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PERCEPTIBILITY IN PRE-HEYTING LOGICS

L.L. MAKSIMOVA, V.F. YUN

Abstract. This paper is dedicated to problems of perceptibility and recognizability in pre-Heyting logics, that is, in extensions of the minimal logic J satisfying the axiom $\neg\neg(\bot \to p)$. These concepts were introduced in [8, 11, 10]. The logic Od and its extensions were studied in [5, 14] and other papers. The semantic characterization of the logic Od and its completeness were obtained in [5]. The formula F and the logic JF were studied in [12]. It was proved that the logic JF has disjunctive and finite-model properties. The logic JF has Craig's interpolation property (established in [17]). The perceptibility of the formula F in well-composed logics is proved in [14]. It is unknown whether the formula F is perceptible over J [8]. We will prove that the formula F is perceptible over the minimal pre-Heyting logic Od and the logic OdF is recognizable over Od.

Keywords: Recognizability, perceptibility, minimal logic, pre-Heyting logic, Johansson algebra, Heyting algebra, superintuitionistic logic, calculus.

In this paper, we investigate the problems of perceptibility and recognizability in pre-Heyting logics, that is, in extensions of the minimal logic J [1] satisfying the axiom $\neg\neg(\bot \to p)$. The mentioned notation was introduced in [8, 10]. For example, a formula A is perceptible over the logic J, if there exists an algorithm that, given any finite system Ax of axioms schemes, checks whether A can be derived in $\mathrm{J}+Ax$.

The logic Od and its extentions were investigated in [5, 14] and other papers. The semantic characterization of the logic Od and its completeness were obtained in [5].

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The formula $F = (\bot \to p \vee q) \to (\bot \to p) \vee (\bot \to q)$ and the logic JF were studied in [12], where it was proved that the logic JF possesses disjunctive and finite-model properties. The fact that the logic JF has Craig's interpolation property CIP was established in [17]. In [14] we proved that the formula F is perceptible in well-composed logics. It is unknown whether F is perceptible over J [8].

In this paper we prove that the formula F is perceptible over the minimal pre-Heyting logic $\mathrm{Od} = \mathrm{J} + \neg\neg(\bot \to \mathrm{p})$, whereas the logic $\mathrm{OdF} = \mathrm{Od} + F$ is recognizable over Od.

Section 1 provides preliminary information. In Section 2, the basic known results concerning pre-Heyting logics are listed. Section 3 is dedicated to proving of theorems on perceptibility of the formula F over the logic Od, and on recognizability and perceptibility of the logic OdF over Od.

1. Preliminary data

The language of Johansson's minimal logic J contains connectives $\&, \vee, \to$ and the propositional constant $\bot$; $\top = \bot \to \bot$, $\neg A = A \to \bot$.

The minimal logic J possesses the same axioms as the positive intuitionistic calculus, and the only rule of inference is modus ponens $R1$: $A, A \to B/\ B$.

J-*logic* is defined as any set of formulas that contains all the axioms of the calculus of J and is closed under $R1$ and substitution. For J-logic L we write $\Gamma \vdash_L A$, if A can be derived from the set $L \cup \Gamma$ using the rule $R1$.

Consider the formulas:

$Int = \bot \to p$,

$Od = \neg\neg(\bot \to p)$,

$X = (\bot \to p) \vee (p \to \bot)$,

$F = (\bot \to p \vee q) \to (\bot \to p) \vee (\bot \to q)$.

Let us introduce the designations for some of the J-logics:

$\mathrm{Int} = \mathrm{J} + Int$,

$\mathrm{Neg} = \mathrm{J} + \bot$,

$\mathrm{Od} = \mathrm{J} + Od$,

$\mathrm{JX} = \mathrm{J} + X$,

$\mathrm{JF} = \mathrm{J} + F$,

$\mathrm{For} = \mathrm{J} + p$.

$\mathrm{OdF} = \mathrm{Od} + F$.

A J-logic is said to be *nontrivial* if it does not coincide with For.

A logic is called *superintuitionistic* (*negative, pre-Heyting, well-composed*) if it contains Int, Neg, Od, JX respectively.

Given the list of variables $\mathbf{p}$, we denote by $A(\mathbf{p})$ a formula all the variables of which belong to $\mathbf{p}$, and by $\mathcal{F}(\mathbf{p})$ – the set of all such formulas.

Suppose that the lists $\mathbf{p}, \mathbf{q}, \mathbf{r}$ do not intersect pairwise.

Craig's interpolation property CIP [2] is defined the following way:

CIP. If $\vdash_L A(\mathbf{p}, \mathbf{q}) \to B(\mathbf{p}, \mathbf{r})$, then there exists a formula $C(\mathbf{p})$, such that $\vdash_L A(\mathbf{p}, \mathbf{q}) \to C(\mathbf{p})$ and $\vdash_L C(\mathbf{p}) \to B(\mathbf{p}, \mathbf{r})$.

Weak interpolation property WIP is defined the following way:

WIP. If $A(\mathbf{p}, \mathbf{q}), B(\mathbf{p}, \mathbf{r}) \vdash_L \bot$, then there exists a formula $A'(\mathbf{p})$, such that $A(\mathbf{p}, \mathbf{q}) \vdash_L A'(\mathbf{p})$ and $A'(\mathbf{p}), B(\mathbf{p}, \mathbf{r}) \vdash_L \bot$.

An algebraic semantics of the logic J is constructed using *Johansson's algebras* (J-*algebras*) $\mathbf{A} =< A; \&, \vee, \rightarrow, \bot, \top >$, where

$< A; \&, \vee, \rightarrow, \bot, \top >$ is a lattice with respect to $\&, \vee$ with the greatest element $\top$, and $\bot$ is an arbitrary element in A,

$z \leq x \rightarrow y \iff z\&x \leq y$.

It is well-known (see, for example, [18]) that the family of J-algebras forms a variety, and there is a one-to-one correspondence between the lattice of J-logics and the lattice of varieties of J-algebras.

If A is a formula and $\mathbf{A}$ is an algebra, then we say that the formula A is valid in $\mathbf{A}$, and write $\mathbf{A} \models A$, if the identity $A = \top$ holds in $\mathbf{A}$.

We denote

$$V(L) = \{\mathbf{A} | \ \mathbf{A} \models L\}.$$

In particular, $V(\mathrm{Int})$ is a variety of Heyting algebras, $V(\mathrm{Neg})$ is a variety of negative algebras, and $V(\mathrm{JX})$ is a variety of well-composed algebras.

The following result is well-known:

Theorem 1.1 (Completeness theorem)**.** *Let L be a* J*-logic. Then $L + A \vdash B$ if and only if $\mathbf{A} \models A$ implies $\mathbf{A} \models B$ for any algebra $\mathbf{A} \in V(L)$.*

Let L be a logic, and Rul – a set of axioms and inference rules. We write $\mathrm{J}+Rul \geq L$, if all the formulas in L are deducible in $(\mathrm{J} + Rul)$; we write $\mathrm{J} + Rul = L$, if L coincides with the set of deducible in $(\mathrm{J} + Rul)$ formulas.

The terms of perceptible formula and recognizable logic were introduced in [8, 10]. Let L_0 be a finitely axiomatizable J-logic. A formula A is *perceptible over* L_0 if there exists an algorithm verifying for any finite system of axioms Ax whether the relation $L_0 + Ax \vdash A$ holds; the formula A is *strongly perceptible over* L_0 if there is an algorithm verifying for any finite system Rul of axioms and inference rules whether the relation $L_0 + Rul \vdash A$ holds.

Let L, L_0 be finitely axiomatizable J-logics, $L \supseteq L_0$.

The logic L is *perceptible over* L_0 if the problem $L_0 + Ax \geq L$ is decidable; L is *recognizable over* L_0 if the problem $L_0 + Ax = L$ is decidable.

The most important J-logics prove to be recognizable over J [8].

Proposition 1.2 ([8])**.** *Let L_0 be a finitely axiomatizable J-logic, $L = L_0 + A$. The logic L is recognizable over L_0 if and only if A is perceptible over L_0 and L is decidable.*

2. Pre-Heyting logics

Let us consider the class of pre-Heyting logics in more detail.

In [16] it is shown that Od is a J-logic for which the analogue of Glivenko's theorem holds [15]: For any formula A

$$\mathrm{Cl} \vdash \neg A \iff \mathrm{Od} \vdash \neg A.$$

Moreover, Od is the smallest of such J-logics [5].

Note that the formula Od is J-conservative [13].

Recall that a formula $A(p)$ with one variable p is called J-*conservative* [7] if

$$A(\bot), A(p), A(q) \vdash_{\mathrm{J}} A(p\&q); \ \ A(\bot), A(p), A(q) \vdash_{\mathrm{J}} A(p \vee q);$$

$$A(\bot), A(p), A(q) \vdash_{\mathrm{J}} A(p \to q).$$

If L is a J-logic and the formula A is J-conservative, then the logic $L + A$ is deduced to L. We have

Lemma 2.1 ([13], [7]). *Suppose that a formula $A(p)$ is* J*-conservative. Then for every logic L and every formula $B(p_1, \ldots, p_k)$*

$$L + A(p) \vdash B(p_1, \ldots, p_k) \iff L \vdash A(\bot) \& A(p_1) \& \ldots \& A(p_k) \to B(p_1, \ldots, p_k).$$

The addition of conservative formulas as new axioms preserves the finite model property FMP and interpolation properties.

Taking into consideration the fact that the logic J possesses the properties FMP and CIP, we get

Theorem 2.2 ([13], [7]). *For any* J*-logic L*

(1) *If L possesses FMP, then $L + Od$ possesses FMP*;

(2) *If L possesses CIP, then $L + Od$ possesses CIP.*

In particular, the logic Od *has the properties CIP and FMP.*

The semantic completeness of the logic Od with respect to modified semantics is established in [5]. This logic is complete with respect to a class of frames that satisfies the condition of maximality and also the condition

(F2) For every maximal in $W - Q$ element x and every $y \in Q$, $x \leq y \Rightarrow y = \infty$.

Similarly to superintuitionistic logics, all pre-Heyting logics possess the weak interpolation property WIP [19].

The characterization of pre-Heyting algebras by nonembeddability is obtained in [6], and the information on strong perceptibility and strong recognizability can be found in [14].

3. Perceptibility of F over Od

The majority of the most important formulas are perceptible over J and, consequently, over any J-logic. In [8], the question of the percebtibility of the formula F over J and the recognizability of the logic JF over J is left open. The decidability of this logic is proved in [12]. The perceptibility of the formula F in the class of well-composed logics is obtained in [14]. In this section, we will prove the perceptibility of the formula F over the logic Od.

Our proof uses algebraic semantics. Every variety is generated by its subdirectly irreducible algebras. Recall [3] that a J-algebra is said to be subdirectly irreducible (s.i.) if and only if it has an *opremum*, that is, the greatest of the elements distinct from $\top$.

The following lemma is known:

Lemma 3.1 ([3]). *If $a \not\leq b$ in a J-algebra $\mathbf{A}$, then there exists a homomorphism h from $\mathbf{A}$ on a subdirectly irreducible J-algebra $\mathbf{B}$ such that $h(a) = \top$, $h(b) = \Omega_{\mathbf{B}}$.*

We will prove the following

Theorem 3.2. *Suppose $Od + A \not\vdash F$. Then there exists a s.i. pre-Heyting algebra with not more than* 14 *elements, satisfying A and refuting F.*

Proof. Suppose $Od + A \not\vdash F$. By Lemma 3.1, there exist s.i. pre-Heyting algebra and elements a_1, b_1 such that

$\bot \leq a_1 \vee b_1$, $(\bot \to a_1) \vee (\bot \to b_1) = \Omega$.

Let $a = \bot \to a_1$, $b = \bot \to b_1$. Then

$\bot \to a = a$, $\bot \to b = b$, $a \vee b = \Omega$.

Let **B** be a subalgebra generated by elements a, b.

Then it is s.i. pre-Heyting, and the elements a, b satisfy the conditions:

$\bot \leq a \vee b$, $(\bot \to a) \vee (\bot \to b) = \Omega$;

$\bot \to a = a$, $\bot \to b = b$;

$a \to \bot = (\bot \to a) \to \bot = \bot$, $b \to \bot = \bot$.

In particular, a, b, $\bot$ are pairwise incomparable.

Also $a\&b$ is incomparable to $\bot$, $\bot\&a$ is incomparable to b, $\bot\&b$. So, the algebra has to include the following elements of the set FM:

1) $a\&b\&\bot$, 2) $a\&b$, 3) $a\&\bot$, 4) $b\&\bot$, 5) a, 6) b, 7) $\bot$, 8) $a \vee \bot$, 9) $b \vee \bot$, 10) $\Omega = a \vee b$, 11) $(a\&b) \vee \bot$, 12) $a\&(b \vee \bot)$, 13) $b\&(a \vee \bot)$, 14) $\top$.

Here 1 is the least, $\top$ is the greatest element, 10 is an opremum. Next, 2, 3, 4 are pairwise incomparable,

$3 < 5 < 8$; $4 < 6 < 9$; $3, 4 < 7 < 8, 9$; $11 \leq 10$;

$8, 9 \leq 10$; $12 \leq 5$; $13 \leq 6$.

The number of elements of the algebra can be estimated with the help of Proposition 3.3.

□

Proposition 3.3. *Suppose* $\mathrm{Od}{+}Ax \not\vdash F$, *then there exists a pre-Heyting s.i. algebra* **B** *with underlying set* $FM = \{1 - 14\}$, *such that* $\mathbf{B} \models Ax$:

1) $a\&b\&\bot$, 2) $a\&b$, 3) $a\&\bot$, 4) $b\&\bot$, 5) a, 6) b, 7) $\bot$, 8) $a \vee \bot$, 9) $b \vee \bot$, 10) $\Omega = a \vee b$, 11) $(a\&b) \vee \bot$, 12) $a\&(b \vee \bot)$, 13) $b\&(a \vee \bot)$, 14) $\top$.

Proof. The set FM contains $\bot$. We want to prove that the set FM is closed under all operations. First we will check whether FM is closed under binary operations. Let us prove that the set FM is closed under &.

It is obvious that $x\&y = x \iff x \leq y$ and $x\&\Omega = x$ for each $x \neq \top$; $\top\&\Omega = \Omega$. Note that $2\&3 = 2\&4 = 2\&7 = 3\&4 = 3\&6 = 1$. The element 4 is comparable to all the elements except 2, 3, 5, 12. The following equalities hold: $4\&5 = 1$, $5\&9 = a\&(b \vee \bot) = 12$, $6\&8 = 13$, $11\&12 = ((a\&b) \vee \bot)\&(a\&(b \vee \bot) = (a \vee \bot)\&(b \vee \bot)\&a\&(b \vee \bot) = a\&(b \vee \bot) = 12$, $11\&13 = 13$, $12\&13 = a\&b = 2$.

The results of the operation & for the elements 1–14 belong to the set FM and are listed in the table below.

&	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2	2	2	2	2	2	2
3	1	1	3	1	3	1	3	3	3	3	3	3	1	3
4	1	1	1	4	1	4	4	4	4	4	4	1	4	4
5	1	2	3	1	5	2	3	5	12	5	12	12	2	5
6	1	2	1	4	2	6	4	13	6	6	13	2	13	6
7	1	1	3	4	3	4	7	7	7	7	7	3	4	7
8	1	2	3	4	5	13	7	8	9	8	11	12	13	8
9	1	2	3	4	12	6	7	9	9	9	11	12	13	9
10	1	2	3	4	5	6	7	8	9	10	11	12	13	10
11	1	2	3	4	12	13	7	11	11	11	11	12	13	11
12	1	2	3	1	12	2	3	12	12	12	12	12	2	12
13	1	2	1	4	2	13	4	13	13	13	13	2	13	13
14	1	2	3	4	5	6	7	8	9	10	11	12	13	14

So, FM is closed under &.

Now, we prove that the set FM is closed under $\vee$.
Note that $(a\&\bot) \vee b = (a \vee b)\&(b \vee \bot) = \Omega\&(b \vee \bot) = b \vee \bot$,
$8 \vee 12 = a \vee \bot \vee a\&(b \vee \bot) = a \vee \bot$,
$11 \vee 12 = (a\&b) \vee \bot \vee (a\&(b \vee \bot)) = (a \vee \bot)\&(a \vee \bot \vee b \vee \bot) = (a \vee \bot)\&\Omega = a \vee \bot$,
$11 \vee 13 = (a\&b) \vee \bot \vee b\&(a \vee \bot) = (a\&b) \vee \bot \vee b\&\bot = (a\&b) \vee \bot = 11$,
$12 \vee 13 = (a\&(b \vee \bot)) \vee (b\&(a \vee \bot)) = (a\&b) \vee (a\&\bot) \vee (a\&b) \vee (b\&\bot) = (a\&b) \vee \bot = 11$.

The results of the operation $\vee$ are listed in the table:

$\vee$	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	2	2	12	13	5	6	11	8	9	10	11	12	13	14
3	3	12	3	7	5	9	7	8	9	10	11	12	11	14
4	4	13	7	4	8	6	7	8	9	10	11	11	13	14
5	5	5	5	8	5	10	8	8	10	10	8	5	8	14
6	6	6	9	6	10	6	9	10	9	10	9	9	6	14
7	7	11	7	7	8	9	7	8	9	10	7	11	11	14
8	8	8	8	8	8	10	8	8	10	10	8	8	8	14
9	9	9	9	9	10	9	9	9	9	10	9	9	9	14
10	10	10	10	10	10	10	10	10	10	10	10	10	10	14
11	11	11	11	11	8	9	7	8	9	10	11	11	11	14
12	12	12	12	11	5	9	11	8	9	10	11	12	11	14
13	13	13	11	13	8	6	11	8	9	10	11	11	13	14
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14

Thus, we have proved that FM is closed under $\vee$.

Now, let us prove that the set FM is closed under $\to$

We have

$$x \leq y \iff x \to y = \top,$$

$$\top \to x = x,$$

$$\Omega \to x = x \text{ if } x < \Omega,\ \Omega \to x = \top \text{ if } x \geq \Omega.$$

$2 \to 3 = a\&b \to a\&\bot = a\&b \to \bot = a \to \bot = \bot = 7$.

$2 \to 4 = \bot = 7$.

Since $2 \leq 5, 2 \leq 6$, then $2 \to 5 = 2 \to 6 = \top$.

$2 \to 7 = a\&b \to \bot = \bot$.

As $2 \leq 8, 9, 10, 11, 12, 13$, then $2 \to 8 = 2 \to 9 = 2 \to 10 = 2 \to 11 = 2 \to 12 = 2 \to 13 = \top$.

Next, $3 \to 2 = a\&\bot \to a\&b = a\&\bot \to b = b$,

$3 \to 4 = a\&\bot \to b\&\bot = a\&\bot \to b = a \to b = b$, $3 \leq 5, 7, 8, 9, 11$.

$3 \to 6 = a\&\bot \to b = b$,

$3 \to 12 = a\&\bot \to (a\&(b \vee \bot) = \top$.

$3 \to 13 = a\&\bot \to b\&(a \vee \bot) = a\&\bot \to b = b$.

For $4 \to k$ the proof is similar.

We note that $5 \to 3 = a \to a\&\bot = a \to \bot = \bot$.

The results of $5 \to 1,\ 5 \to 2,\ 5 \to 4$ belong to the set FM because 1–14 are closed under conjunction, as $x \to y\&z = (x \to y)\&(x \to z)$.

We have $5 \to 6 = a \to b = b$,

$5 \to 7 = \bot$.

Since $5 \leq 8$, then $5 \to 8 = \top$.

Next, $5 \to 9 = a \to b \vee \bot = \Omega \to b \vee \bot$ holds.

It is easy to show that $5 \to 10,\ 5 \to 11,\ 5 \to 12,\ 5 \to 13 \in$1–14.

For $6 \to k$ the proof is similar.

We note that $7 \to 5 = \bot \to a = a$, $7 \to 6 = \bot \to b = b$, also $7 \leq 8, 9, 10, 11$.

The results of $7 \to 12,\ 7 \to 14$ belong to the set FM because FM is closed under &, as $x \to y\&z = (x \to y)\&(x \to z)$.

Similarly, $8 \to k, 9 \to k, 11 \to k$ belong to the set FM due to the fact that FM is closed under &.

Since $12 \to k = a\&(b \vee \bot) \to k = a \to (b \vee \bot \to k)$, then $12 \to k$ belongs to the set FM due to the already proved fact that the elements $(b \vee \bot) \to k$, $a \to k$ belong to the set FM. Similarly, it can be shown that $13 \to k \in FM$.

So, we proved that the set FM is closed under $\to$.

$\square$

Theorem 3.4. *The formula F is perceptible over* Od, *and the logic* OdF *is recognizable over* Od.

Proof. Let Ax be an arbitrary finite set of axioms. If $\mathrm{Od} + Ax \nvdash F$, then we apply Theorem 3.2. Conversely, if $\mathbf{B} \models Ax$ and $\mathbf{B} \not\models F$ for some pre-Heyting algebra $\mathbf{B}$, then $\mathrm{Od} + Ax \nvdash F$. Therefore, F is perceptible over Od.

The decidability of the logic OdF follows from the finite model property of the logic JF [12] and the fact that the formula Od is J-conservative. Thus, the logic OdF is perceptible over Od.

□

Note that the 14 elements in the obtained algebra do not have to be distinct.

Let us consider, for example, the following cases:

(1) $a \vee \perp = b \vee \perp = \Omega$.

The elements 8, 9, 11, 12, 13 stick together with the others, and 9 elements remain.

(2) $a \vee \perp = \Omega$, $b \vee \perp < \Omega$.

There are 11 elements remaining in the algebra.

(3) $b \vee \perp = \Omega$, $a \vee \perp < \Omega$.

There are also 11 elements that remain in the algebra.

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Larisa L'vovna Maksimova
Sobolev Institute of Mathematics,
4, Koptyuga ave.,
Novosibirsk, 630090, Russia
Email address: lmaksi@math.nsc.ru

Veta Fedorovna Yun
Sobolev Institute of Mathematics,
4, Koptyuga ave.,
Novosibirsk ,630090, Russia
Email address: yun@math.nsc.ru