1981), 1981, pp. 1–22.

Conjugacy of orientation preserving Morse–Smale diffeomorphisms graphs

A. Morozov (Nizhny Novgorod)

In the present paper we consider preserving orientation Morse–Smale diffeomorphisms on surfaces. We will consider class $MS(M^2)$ o.p. Morse–Smale diffeomorfisms on M^2 – smooth closed connected orientamle 2–dimensional manifold. Using the combinatorics theory and theory of knots and links we will show, that conjugacy of two graphs G, G' built on the two mapping f, f' is enough to say, that f, f' are conjugated.

Definition 1 (Orientable heteroclinic intersections)**.** *Let $f \in MS(M^2)$, σ_i, σ_j — saddle points of diffeomorphism f, such that $W^s_{\sigma_i} \cap W^u_{\sigma_j} \neq \emptyset$. For any heteroclinic point $x \in W^s_{\sigma_i} \cap W^u_{\sigma_j}$ define an ordered pair of vectors $(\vec{v}^u_x, \vec{v}^s_x)$, where:*

- *$\vec{v}^u_x$ — the tangent vector to the unstable manifold of the point σ_j at the point x;*
- *$\vec{v}^s_x$ — the tangent vector to the stable manifold of the point σ_i at the point x.*

Heteroclinics intersections of diffeomorphism f called ***orientable****, if an ordered pairs of vectors $(\vec{v}^u_x, \vec{v}^s_x)$ set the same orientation of the bearing surface M^2. Otherwise heteroclinic intersection is called non-orientable.*

Using , introduced by S. Smale [3], partial orderliness relation $\prec$ we shatter Σ_f (the set of periodic mapping orbits of f) on a subset Σ_i as follows:

- Σ_0 — the set of all stable orbits ω;
- Σ_1 — the set of all saddle orbits σ_i, such $W^u_{\sigma_i}$ does not contain heteroclinic points.
- Σ_2 — the set of remaining saddle orbits of the system.
- Σ_3 — the set of all unstable orbits α

Sets Σ_i is ordered in the following way:

$$\Sigma_0 \prec \Sigma_1 \prec \Sigma_2 \prec \Sigma_3$$

Let G is class of maps f, such $f : M^2 \to M^2$ - orientation preserving Morse–Smale diffeomorphism on smooth closed connected orientable 2–dimensional manifold.

In papers [1] and [2] shown, that $beh(f) = 1$ (have finite number of heteroclinic orbits). In present paper we will show that the classification is reduced to the combinatorial problem of distinguishing graphs.

Set that $\mathcal{A}_f = \Sigma_0 \cup W^u_{\Sigma_1}$ — is the attractor of the system, then $\mathcal{R}_f = \Sigma_3 \cup W^s_{\Sigma_2}$ — repeller of the system. In paper [4] shown, that the chosen sets are the attractor and repeller of the system respectively.

Set that $V_f = M^2 \setminus (\mathcal{A}_f \cup \mathcal{R}_f)$, then make factorization we get the quotient spase $\hat{V}_f = V_f/f$. Let n — the number of tori of which is consist the quotient space $\hat{V}_f$ [], i. e. $\hat{V}_f = \bigsqcup\limits_{i=1}^{n} \hat{V}_f^i$.

Let is introduce the canonical projection $p_f : V_f \to \hat{V}_f$, such $p_f(L^u) = \hat{L}^u$, $p_f(L^s) = \hat{L}^s$, where L^u, L^s – families of unstable and stable separatrices.

Also for any $f \in G$ exist scheme $S_f = (\hat{V}_f, \hat{L}^u, \hat{L}^s)$. From paper [] follows, that $f, f' \in G$ are topologically conjugatd $\Leftrightarrow S_f$ is equivalent to $S_{f'}$

Now we can put each scheme in accordance with the graph. We define the graph P as follows:

$\bigsqcup\limits_{i=1}^{n} B_i(\hat{V}_f^i, \hat{L}_i^s, \hat{L}_i^u)$ – the set of vertices of a graph to which we add one vertex for each separatrix of $\hat{L}^s, \hat{L}^u$ and one vertex ϱ_i, that denoting belonging to the torus.

$E(\hat{V}_f, \hat{L}_i^s, \hat{L}_i^u)$ — the set containing edges of several types:

- Edges corresponding to connectivity components of $\hat{V}_i \setminus W_i^s$, that connect vertices corresponding to separatrices $\hat{L}_i^s$.
- Edges corresponding to connectivity components of $\hat{V}_i \setminus W_i^u$, that connect vertices corresponding to separatrices $\hat{L}_i^u$.
- Edges corresponding to connectivity components of $W_\sigma^s \setminus \sigma(\sigma \in \Sigma_1)$, that connect vertices corresponding to separatrices $\hat{L}^s$, which can also belong to different tori.
- Edges corresponding to connectivity components of $W_\sigma^u \setminus \sigma(\text{where } \sigma \in \Sigma_2)$, that connect vertices corresponding to separatrices $\hat{L}^s$, which can also belong to different tori.
- Edges that connect all vertices of the set $B_i(\hat{V}_f^i, \hat{L}_i^s, \hat{L}_i^u)$ and vertex ϱ_i.

Theorem 1. *If f topologically conjugated to f', then graphs P and P' are isomorphic*

Reference

[1] A.N. Bezdenezhnykh and V.Z. Grines, *Diffeomorphisms with orientable heteroclinic sets on two–dimensional manifolds.* Methods of the quantitative theory of differential equations, Gorkii State University, Gorkii. (1985), 139–152.

[2] A.I. Morozov and O.V. Pochinka, *Morse–Smale surfaced diffeomorphisms with orientable heteroclinic.* arXiv:1909.13149. (28.09.2019).

[3] S. Smale, *Diferentiable dynamical systems,*Bull. Amer. Math. Soc., 1967, Vol. 73(6), pp. 747–817.

[4] V. Grines, O. Pochinka, E. Zhuzhoma, V. Medvedev, *Global attractor and repeller of Morse–Smale diffeomorphisms* Proceedings of the Steklov Institute of Mathematics, 2010, Vol. 271(1), pp. 103–124.

Решение 33–ей проблемы Палиса–Пью для градиентно–подобных диффеоморфизмов двумерной сферы

Е. В. Ноздринова (Нижний Новгород)

Проблема существования дуги с не более, чем счетным (конечным) числом бифуркаций, соединяющей структурно устойчивые системы (системы Морса–Смейла) на многообразиях вошла в список пятидесяти проблем Палиса–Пью [6] под номером 33. Настоящий доклад посвящен решению этой проблемы для градиентно–подобных диффеоморфизмов двумерной сферы.

В 1976 году Ш. Ньюхаусом, Дж. Палисом, Ф. Такенсом [3] было введено понятие устойчивой дуги, соединяющей две структурно устойчивые системы на многообразии. Такая дуга не меняет своих качественных свойств при малом шевелении. В том же году Ш. Ньюхаус и М. Пейшото [4] доказали существование простой дуги (содержащей лишь элементарные бифуркации) между любыми двумя потоками Морса–Смейла. Из результата работы Ж. Флейтас [1] вытекает, что простую дугу, построенную Ньюхаусом и Пейшото всегда можно заменить на устойчивую. Для диффеоморфизмов Морса–Смейла, заданных на многообразиях любой размерности известны примеры систем, которые не могут быть соединены устойчивой дугой. В связи с этим естественно возникает вопрос о нахождении инварианта, однозначно определяющего класс эквивалентности диффеоморфизма Морса–Смейла относительно отношения связанности устойчивой дугой (*компоненту устойчивой связанности*).

Согласно [2], для диффеоморфизмов замкнутого многообразия M^n с конечным предельным множеством, устойчивость дуги $\{f_t \in Diff(M^n),\, t \in [0,1]\}$ характеризуется конечным числом бифуркационных значений $0 < b_1 < \cdots < b_k < 1$, при этом бифуркационный диффеоморфизм $\varphi_{b_i}, i \in \{1, \ldots, m\}$ обладает следующими свойствами:

1) все инвариантные многообразия периодических точек диффеоморфизма φ_{b_i} пересекаются трансверсально;

2) диффеоморфизм φ_{b_i} не имеет циклов и имеет ровно одну негиперболическую периодическую орбиту, а именно флипп или некритический седло–узел, при этом дуга проходит через бифуркационное значение типично.

Будем говорить, что диффеоморфизмы $f_0, f_1 : M^n \to M^n$ принадлежат одному и тому же классу *устойчивой изотопической связанности*, если в пространстве диффеоморфизмов они могут быть соединены дугой с описанными выше свойствами.

Классификация с точки зрения введенного отношения эквивалентности нетривиальна уже для сохраняющих ориентацию диффеоморфизмов окружности S^1, где появляется счетное множество таких классов, каждый из которых однозначно определяется числом вращения грубого преобразования окружности [5], которое равно $\frac{k}{m}$, где $k \in (\mathbb{N} \cup 0), m \in \mathbb{N}, k < m, (k, m) = 1$.

Рассмотрим S^1 как экватор сферы S^2. Тогда структурно устойчивый диффеоморфизм окружности в точности с двумя периодическими орбитами периода $m \in \mathbb{N}$ и числом вращения $\frac{k}{m}$, $k < m/2$ может быть продолжен до диффеоморфизма $\phi_{k,m} : S^2 \to S^2$, имеющего два неподвижных источника в северном и южном полюсах. Обозначим через $C_{k,m}$ компоненту устойчивой изотопической связанности диффеоморфизма $\phi_{k,m}$.

Основным результатом являются следующие теоремы.

Theorem 2. *Компоненты* $C_{k,m}$, $k \in (\mathbb{N} \cup 0)$, $m \in \mathbb{N}$, $k < m/2$, $(k, m) = 1$ *попарно не пересекаются.*

Theorem 3. *Любой сохраняющий ориентацию градиентно–подобный диффеоморфизм 2–сферы принадлежит одной из компонент* $C_{k,m}$, $k \in (\mathbb{N} \cup 0)$, $m \in \mathbb{N}$, $k < m/2$, $(k, m) = 1$.

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Литература

[1] G. Fleitas, *Replacing tangencies by saddle–nodes*, Bol. Soc. Brasil. Mat., 1977, Vol. 8, No. 1, pp. 47–51.

[2] S.E. Newhouse, J. Palis, F. Takens, *Publications mathematiques de l' I.H.E.S*, 1983, Vol. 57, pp. 5–71.

[3] S. Newhouse, J. Palis, and F. Takens, *Stable arcs of diffeomorphisms*, Bull. Amer. Math. Soc., 1976, Vol. 82, No. 3, pp. 499–502.

[4] S. Newhouse, M. Peixoto, *There is a simple arc joining any two Morse–Smale fows*, Asterisque, 1976, Vol. 31, pp. 15–41.

[5] E. Nozdrinova, *Rotation number as a complete topological invariant of a simple isotopic class of rough transformations of a circle*. Russian Journal of Nonlinear Dynamics, 2018, Vol. 14, No. 4, pp. 543–551.

[6] J. Palis, C. Pugh, Lecture Notes in Math., 1975, Vol. 468, pp. 345–353.

Как интегралы по матрицам перечисляют накрытия римановых и клейновых поверхностей

А. Орлов (Москва)

Я расскажу про связь интегралов по матрицам с числами Гурвица и покажу, как фейнмановские диаграммы перечисляют разветвленные накрытия римановых и клейновых поверхностей любого рода (с произвольным числом критических точек). По совместному обзору с С.М. Натанзоном.

Discrete Heisenberg groups, algebraic surfaces and theta functions

D. Osipov (Moscow)

The discrete Heisenberg group $\mathrm{Heis}(3,\mathbb{Z})$ is the group of matrices

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

where a, b, c are integers.

The extended discrete Heisenberg group G is the group of matrices

$$\begin{pmatrix} 1 & m & a & c \\ 0 & 1 & b & \frac{1}{2}b(b-1) \\ 0 & 0 & 1 & b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where a, b, c, m are integers. The group $\mathrm{Heis}(3,\mathbb{Z})$ is a subgroup of the group G, and moreover $G = \mathrm{Heis}(3,\mathbb{Z}) \rtimes \mathbb{Z}$.

I will speak how the groups $\mathrm{Heis}(3,\mathbb{Z})$ and G naturally appear from a data: an algebraic surface X, a point $x \in X$, and the stalk $C \subset X$ of an irreducible curve such that $x \in C$.

In case, X is a surface over a finite field $\mathbb{F}_q$, I will describe the family of infinite-dimensional irreducible complex representations of the group $\mathrm{Heis}(3,\mathbb{Z})$ which are parameterized by the elliptic curve $\mathbb{C}^*/q^{\mathbb{Z}}$. The group G also acts on each representation from this family, and there are traces of some elements of G in this representation which are classical theta functions.

The talk is based on joint papers with A. N. Parshin.

Зеркальная симметрия и структуры Ходжа

В.В. Пржиялковский (Москва)

В докладе я расскажу о гипотезах зеркальной симметрии, связанных со структурами Ходжа. Мы обсудим гипотезы Кацаркова–Концевича–Пантева и P=W, их связь и доказательства в размерности 2 и 3.

Rank 3 Killing tensor fields on a Riemannian 2–torus

V. Sharafutdinov (Novosibirsk)

For a Riemannian manifold (M, g), let $C^\infty(S^m\tau'_M)$ be the space of smooth covariant symmetric tensor fields of rank m on M. The differential operator $d = \sigma\nabla : C^\infty(S^m\tau'_M) \to C^\infty(S^{m+1}\tau'_M)$, where ∇ is the covariant derivative with respect to the Levi–Civita connection and σ is the symmetrization, is called the inner derivative. We say $f \in C^\infty(S^m\tau'_M)$ is a Killing tensor field if $df = 0$. Being written in coordinates, the latter equation represents a system of $\binom{n+m}{m+1}$ linear first order differential equations in $\binom{n+m-1}{m}$ coordinates of f, where $n = \dim M$. Since the system is overdetermined, not every Riemannian manifold admits nonzero Killing tensor fields. The two–dimensional case is of the most interest since the degree of the overdetermination is equal to 1 in this case. In the 2D–case, we obtain one equation on the metric g after eliminating all coordinates of f from the system, although the possibility of such elimination is problematic. Points of the tangent bundle TM are denoted by pairs (x, ξ), where $x \in M$ and $\xi \in T_xM$. Given a tensor field $f \in C^\infty(S^m\tau'_M)$, let $F \in C^\infty(TM)$ be defined in coordinates by $F(x, \xi) = f_{i_1\dots i_m}(x)\,\xi^{i_1}\dots\xi^{i_m}$. The correspondence $f \mapsto F$ identifies $C^\infty(S^m\tau'_M)$ with the subspace of $C^\infty(TM)$ consisting of functions that are homogeneous polynomials of degree m in ξ. Let H be the vector field on TM generating the geodesic flow. The operators d and H are related as follows: if $f \in C^\infty(S^m\tau'_M)$ and $F \in C^\infty(TM)$ is the corresponding polynomial, then $HF = (df)_{i_1\dots i_{m+1}}\,\xi^{i_1}\dots\xi^{i_{m+1}}$. In particular, f is a Killing tensor field if and only if $HF = 0$, i.e., if F is a first integral for the geodesic flow. Thus, the problem of finding Killing tensor fields is equivalent to the problem of finding first integrals of the geodesic flow which polynomially depend on ξ. Because of the relation to integrable dynamical systems, the problem has been considered by many mathematicians, starting with classical works of G. Darboux and J. Birkhoff. There exist global isothermal coordinates on a two-dimensional torus $\mathbb{T}^2$ endowed with a Riemannian metric g. More precisely, there exists a lattice $\Gamma \subset \mathbb{R}^2 = \mathbb{C}$ such that $\mathbb{T}^2 = \mathbb{C}/\Gamma$ and $g = \lambda|dz|^2$, where $\lambda(z)$ is a Γ-periodic smooth positive function on the plane. An easy analysis of equation (4.7) of [1] results the following

Theorem 4. *Assume a Riemannian 2–torus* $(\mathbb{T}^2, g)$ *do not admit a nonzero Killing vector field. The torus admits a nonzero rank 3 tensor field if and only if there exists a lattice* $\Gamma \subset \mathbb{C}$ *such that* $(\mathbb{T}^2, g) = (\mathbb{C}/\Gamma, \lambda|dz|^2)$*, where* $\lambda \in C^\infty(\mathbb{C})$ *is a* Γ*–periodic positive function satisfying the equation*

$$\frac{\partial}{\partial z}\Big(\lambda(\Delta^{-1}\lambda_{zz} + a)\Big) + \frac{\partial}{\partial \bar z}\Big(\lambda(\Delta^{-1}\lambda_{\bar z\bar z} + \bar a)\Big) = 0$$

with a complex constant a.

Reference

[1] V.A. Sharafutdinov, *Killing tensor fields on the 2–torus*, Siberian Math. J., 2016, Vol. 57, No 1, pp. 155–173.

Automorphisms of elliptic surfaces

C. Shramov (Moscow)

I will discuss automorphism groups acting on compact complex surfaces that have a structure of an elliptic fibration, and stabilizers of points therein. In particular, we will see that the image of an automorphism group of a surface of Kodaira dimension 1 in the automorphism group of the base of its pluricanonical fibration is finite. I will also speculate on possible higher dimensional generalizations.

О трехмерных диффеоморфизмах Морса–Смейла с единственной некомпактной гетероклинической кривой

В.И. Шмуклер (Нижний Новгород)

Рассмотрим класс G сохраняющих ориентацию диффеоморфизмов Морса–Смейла f, заданных на замкнутом многообразии M^3, неблуждающее множество которых состоит в точности из четырех точек $\omega, \sigma_1, \sigma_2, \alpha$ с индексами Морса $0, 1, 2, 3$, соответственно. В работе [1] доказано, что для любого диффеоморфизма $f \in G$ множество $H_f = W^s_{\sigma_1} \cap W^u_{\sigma_2}$ не пусто и содержит как минимум одну некомпактную гетероклиническую кривую γ (см. рисунок 1). Кроме того, диффеоморфизмы рассматриваемого класса допускает сфера S^3 и все линзовые пространства. Однако вопрос о полном списке объемлющих многообразий для диффеоморфизмов $f \in G$ является открытым.

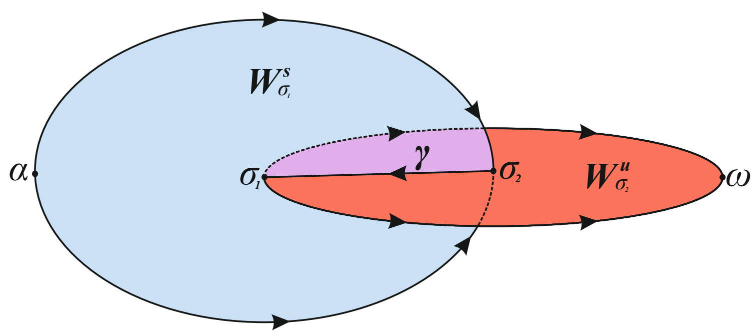

Рис. 1. Некомпактная гетероклиническая кривая

В настоящей работе будет установлен следующий факт.

Теорема 1. *Пусть $f \in G$ и множество H_f связно. Тогда M^3 диффеоморфно 3–сфере.*

Доказательство теоремы 1 основано на построении следующей дуги диффеоморфизмов.

Теорема 2. *Пусть $f \in G$ и множество H_f связно. Тогда f соединяется устойчивой дугой $\varphi_t : M^3 \to M^3, t \in [0,1]$ с диффеоморфизмом "источник-сток", причем φ_t имеет единственную бифуркационную точку типа седло–узел.*

Литература

[1] V.Z. Grines, E.V. Zhuzhoma, V.S. Medvedev, *On Morse–Smale Diffeomorphisms with Four Periodic Points on Closed Orientable Manifolds*, Mathematical Notes, 2003, T. 74, C. 352–366.

On 4–dimensional flows with wildly embedded invariant manifolds of a periodic orbit

O. Pochinka, D. Shubun (Nizhny Novgorod)

Qualitative study of dynamical systems reveals various topological constructions naturally emerged in the modern theory. For example, Cantor set with cardinality of continuum and Lebesgue measure zero as expanding attractor or contracting repeller. Also, a curve in 2–torus with irrational winding number, which is not a topological submanifold but is injectively immersed subset, can be found being invariant manifold of Anosov toral diffeomorphism's fixed point.

Another example of intersection of topology and dynamics is the Artin–Fox arc [1] appeared in work by D. Pixton [2] as the closure of a saddle separatrix of a Morse–Smale diffeomorphism on the 3–sphere. A wild behaviour of the Artin–Fox arc complicates the classification of dynamical systems, it does not admit already a combinatorial description like to Peixoto's graph [3] for 2–dimensional Morse–Smale flows. It is well known that there are no wild arcs in dimension 2. In dimension 3 they exist and can be realize as invariant set for a discrete dynamics, in different from regular 3–dimensional flows, which do not possess wild invariant sets. The dimension 4 is very rich. Here wild objects appear both for discrete and continuous dynamics. Despite the fact that there are no wild arcs in this dimension, there are wild objects of co-dimension 2. So the closure of 2–dimensional saddle separatrix can be wild for 4–dimensional Morse–Smale system (diffeomorphism or flow). Such examples recently were constructed by V. Medvedev and E. Zhuzoma [4]. T. Medvedev and O. Pochinka [5] shown as wild Artin–Fox 2–dimension sphere appears as closure of heteroclinic intersection of Morse–Smale 4–diffeomorphism.

In the present paper we prove that the suspension under a non–trivial Pixton's diffeomorphism provides a 4–flow with wildly embedded 2–dimensional invariant manifold of a periodic orbit. Moreover, we show that there are countable many different wild suspensions.

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Reference

[1] R. Fox, E. Artin, *Some wild cells and spheres in three–dimensional space*, Ann of Math., 1948, pp. 979–990.

[2] D. Pixton, *Wild unstable manifolds*, Topology, 1977, Vol. 16(2), pp. 167–172.

[3] M. Peixoto, *On the classification of flows on two manifolds*, Dynamical systems Proc., 1971.

[4] E. V. Zhuzhoma, V. Medvedev, E. V. Zhuzhoma, V. Medvedev, *Morse–Smale systems with few non–wandering points*, Topology and its Applications, 2013, Vol. 160. No. 3. pp. 498–507.

[5] T.V. Medvedev, O.V. Pochinka, *The wild Fox–Artin arc in invariant sets of dynamical systems*, Dynamical Systems, 2018, Vol. 33, No. 4. pp. 660–666.

Self–Similar Sets with Finite Intersection Property

A. Tetenov (Gorno–Altaisk)

Let $\mathcal{S} = \{S_1, S_2, \dots, S_m\}$ be a system of injective contraction maps on $\mathbb{R}^n$. A nonempty compact set $K \subset \mathbb{R}^n$ is called the attractor of the system $\mathcal{S}$, if $K = \bigcup\limits_{i=1}^{m} S_i(K)$. We call the sets $K_i = S_i(K)$ the pieces of K and denote by $\mathcal{A} = \{K_1, ..., K_m\}$ and by $G(\mathcal{S})$ the semigroup generated by $S_1, S_2, \dots, S_m$.

We say that the system $\mathcal{S}$ has *finite intersection property* (FIP) if the set $P = \bigcup\limits_{i \neq j} (K_i \cap K_j)$ is finite. The *intersection graph* of the system $\mathcal{S}$ is a bipartite graph $\Gamma(\mathcal{S}) = (\mathcal{A}, P; E)$ in which $\{K_i, p\} \in E$ iff $p \in K_i$.

The simplest examples of FIP systems are polygaskets studied by R.Strichartz [2] and polygonal systems studied in [3].

We prove a general condition under which the attractor K of a FIP system $\mathcal{S}$ is a dendrite.

Theorem 1. *Let $\mathcal{S} = \{S_1, ..., S_m\}$ be a FIP system of injective contraction maps in $\mathbb{R}^n$*
The intersection graph $\Gamma(\mathcal{S})$ is a tree iff the attractor K of the system $\mathcal{S}$ is a dendrite.

We say that a system $\mathcal{S}$ of similarities in $\mathbb{R}^n$ has a Weak Separation Property (WSP) [1, 4] if Id is an isolated point in the set $G(\mathcal{S})^{-1} \circ G(\mathcal{S})$.
In the case when the system $\mathcal{S}$ has WSP, we prove the finiteness of the order of the ramification points of its attractor K.

Theorem 2. *Let $\mathcal{S} = \{S_1, ..., S_m\}$ be a system of contraction similarities in $\mathbb{R}^n$ which has finite intersection and weak separation properties.*
Then for any point $x \in K$ there is a neighbourhood base $\mathcal{W}(x) = \{W_k, k \in \mathbb{N}\}$ such that the number n_k of the components of $W_k \setminus \{x\}$ is bounded by some $M > 0$ independent of $x \in K$.
Particularly, if K is a dendrite, then $\mathrm{Ord}(x, K) \leq M$

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References

[1] R. Fox, E. Artin, *Some wild cells and spheres in three–dimensional space*, Ann of Math., 1948, pp. 979–990.

[1] Ch. Bandt, S. Graf, *Self–similar sets 7. A characterization of self–similar fractals with positive Hausdorff measure*, Proc. Amer. Math. Soc., 1992, Vol. 114, No. 4, pp. 995–1001, MR1100644.

[2] R. S. Strichartz, *Isoperimetric estimates on Sierpinski gasket type fractals*, Trans. Amer.Math. Soc., 1999, Vol. 351, pp. 1705—1752.

[3] M. Samuel, A. Tetenov, D. Vaulin, *Self–similar dendrites generated by polygonal systems in the plane*, Sib. El. Math. Rep., 2017, Vol. 14, pp. 737–751, MR3693741.

[4] M. P. W. Zerner, *Weak separation properties for self–similar sets*, Proc. Amer. Math. Soc., 1996, Vol. 124, No. 11, pp. 3529–3539.

О задаче дирихле для эллиптического функционально–дифференциального уравнения с аффинным преобразованием аргумента

Л. Е. Россовский, А. А. Товсултанов (Грозный)

В работе рассматривается задача Дирихле для эллиптического функционально–дифференциального уравнения, содержащего комбинацию сдвигов и сжатия аргумента неизвестной функции под знаком оператора Лапласа. Установлены достаточные условия однозначной разрешимости. Показано также, что задача может иметь бесконечномерное многообразие решений.

Примеры лагранжевых торов в грассманианах $Gr(1, n)$

А.Н. Тюрин, (Москва)

Обобщая технику построения лагранжевых торов с помощью псевдоторической геометрии, предлагается конструкция семейства лагранжевых торов в многообразиях Грассмана $Gr(1, n)$. Так как геометрически это многообразие прямых в $\mathbb{CP}^n$, на котором имеется стандартное торическое действие T^n, мы используем индуцированное торическое действие, предварительно расслаивая $Gr(1, n)\backslash Gr(1, n-1)$ над $\mathbb{CP}^{n-1}$ так что индуцированное действие естественное действует на слоях. На каждом слое выделяется дивизор, Вейнстейнов скелет дополнения к которому есть гладкий лагранжев тор в слое; собирая вместе эти скелеты над точками стандартного тора базы $\mathbb{CP}^{n-1}$, получаем лагранжев тор во всем $Gr(1, n)$. Возможно, таким путем можно получать минимальные и H – минимальные торы в $Gr(1, n)$. "Examples of lagrangian tori in Grassmanians $Gr(1, n)$"A generalization of the psedutoric technique for constructions of lagrangian tori presents a way how one can construct a family of lagrangian tori in the Grassmann varieties $Gr(1, n)$. Since geometrically it is the variety of lines in $\mathbb{CP}^n$, where one has a standard toric action of T^n, we exploit the induced toric action for the fibration of $Gr(1, n)\backslash Gr(1, n-1)$ over $\mathbb{CP}^{n-1}$ such that the induced toric action interchanges the fibers. On each fiber one cuts a divisor, and the Weinstein skeleton of the complement contains a smooth lagrangian torus in the fiber; combinig together these skeletons over the points of a standard lagrangian torus in the base $\mathbb{CP}^{n-1}$, one gets a lagrangian torus in the ambient $Gr(1, n)$. It seems that this way leads to the construction of minimal of H – minimal lagrangian tori in $Gr(1, n)$.

Пары лакса для линейных гамильтоновых систем

А.Б. Жеглов (Москва)

В докладе речь пойдет о способе построения пар Лакса для линейных гамильтоновых систем дифференциальных уравнений. Получающиеся из этих пар первые интегралы имеют явную связь с уже известными первыми интегралами, что дает, в частности, новое простое доказательство теоремы Вильямсона. Доклад основан на недавней совместной работе с Д.В. Осиповым, отправной точкой для появления которой был доклад В.В. Козлова на семинаре отдела алгебры и алгебраической геометрии в МИАНе.

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