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СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru

Том 17, стр. 1183–1216 (2020) УДК 514.765
DOI 10.33048/semi.2020.17.089 MSC 53B99

ON FOUR-DIMENSIONAL LOCALLY HOMOGENEOUS PSEUDO-RIEMANNIAN MANIFOLDS WITH ISOTROPIC WEYL TENSOR

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Abstract. The papers of many mathematicians are devoted to studying of (pseudo)Riemannian manifolds with zero Weyl tensor (i.e. conformally flat manifolds). Moreover, one can consider manifolds whose Weyl tensor has zero squared length, and itself is not zero. Also such manifolds are called manifolds with isotropic Weyl tensor. In the case of a Riemannian metric, the squared length of a tensor is the sum of the squares of all components in some orthonormal basis, and it is zero iff the tensor itself is trivial. Therefore, it is natural to consider only the case of a pseudo-Riemannian metric. In the case of dimension 3, the Weyl tensor is trivial, and the Schouten–Weyl tensor (also known as the Cotton tensor) is the analogue of the Weyl tensor. The Schouten–Weyl tensor was investigated for a left-invariant Lorentzian metric on three-dimensional Lie groups, including the problem of its isotropy, in the work of E.D. Rodionov, V.V. Slavskii, L.N. Chibrikova. In this paper results on the investigation of four-dimensional locally homogeneous spaces with nontrivial isotropy subgroup and with invariant pseudo-Riemannian metric and an isotropic Weyl tensor are presented.
Keywords: (pseudo)Riemannian manifold, isotropic Weyl tensor, systems of computer mathematics.

1. Introduction.

The papers of many mathematicians are devoted to studying of (pseudo)Riemannian manifolds with zero Weyl tensor (i.e. conformally flat manifolds). Moreover,

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Klepikova, S.V., On four-dimensional locally homogeneous pseudo-Riemannian manifolds with isotropic Weyl tensor.

Received August, 26, 2018, published August, 24, 2020.

one can consider manifolds whose Weyl tensor has zero squared length, and itself is not zero. Also such manifolds are called manifolds with isotropic Weyl tensor [1].

In the case of a Riemannian metric, the squared length of the tensor is the sum of the squares of all components in some orthonormal basis, and it is zero if the tensor itself is trivial. Therefore, it is natural to consider only the case of a pseudo-Riemannian metric. In the case of dimension 3, the Weyl tensor is trivial, and the Schouten–Weyl tensor (also known as the Cotton tensor) is the analogue of the Weyl tensor. The Schouten–Weyl tensor was studied for left-invariant Lorentzian metric on three-dimensional Lie groups [2, 3]. This investigation is the continuation of the research of J. Milnor for left-invariant Riemannian metrics on three-dimensional Lie groups [4].

In this paper results on the investigation of four-dimensional locally homogeneous spaces with nontrivial isotropy subgroup and with invariant pseudo-Riemannian metric and an isotropic Weyl tensor are presented. This paper continues the research, begun in [3, 5].

2. Basic notations and facts.

Let $(M = G/H, g)$ is a locally homogeneous pseudo-Riemannian manifold of dimension n, $\mathfrak{g}$ is a Lie algebra of the group G, $\mathfrak{h}$ is a isotropy subalgebra, $\mathfrak{m} = \mathfrak{g}/\mathfrak{h}$ is a subspace of $\mathfrak{g}$ complementary to $\mathfrak{h}$.

The pair $(\mathfrak{g}, \mathfrak{h})$ uniquely determines the isotropy representation $\psi : \mathfrak{h} \to \mathfrak{gl}(\mathfrak{m})$ by the rule $\psi_X(Y) = [X, Y]_{\mathfrak{m}}\ \forall X \in \mathfrak{h}, Y \in \mathfrak{m}$. An invariant (pseudo)Riemannian metric on G/H corresponds to a non-degenerate bilinear form g on $\mathfrak{m}$, such that

$$(\psi_X)^t \cdot g + g \cdot \psi_X = 0, \quad \forall X \in \mathfrak{h},$$

where $(\psi_X)^t$ is a transposed matrix.This form uniquely identifies the Levi-Civita connection $\nabla : \mathfrak{g} \to \mathfrak{gl}(\mathfrak{m})$ by the rule

$$\nabla_X(Y_{\mathfrak{m}}) = \frac{1}{2}[X, Y]_{\mathfrak{m}} + v(X, Y),$$

where $v : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{m}$ defined by the formula

$$2g(v(X, Y), Z_{\mathfrak{m}}) = g(X_{\mathfrak{m}}, [Z, Y]_{\mathfrak{m}}) + g(Y_{\mathfrak{m}}, [Z, X]_{\mathfrak{m}}).$$

The connection ∇ corresponds to a curvature tensor $R : \mathfrak{m} \times \mathfrak{m} \to \mathfrak{gl}(\mathfrak{m})$ such that

$$R(X, Y) = [\nabla_Y, \nabla_X] + \nabla_{[X,Y]}.$$

The Ricci tensor r and scalar curvature s are determined by formulas

$$r(X, Y) = \mathrm{tr}(Z \to R(X, Z)Y), \quad s = tr_g(r).$$

The Weyl tensor (i.e. the conformal curvature tensor) W has the following form:

$$W = R - A \bigcirc\!\!\!\!\wedge\, g,$$

where

$$A = \frac{1}{n-2}\left(r - \frac{sg}{2(n-1)}\right) \text{ is the one-dimensional curvature tensor,}$$

$$(A \bigcirc\!\!\!\!\wedge\, g)(X, Y, Z, V) = A(X, Z)g(Y, V) + A(Y, V)g(X, Z) - A(X, V)g(Y, Z) \\ - A(Y, Z)g(X, V) \text{ is the Kulkarni–Nomizu product [6].}$$

The square of length of the Weyl tensor calculated by the following formula

$$\|W\|^2 = W_{ijkt} W_{lmqr} g^{il} g^{jm} g^{kq} g^{tr}.$$

Definition 1. *Pseudo-Riemannian manifold (M, g) is called the manifold with isotropic Weyl tensor, if simultaneously $\|W\|^2 = 0$ and $W \neq 0$ for metric g.*

Application of computer mathematic systems for studying the locally homogeneous (pseudo)Riemannian manifolds with isotropic Weyl tensor becomes possible with a sufficiently low dimension. Further we present a mathematical model that allows us to calculate the squared length of the Weyl tensor on a locally homogeneous (pseudo)Riemannian space [7, 8, 9].

Let, as earlier, $(M = G/H, g)$ is a homogeneous (pseudo)Riemannian manifold of dimension n, $\mathfrak{g}$ is a Lie algebra of the group G, $\mathfrak{h}$ is a isotropy subalgebra, $\mathfrak{m} = \mathfrak{g}/\mathfrak{h}$ is a subspace of $\mathfrak{g}$ complementary to $\mathfrak{h}$, $h = \dim \mathfrak{h}$, $m = \dim \mathfrak{m}$.

Let $\{e_1, e_2, \ldots, e_h, u_1, u_2, \ldots, u_m\}$ is a basis $\mathfrak{g}$, where $\{e_i\}$ и $\{u_i\}$ are bases of $\mathfrak{h}$ and $\mathfrak{m}$ respectively. Accept, that

$$[u_i, u_j]_{\mathfrak{m}} = c_{ij}^k u_k, \quad [u_i, u_j]_{\mathfrak{h}} = C_{ij}^k e_k, \quad [h_i, u_j]_{\mathfrak{m}} = \bar{c}_{ij}^k u_k,$$

where c_{ij}^k, C_{ij}^k are $\bar{c}_{ij}^k$ are arrays of appropriate sizes.

At first we calculate the isotropy representation ψ on basis vectors $\mathfrak{h}$:

$$(\psi_i)_j^k = (\psi(e_i))_j^k = \bar{c}_{ij}^k, \tag{1}$$

and write the invariance condition for the metric tensor g:

$$(\psi_i)^t \cdot g + g \cdot \psi_i = 0, \quad i = 1, \ldots, h, \tag{2}$$

where $(\psi_i)^t$ is a transposed matrix.

Further, we find the components of the Levi-Civita connection ∇, using the structure constants and the matrix of metric tensor:

$$\Gamma_{ij}^k = \frac{1}{2}\left(c_{ij}^k + g^{sk} c_{sj}^l g_{il} + g^{sk} c_{si}^l g_{jl}\right), \quad \bar{\Gamma}_{ij}^k = \frac{1}{2}\bar{c}_{ij}^k - \frac{1}{2} g^{sk} \bar{c}_{is}^l g_{jl} :$$

where $\nabla_{u_i} u_j = \Gamma_{ij}^k u_k$, $\nabla_{h_i} u_j = \bar{\Gamma}_{ij}^k u_k$ и $\{g^{ij}\}$ is a matrix inverse to the matrix $\{g_{ij}\}$.

Next step of the algorithm is a calculating the components of curvature tensor R, Ricci tensor r and scalar curvature s:

$$R_{ijks} = \left(\Gamma_{ik}^l \Gamma_{jl}^p - \Gamma_{jk}^l \Gamma_{il}^p + c_{ij}^l \Gamma_{lk}^p + C_{ij}^l \bar{\Gamma}_{lk}^p\right) g_{ps},$$
$$r_{ik} = R_{ijks} g^{js}, \quad s = r_{ik} g^{ik}.$$

Further, we can find the components of the one-dimensional curvature tensor, the Weyl tensor and the square of length of the Weyl tensor

$$A_{ij} = \frac{1}{n-2}\left(r_{ij} - \frac{s g_{ij}}{2(n-1)}\right),$$
$$W_{ijkt} = R_{ijkt} - A_{ik} g_{jt} - A_{jt} g_{ik} + A_{it} g_{jk} + A_{jk} g_{it},$$
$$\|W\|^2 = W_{ijkt} W_{\alpha\beta\gamma\delta} g^{i\alpha} g^{j\beta} g^{k\gamma} g^{t\delta}.$$

The aim of this paper is proof of the following

Theorem 1. *Let $(M = G/H, g)$ is a locally homogeneous manifold of dimension 4 with nontrivial isotropy subgroup. Then $(M = G/H, g)$ has isotropic Weyl tensor iff the Lie algebra of group G contained in tables 1–6.*

Table 1. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor

№	Lie brackets	№ g	Restrictions
$1.1^1.1$	$[e_1,u_1]=u_1$, $[e_1,u_3]=-u_3$, $[u_1,u_3]=u_2$, $[u_2,u_4]=u_2$, $[u_3,u_4]=u_3$	1	$\alpha_{22}=\frac{\alpha_{13}^2(13\pm3\sqrt{17})+8\alpha_{24}^2}{8\alpha_{44}}$
$1.1^2.1$	$[e_1,u_1]=u_3$, $[e_1,u_3]=-u_1$, $[u_1,u_3]=-u_2$, $[u_1,u_4]=u_1$, $[u_2,u_4]=2u_2$, $[u_3,u_4]=u_3$	3	$\alpha_{22}=\frac{2\alpha_{24}^2-\alpha_{33}^2(13\pm3\sqrt{17})}{2\alpha_{44}}\neq 0$
$1.3^1.1$	$[e_1,u_1]=e_1$, $[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_2]=-\frac{1}{2}u_2$, $[u_1,u_3]=u_3$, $[u_1,u_4]=\frac{1}{2}u_4$, $[u_2,u_3]=\frac{1}{2}u_4$	5	$\alpha_{44}=0$, $\alpha_{33}^2+\alpha_{34}^2\neq 0$
$1.3^1.2$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=-\lambda e_1+(\lambda+1)u_1+\lambda u_2$, $[u_2,u_4]=u_2$, $\lambda\in[-1,1]$	5	$\alpha_{44}\neq 0$, $\lambda\neq 0$
$1.3^1.3$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=u_1$, $[u_2,u_4]=u_2$, $[u_3,u_4]=e_1$	5	—
$1.3^1.4$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=-(1+\lambda^2)e_1+2\lambda u_1+(1+\lambda^2)u_2$, $[u_2,u_4]=u_2$, $\lambda\geqslant 0$	5	$\alpha_{44}\neq 0$
$1.3^1.5$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=-\frac{\lambda^2+\mu}{\mu-1}e_1+\frac{1+\lambda^2}{\mu-1}u_2$, $[u_1,u_4]=-\lambda e_1+u_1+\lambda u_2$, $[u_2,u_3]=-\lambda e_1+u_1+\lambda u_2$, $[u_2,u_4]=-\mu e_1+(\mu+1)u_2$, $\lambda\geqslant 0$, $\mu\neq 1$	5	$(\mu-1)(2\lambda\alpha_{34}-\mu\alpha_{33})\neq$ $\neq\alpha_{44}(\lambda^2+\mu)$
$1.3^1.6$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=-u_2$, $[u_1,u_4]=u_1$, $[u_2,u_3]=u_1$, $[u_2,u_4]=u_2$, $[u_3,u_4]=e_1$	5	—
$1.3^1.7$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_1,u_3]=\frac{1}{1+\lambda}e_1+\frac{\lambda}{1+\lambda}u_1-\frac{1}{1+\lambda}u_2$, $[u_1,u_4]=-\frac{1}{1+\lambda}e_1+\frac{1}{1+\lambda}u_1+\frac{1}{1+\lambda}u_2$, $[u_2,u_3]=-\frac{1}{1+\lambda}e_1+\frac{1}{1+\lambda}u_1+\frac{1}{1+\lambda}u_2$, $[u_2,u_4]=-\frac{\lambda}{1+\lambda}e_1+\frac{\lambda}{1+\lambda}u_1+\frac{1+2\lambda}{1+\lambda}u_2$, $\lambda\neq -1$	5	$\alpha_{44}\neq\lambda\alpha_{33}-2\alpha_{34}$
$1.3^1.8$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_2,u_3]=u_1$, $[u_2,u_4]=u_2$, $[u_3,u_4]=-u_3$	5	$\alpha_{33}\neq 0$
$1.3^1.9$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_2,u_3]=\lambda u_1$, $[u_2,u_4]=-\lambda e_1+(\lambda+1)u_2$, $[u_3,u_4]=-\lambda u_3$	5	$\alpha_{33}\neq 0$, $\lambda\neq -1$, $\lambda\neq 0$
$1.3^1.10$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_2,u_4]=u_2$, $[u_3,u_4]=e_1$	5	—
$1.3^1.11$	$[e_1,u_3]=u_1$, $[e_1,u_4]=u_2$, $[u_2,u_3]=-u_1$, $[u_2,u_4]=e_1$, $[u_3,u_4]=e_1+u_3$	5	—

The proof of this theorem will consist of a consideration of several propositions below.

In this paper, we will use the classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds obtained in [10]. This classification given at the end of this paper in tables 8–14. The Lie brackets, defining the Lie algebra of group G are indicated for each case. The parameters can take any real values, unless otherwise indicated, $\mathfrak{h}=\operatorname{span}(e_i)$, $\mathfrak{m}=\operatorname{span}(u_i)$ in all cases. Further along the text we will refer to cases from this classification by their number (for example, "$2.1^3.5$").

Below in the text, when specifying the type of invariant metric, we will refer to the table 7. For example, the phrase "The metric tensor has the form 4" means that the matrix of the metric tensor has the form, shown in the table 7 under number 4 together with the corresponding restrictions on the components of the metric tensor.

Table 2. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor. Continuation

№	Lie brackets	№ g	Restrictions
$1.3^1.12$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\mu u_1,$ $[u_2,u_4]=-\lambda\mu e_1+(\lambda+\mu)u_2,\ [u_3,u_4]=(1-\mu)u_3$	5	$\alpha_{33}\neq 0,\ \mu\neq\frac{1}{2},$ $\mu\neq-(\lambda+1)$
$1.3^1.13$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\frac{1}{2}u_1,$ $[u_2,u_4]=-\frac{\lambda}{2}e_1+\left(\lambda+\frac{1}{2}\right)u_2,\ [u_3,u_4]=e_1+\frac{1}{2}u_3$	5	—
$1.3^1.14$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=(1-\lambda)u_1,$ $[u_2,u_4]=\lambda(\lambda-1)e_1+u_2,\ [u_3,u_4]=e_1+\lambda u_3,\ \lambda\neq\frac{1}{2}$	5	—
$1.3^1.15$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-e_1+2u_1,\ [u_1,u_4]=u_2,$ $[u_2,u_3]=u_2,\ [u_2,u_4]=-e_1+u_1$	5	$\alpha_{33}\neq-\alpha_{44}$
$1.3^1.16$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-e_1+2u_1,\ [u_1,u_4]=u_2,$ $[u_2,u_3]=u_2,\ [u_2,u_4]=e_1-u_1$	5	$\alpha_{33}\neq\alpha_{44}$
$1.3^1.17$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_4]=u_1,\ [u_3,u_4]=e_1$	5	—
$1.3^1.19$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=u_1,$ $[u_2,u_4]=-e_1+u_1+2u_2$	5	$\alpha_{33}\neq 0$
$1.3^1.20$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_3]=u_1,\ [u_2,u_4]=u_2-u_1,$ $[u_3,u_4]=-u_3$	5	$\alpha_{33}\neq 0$
$1.3^1.21$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\lambda u_1,$ $[u_2,u_4]=-\lambda e_1+(1-\lambda)u_1+(1+\lambda)u_2,\ [u_3,u_4]=(1-\lambda)u_3,$ $\lambda\neq 1$	5	$\alpha_{33}\neq 0,\ \lambda\neq 0,$ $\lambda\neq\frac{1}{2}$
$1.3^1.22$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\frac{1}{2}u_1,$ $[u_2,u_4]=-\frac{1}{2}e_1+\frac{1}{2}u_1+\frac{3}{2}u_2,\ [u_3,u_4]=e_1+\frac{1}{2}u_3$	5	—
$1.3^1.23$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_1+u_2,$ $[u_3,u_4]=e_1+u_3$	5	—
$1.3^1.24$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=(1-2\lambda)e_1+2\lambda u_1,$ $[u_1,u_4]=(2\lambda-1)u_2,\ [u_2,u_3]=\lambda u_2,$ $[u_2,u_4]=\frac{2\lambda-1}{2\lambda-2}e_1-\frac{1}{2\lambda-2}u_1,\ [u_3,u_4]=(\lambda-1)u_4,\ \lambda\neq 1$	5	$\lambda\neq\frac{2}{3},$ $\alpha_{33}\neq 2\lambda\alpha_{44}(\lambda-1)$
$1.3^1.25$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=(1-2\lambda)e_1+2\lambda u_1,$ $[u_1,u_4]=(2\lambda-1)u_2,\ [u_2,u_3]=\lambda u_2,$ $[u_2,u_4]=\frac{1-2\lambda}{2\lambda-2}e_1+\frac{1}{2\lambda-2}u_1,\ [u_3,u_4]=(\lambda-1)u_4,\ \lambda\neq 1$	5	$\lambda\neq\frac{2}{3},$ $\alpha_{33}\neq 2\lambda\alpha_{44}(1-\lambda)$

3. Isotropy of the Weyl tensor.

The classification of locally homogeneous conformally flat (pseudo)Riemannian 4-manifolds with a nontrivial isotropy subgroup was obtained in [9]. In particular, the following was proved

Proposition 1. [9] *Let $(M=G/H,g)$ is a locally homogeneous (pseudo)Riemannian manifold of dimension 4 with a nontrivial isotropy subgroup. Then the Weyl tensor of $(G/H,g)$ is equal to zero for any invariant metric g, iff G/H contains in following list:*

$1.1^1.(9,10)$, $1.1^2.12$, $1.1^3.1$, $1.1^4.1$, $1.1^5.1$,
$1.1^6.1$, $1.2^1.1$, $1.2^2.1$, $1.3^1.(18,32)$, $1.4^1.(8,23,26)$,
$2.1^1.3$, $2.1^2.6$, $2.1^3.6$, $2.1^4.2$, $2.2^1.(2,3,6,7)$,
$2.2^2.4$, $2.2^3.1$, $2.3^1.1$, $2.4^1.$(1-3), $2.5^1.(4,6,9,10,14)$,
$2.5^2.7$, $3.1^1.1$, $3.1^2.1$, $3.2^1.(1,2,4)$, $3.2^2.(1,2)$,
$3.3^1.$(1-4), $3.3^2.$(1-4), $3.4^1.1$, $3.4^2.1$, $3.5^1.$(1-4),
$3.5^2.$(1-4), $4.1^1.1$, $4.1^2.1$, $4.2^1.2$, $4.2^2.3$,
$4.2^3.2$, $4.3^1.2$, $5.1^1.1$, $6.1^1.(1,2)$, $6.1^2.$(1-3),
$6.1^3.$(1-3).

TABLE 3. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor. Continuation

№	Lie brackets	№ g	Restrictions
$1.3^1.26$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\frac{1}{3}e_1+\frac{4}{3}u_1,$ $[u_1,u_4]=\frac{1}{3}u_2,\ [u_2,u_3]=\frac{2}{3}u_2,\ [u_2,u_4]=-\frac{1}{2}e_1+\frac{3}{2}u_1,$ $[u_3,u_4]=e_1-\frac{1}{3}u_4$	5	—
$1.3^1.27$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\frac{1}{3}e_1+\frac{4}{3}u_1,$ $[u_1,u_4]=\frac{1}{3}u_2,\ [u_2,u_3]=\frac{2}{3}u_2,\ [u_2,u_4]=\frac{1}{2}e_1-\frac{3}{2}2u_1,$ $[u_3,u_4]=e_1-\frac{1}{3}u_4$	5	—
$1.3^1.28$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=2u_1,$ $[u_1,u_4]=2u_2,\ [u_2,u_3]=u_2,\ [u_2,u_4]=e_1-\frac{1}{2}u_1,$ $[u_3,u_4]=u_4$	5	$\alpha_{33}\neq 2\alpha_{44}$
$1.3^1.29$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=2u_1,$ $[u_1,u_4]=2u_2,\ [u_2,u_3]=u_2,\ [u_2,u_4]=-e_1+\frac{1}{2}u_1,$ $[u_3,u_4]=u_4$	5	$\alpha_{33}\neq -2\alpha_{44}$
$1.3^1.30$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,$ $[u_1,u_3]=\frac{\lambda\mu(\lambda-1)}{\lambda+\mu-\lambda\mu}e_1+\frac{\lambda^2+\mu-\lambda^2\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda(1-\lambda)}{\lambda+\mu-\lambda\mu}u_2,$ $[u_1,u_4]=-\frac{\lambda\mu}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda}{\lambda+\mu-\lambda\mu}u_2,$ $[u_2,u_3]=-\frac{\lambda\mu}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda}{\lambda+\mu-\lambda\mu}u_2,$ $[u_2,u_4]=\frac{\lambda\mu(\mu-1)}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu(1-\mu)}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda+\mu^2-\mu^2\lambda}{\lambda+\mu-\lambda\mu}u_2,$ $\lambda+\mu-\lambda\mu\neq 0,\ 1\leqslant\mu\leqslant\lambda,\ \lambda\mu>0$	5	$\alpha_{34}\neq\frac{\alpha_{33}(1-\mu)+\alpha_{44}(1-\lambda)}{2}$
$1.3^1.31$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_3,u_4]=e_1$	5	—
$1.4^1.1$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_2]=u_1,$ $[u_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=u_3,\ [u_3,u_4]=-u_3$	6	$\alpha_{33}\neq 0$
$1.4^1.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=pu_1,$ $[u_2,u_4]=(p-1)u_2,\ [u_3,u_4]=(p-2)u_3$	6	$\alpha_{33}\neq 0,\ p\neq 3$
$1.4^1.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=2u_1,$ $[u_2,u_3]=e_1,\ [u_2,u_4]=u_2$	6	$\alpha_{33}\neq\alpha_{44}$
$1.4^1.4$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=2u_1,$ $[u_2,u_3]=-e_1,\ [u_2,u_4]=u_2$	6	$\alpha_{33}\neq -\alpha_{44}$

Remark. *A notation like "$1.1^1.(9,10)$" means "$1.1^1.9$, $1.1^1.10$". In total, the above list contains 74 locally homogeneous (pseudo)Riemannian manifolds.*

Proposition 2. *Let $(M=G/H,g)$ is the locally homogeneous (pseudo)Riemannian manifold of dimension 4 with a nontrivial isotropy subgroup. If G/H contains in the below list, then the square of length of the Weyl tensor $\|W\|^2$ is not equal to zero for any invariant metric g:*

$$1.1^1.(7,8)\quad 1.1^2.(9\text{-}11)\quad 2.1^1.2\quad 2.1^2.(3\text{-}5)$$
$$2.1^3.(4,5)\quad 4.2^1.1\quad 4.2^2.(1,2)\quad 4.2^3.1$$

Proof. Sequentially consider all the cases that are given above.

Case $1.1^1.7$**.** In this case the Lie brackets are as follows:

$$[e_1,u_1]=u_1,\qquad [e_1,u_3]=-u_3,\qquad [u_1,u_3]=e_1,$$

where $\mathfrak{h}=\operatorname{span}(e_1)$, $\mathfrak{m}=\operatorname{span}(u_1,u_2,u_3,u_4)$.

TABLE 4. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor. Continuation

№	Lie brackets	№ g	Restrictions
$1.4^1.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_2]=u_1,\ [u_1,u_3]=u_2,$ $[u_2,u_3]=u_3$	6	$\alpha_{33}\neq 0$
$1.4^1.6$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,$ $[u_3,u_4]=u_1+u_3$	6	—
$1.4^1.7$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,$ $[u_3,u_4]=-u_1+u_3$	6	—
$1.4^1.9$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,$ $[u_2,u_3]=re_1+u_2+u_4,\ [u_3,u_4]=pu_4$	6	$\alpha_{44}\neq 0,$ $\alpha_{22}(p(p+1)-r)\neq 0$
$1.4^1.10$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2,$ $[u_3,u_4]=pu_4$	6	$r\neq p(p+1)$
$1.4^1.11$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,$ $[u_2,u_3]=re_1+u_2+u_4,\ [u_3,u_4]=u_1-u_4$	6	—
$1.4^1.12$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2,$ $[u_3,u_4]=u_1-u_4$	6	$r\neq 0$
$1.4^1.13$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=re_1+u_4,\ [u_3,u_4]=u_4$	6	—
$1.4^1.14$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=re_1,\ [u_3,u_4]=u_4$	6	$r\neq 1$
$1.4^1.15$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1+u_4,\ [u_3,u_4]=u_1$	6	$\alpha_{22}\neq-\alpha_{44}$
$1.4^1.16$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1+u_4,\ [u_3,u_4]=u_1$	6	$\alpha_{22}\neq\alpha_{44}$
$1.4^1.17$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=u_4,\ [u_3,u_4]=u_1$	6	—
$1.4^1.18$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1+u_4$	6	$\alpha_{22}\neq-\alpha_{44}$
$1.4^1.19$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1+u_4$	6	$\alpha_{22}\neq\alpha_{44}$
$1.4^1.20$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=u_4$	6	—
$1.4^1.21$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1,\ [u_3,u_4]=u_1$	6	—
$1.4^1.22$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1,\ [u_3,u_4]=u_1$	6	—
$1.4^1.24$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1$	6	—
$1.4^1.25$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1$	6	—

We calculate the isotropy representation (1):

$$\psi_1=\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix}.$$

Let the metric tensor has the form:

$$g=\begin{pmatrix}\alpha_{11}&\alpha_{12}&\alpha_{13}&\alpha_{14}\\\alpha_{12}&\alpha_{22}&\alpha_{23}&\alpha_{24}\\\alpha_{13}&\alpha_{23}&\alpha_{33}&\alpha_{34}\\\alpha_{14}&\alpha_{24}&\alpha_{34}&\alpha_{44}\end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{12}=0,\quad \alpha_{14}=0,\quad \alpha_{11}=0,\quad \alpha_{23}=0,\quad \alpha_{33}=0,\quad \alpha_{34}=0.$$

Thus, the invariant metric tensor has the form 1.

The nontrivial components of the Weyl tensor are:

$$W_{1234}=-W_{2314}=-\frac{\alpha_{24}}{6},\quad W_{1223}=-\frac{\alpha_{22}}{6},$$

$$W_{1434}=\frac{\alpha_{44}}{6},\quad W_{1313}=\frac{\alpha_{13}}{3},\quad W_{2424}=-\frac{\alpha_{22}\alpha_{44}-\alpha_{24}^2}{3\alpha_{13}}.$$

TABLE 5. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor. Continuation

№	Lie brackets	№ g	Restrictions
$2.2^1.4$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,$ $[e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [u_1,u_3]=e_2,\ [u_2,u_3]=e_1,\ [u_2,u_4]=e_2$	10	$\alpha_{23}\neq 0$
$2.2^1.5$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,$ $[e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [u_2,u_3]=e_2$	10	—
$2.2^2.1$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,$ $[e_2,u_3]=u_1,\ [e_2,u_4]=u_2,\ [u_1,u_3]=e_2,\ [u_2,u_4]=e_2,\ [u_3,u_4]=-e_1$	11	$\alpha_{44}\neq 0$
$2.2^2.2$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,$ $[e_2,u_3]=u_1,\ [e_2,u_4]=u_2,\ [u_1,u_3]=-e_2,\ [u_2,u_4]=-e_2,\ [u_3,u_4]=e_1$	11	$\alpha_{44}\neq 0$
$2.2^2.3$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,$ $[e_2,u_3]=u_1,\ [e_2,u_4]=u_2,\ [u_3,u_4]=e_2$	11	—
$2.5^1.1$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_1,u_4]=-2e_1,\ [e_2,u_2]=-2e_2,$ $[e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_1,u_2]=2e_2-u_1,\ [u_1,u_3]=u_2+u_4,$ $[u_1,u_4]=2e_1-u_1,\ [u_2,u_3]=-2u_3,\ [u_2,u_4]=u_2-u_4,\ [u_3,u_4]=2u_3$	12	$\alpha_{33}\neq 0$
$2.5^1.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_2]=-2e_2,\ [e_2,u_3]=-u_2,$ $[e_2,u_4]=u_1,\ [u_1,u_2]=-u_1,\ [u_1,u_3]=u_4,\ [u_2,u_3]=-2u_3,$ $[u_2,u_4]=-u_4$	12	$\alpha_{33}\neq 0$
$2.5^1.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_1,u_3]=u_1,\ [u_2,u_3]=e_1+pe_2+(1-q)u_2,\ [u_2,u_4]=qu_1,$ $[u_3,u_4]=-(p+q)e_1+\lambda e_2-(1+q)u_4,\ q\geqslant 0\ (if\ \lambda\neq 0),$ $q\in\mathbb{R}\ (if\ \lambda=0)$	12	—
$2.5^1.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1+qe_2-u_2,\ [u_2,u_4]=u_1,\ [u_3,u_4]=-qe_1-\lambda e_2-u_4$	12	—
$2.5^1.7$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1+e_2,\ [u_3,u_4]=-e_1+\lambda e_2$	12	—

The square of length of the Weyl tensor has the following form:

$$\|W\|^2=\frac{4}{3\alpha_{13}^2}.$$

Obviously, it's not equal to zero for any invariant metric g.

Because the proof algorithm is uniform in all cases, then further we give only the number of type the metric tensor g (from table 7), the nontrivial components of Weyl tensor W and the square of length of the Weyl tensor $\|W\|^2$ for each case.

Case $1.1^1.8$.

g	W	$\|W\|^2$
2	$W_{1234}=W_{2314}=-\frac{\alpha_{24}}{2},\ W_{1324}=-\alpha_{24}$	$\frac{12}{\alpha_{13}^2}$

Case $1.1^2.9$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=-\frac{\alpha_{24}}{6},\ W_{1212}=W_{2323}=-\frac{\alpha_{22}}{6},$ $W_{1414}=W_{3434}=-\frac{\alpha_{44}}{6},\ W_{1313}=\frac{\alpha_{33}}{3},\ W_{2424}=\frac{\alpha_{22}\alpha_{44}-\alpha_{24}^2}{3\alpha_{33}}$	$\frac{4}{3\alpha_{33}^2}$

Case $1.1^2.10$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=\frac{\alpha_{24}}{6},\ W_{1212}=W_{2323}=\frac{\alpha_{22}}{6},$ $W_{1414}=W_{3434}=\frac{\alpha_{44}}{6},\ W_{1313}=-\frac{\alpha_{33}}{3},\ W_{2424}=-\frac{\alpha_{22}\alpha_{44}-\alpha_{24}^2}{3\alpha_{33}}$	$\frac{4}{3\alpha_{33}^2}$

TABLE 6. The locally homogeneous pseudo" Riemannian 4" manifolds with nontrivial isotropy subgroup and isotropic Weyl tensor. Continuation

№	Lie brackets	№ g	Restrictions
$2.5^1.8$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1-e_2,\ [u_3,u_4]=e_1+\lambda e_2$	12	—
$2.5^1.11$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1,\ [u_3,u_4]=e_2$	12	—
$2.5^1.12$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1,\ [u_3,u_4]=-e_2$	12	—
$2.5^1.13$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_2,u_3]=e_1$	12	—
$2.5^2.1$	$[e_1,u_2]=-e_1+u_1,\ [e_1,u_3]=-u_2,\ [e_1,u_4]=e_2,\ [e_2,u_2]=-e_2,$ $[e_2,u_3]=u_4,\ [e_2,u_4]=-e_1-u_1,\ [u_1,u_2]=e_1-u_1,\ [u_1,u_3]=u_2,$ $[u_1,u_4]=-e_2,\ [u_2,u_3]=-2u_3,\ [u_2,u_4]=-u_4$	6	$\alpha_{33}\neq 0$
$2.5^2.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,$ $[u_1,u_3]=u_1,\ [u_2,u_3]=(p+s)e_1+re_2+u_2-2ru_4,\ [u_2,u_4]=2ru_1,$ $[u_3,u_4]=-re_1+(p-s)e_2-2ru_2-u_4,\ r\geqslant 0,\ s\geqslant 0$	13	$s\neq 0$
$2.5^2.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,$ $[u_2,u_3]=-(r+s)e_1-u_4,\ [u_2,u_4]=u_1,\ [u_3,u_4]=(s-r)e_2-u_2,$ $s\geqslant 0$	13	$s\neq 0$
$2.5^2.4$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,$ $[u_2,u_3]=(1+s)e_1,\ [u_3,u_4]=(1-s)e_2,\ s\geqslant 0$	13	$s\neq 0$
$2.5^2.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,$ $[u_2,u_3]=-(1+s)e_1,\ [u_3,u_4]=(s-1)e_2,\ s\geqslant 0$	13	$s\neq 0$
$2.5^2.6$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,$ $[u_2,u_3]=e_2,\ [u_3,u_4]=e_1$	13	—
$3.2^1.3$	$[e_1,e_3]=2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,$ $[e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,$ $[e_3,u_4]=u_1,\ [u_2,u_3]=e_2$	14	—
$4.3^1.1$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,$ $[e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,$ $[e_2,u_3]=-u_4,\ [e_3,u_1]=u_2,\ [e_3,u_4]=-u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=u_1,\ [u_3,u_4]=e_4$	14	—

Case $1.1^2.11$**.**

g	W	$\|W\|^2$
4	$W_{1234}=W_{2314}=-\alpha_{44},\ W_{1324}=-2\alpha_{44}$	$\frac{48}{\alpha_{33}^2}$

Case $2.1^1.2$**.**

g	W	$\|W\|^2$
2	$W_{1234}=-W_{2314}=-\frac{\alpha_{24}}{6},\ W_{1313}=\frac{\alpha_{13}}{2},\ W_{2424}=-\frac{\alpha_{24}^2}{3\alpha_{13}}$	$\frac{4}{3\alpha_{13}^2}$

Case $2.1^2.3$**.**

g	W	$\|W\|^2$
7	$W_{1223}=-W_{1434}=-\frac{\alpha_{44}}{6},\ W_{1313}=\frac{\alpha_{13}}{3},\ W_{2424}=-\frac{\alpha_{44}^2}{3\alpha_{13}}$	$\frac{4}{3\alpha_{13}^2}$

Case $2.1^2.4$**.**

g	W	$\|W\|^2$
7	$W_{1223}=-W_{1434}=-\frac{\alpha_{13}}{6},\ W_{1313}=-\frac{\alpha_{13}^2}{3\alpha_{44}},\ W_{2424}=\frac{\alpha_{44}}{3}$	$\frac{4}{3\alpha_{44}^2}$

TABLE 7. The form of the invariant metric tensor

№	The matrix of metric tensor	Restrictions	№	The matrix of metric tensor	Restrictions
1	$\begin{pmatrix} 0 & 0 & \alpha_{13} & 0 \\ 0 & \alpha_{22} & 0 & \alpha_{24} \\ \alpha_{13} & 0 & 0 & 0 \\ 0 & \alpha_{24} & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{13} \neq 0$, $\alpha_{24}^2 \neq \alpha_{22}\alpha_{44}$	9	$\begin{pmatrix} 0 & 0 & \alpha_{24} & 0 \\ 0 & \alpha_{22} & 0 & \alpha_{24} \\ \alpha_{24} & 0 & 0 & 0 \\ 0 & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{24} \neq 0$
2	$\begin{pmatrix} 0 & 0 & \alpha_{13} & 0 \\ 0 & 0 & 0 & \alpha_{24} \\ \alpha_{13} & 0 & 0 & 0 \\ 0 & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{13} \neq 0$, $\alpha_{24} \neq 0$	10	$\begin{pmatrix} 0 & 0 & \alpha_{24} & 0 \\ 0 & 0 & \alpha_{23} & \alpha_{24} \\ \alpha_{24} & \alpha_{23} & 0 & 0 \\ 0 & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{24} \neq 0$
3	$\begin{pmatrix} \alpha_{33} & 0 & 0 & 0 \\ 0 & \alpha_{22} & 0 & \alpha_{24} \\ 0 & 0 & \alpha_{33} & 0 \\ 0 & \alpha_{24} & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{33} \neq 0$, $\alpha_{24}^2 \neq \alpha_{22}\alpha_{44}$	11	$\begin{pmatrix} 0 & 0 & 0 & -\alpha_{23} \\ 0 & 0 & \alpha_{23} & 0 \\ 0 & \alpha_{23} & \alpha_{44} & 0 \\ -\alpha_{23} & 0 & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{23} \neq 0$
4	$\begin{pmatrix} \alpha_{33} & 0 & 0 & 0 \\ 0 & \alpha_{44} & 0 & 0 \\ 0 & 0 & \alpha_{33} & 0 \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{33} \neq 0$, $\alpha_{44} \neq 0$	12	$\begin{pmatrix} 0 & 0 & \alpha_{24} & 0 \\ 0 & 0 & 0 & \alpha_{24} \\ \alpha_{24} & 0 & \alpha_{33} & 0 \\ 0 & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{24} \neq 0$
5	$\begin{pmatrix} 0 & 0 & 0 & -\alpha_{23} \\ 0 & 0 & \alpha_{23} & 0 \\ 0 & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ -\alpha_{23} & 0 & \alpha_{34} & \alpha_{44} \end{pmatrix}$	$\alpha_{23} \neq 0$	13	$\begin{pmatrix} 0 & 0 & \alpha_{44} & 0 \\ 0 & \alpha_{44} & 0 & 0 \\ \alpha_{44} & 0 & \alpha_{33} & 0 \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{44} \neq 0$
6	$\begin{pmatrix} 0 & 0 & -\alpha_{22} & 0 \\ 0 & \alpha_{22} & 0 & 0 \\ -\alpha_{22} & 0 & \alpha_{33} & \alpha_{34} \\ 0 & 0 & \alpha_{34} & \alpha_{44} \end{pmatrix}$	$\alpha_{22} \neq 0$, $\alpha_{44} \neq 0$	14	$\begin{pmatrix} 0 & 0 & \alpha_{24} & 0 \\ 0 & 0 & 0 & \alpha_{24} \\ \alpha_{24} & 0 & 0 & 0 \\ 0 & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{24} \neq 0$
7	$\begin{pmatrix} 0 & 0 & \alpha_{13} & 0 \\ 0 & \alpha_{44} & 0 & 0 \\ \alpha_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{13} \neq 0$, $\alpha_{44} \neq 0$	15	$\begin{pmatrix} \alpha_{44} & 0 & 0 & 0 \\ 0 & \alpha_{44} & 0 & 0 \\ 0 & 0 & \alpha_{44} & 0 \\ 0 & 0 & 0 & \alpha_{44} \end{pmatrix}$	$\alpha_{44} \neq 0$
8	$\begin{pmatrix} 0 & 0 & -\alpha_{24} & \alpha_{23} \\ 0 & 0 & \alpha_{23} & \alpha_{24} \\ -\alpha_{24} & \alpha_{23} & 0 & 0 \\ \alpha_{23} & \alpha_{24} & 0 & 0 \end{pmatrix}$	$\alpha_{23}^2 + \alpha_{24}^2 \neq 0$			

Case $2.1^2.5$**.**

g	W	$\lVert W \rVert^2$
7	$W_{1223} = -W_{1434} = -\frac{\alpha_{13}}{6}$, $W_{1313} = \frac{\alpha_{13}^2}{3\alpha_{44}}$, $W_{2424} = -\frac{\alpha_{44}}{3}$	$\frac{4}{3\alpha_{44}^2}$

Case $2.1^3.4$**.**

g	W	$\lVert W \rVert^2$
4	$W_{1212} = W_{2323} = -\frac{\alpha_{44}}{6}$, $W_{1414} = W_{3434} = -\frac{\alpha_{44}}{6}$, $W_{1313} = \frac{\alpha_{33}}{3}$, $W_{2424} = -\frac{\alpha_{44}^2}{3\alpha_{33}}$	$\frac{4}{3\alpha_{33}^2}$

Case $2.1^3.5$**.**

g	W	$\lVert W \rVert^2$
4	$W_{1212} = W_{2323} = \frac{\alpha_{44}}{6}$, $W_{1414} = W_{3434} = \frac{\alpha_{44}}{6}$, $W_{1313} = -\frac{\alpha_{33}}{3}$, $W_{2424} = -\frac{\alpha_{44}^2}{3\alpha_{33}}$	$\frac{4}{3\alpha_{33}^2}$

Case $4.2^1.1$.

g	W	$\|W\|^2$
14	$W_{1234} = W_{1313} = W_{1324} = W_{2424} = 2\alpha_{24}$	$\frac{96}{\alpha_{24}^2}$

Case $4.2^2.1$.

g	W	$\|W\|^2$
15	$W_{1212} = W_{2323} = -\alpha_{44}$, $W_{1234} = W_{2314} = W_{1414} = W_{3434} = \alpha_{44}$, $W_{1313} = W_{1324} = W_{2424} = 2\alpha_{44}$	$\frac{96}{\alpha_{44}^2}$

Case $4.2^2.2$.

g	W	$\|W\|^2$
15	$W_{1212} = W_{2323} = \alpha_{44}$, $W_{1234} = W_{2314} = W_{1414} = W_{3434} = -\alpha_{44}$, $W_{1313} = W_{1324} = W_{2424} = -2\alpha_{44}$	$\frac{96}{\alpha_{44}^2}$

Case $4.2^3.1$.

g	W	$\|W\|^2$
14	$W_{1234} = W_{1313} = W_{1324} = W_{2424} = -\alpha_{44}$, $W_{1212} = W_{3434} = -3\alpha_{24}$	$\frac{96}{\alpha_{24}^2}$

□

Proposition 3. *If $(M = G/H, g)$ is locally homogeneous (pseudo)Riemannian manifold of dimension 4 with a nontrivial isotropy subgroup (besides those, which are given in Proposition 1) and G/H contains in the below list, then if the square of length of the Weyl tensor $\|W\|^2$ is vanished for some invariant metric g, then the Weyl tensor W itself vanishes:*

$1.1^1.(2\text{-}6)$ $1.1^2.(2\text{-}8)$ $2.1^1.1$ $2.1^2.(1,2)$ $2.1^3.(1\text{-}3)$ $2.1^4.1$ $2.2^1.1$

Proof. Sequentially consider all the cases that are given above.

Case $1.1^1.2$. In this case the Lie brackets are as follows:

$$[e_1, u_1] = u_1, \qquad [e_1, u_3] = -u_3, \qquad [u_2, u_4] = pu_2, \qquad [u_3, u_4] = u_3,$$

where $\mathfrak{h} = \operatorname{span}(e_1)$, $\mathfrak{m} = \operatorname{span}(u_1, u_2, u_3, u_4)$, $p \in \mathbb{R}$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{12} = 0, \quad \alpha_{14} = 0, \quad \alpha_{11} = 0, \quad \alpha_{23} = 0, \quad \alpha_{33} = 0, \quad \alpha_{34} = 0.$$

Thus, the invariant metric tensor has the form 1.

The nontrivial components of the Weyl tensor are:

$$W_{1234} = -W_{2314} = \frac{\alpha_{22}\alpha_{24}\alpha_{13}p(2p-1)}{12\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)},$$

$$W_{1223} = -\frac{\alpha_{22}^2\alpha_{13}p(2p-1)}{12\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)} \qquad W_{1434} = \frac{\alpha_{22}\alpha_{13}\alpha_{44}p(2p-1)}{12\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)},$$

$$W_{1313} = \frac{\alpha_{22}\alpha_{13}^2 p(2p-1)}{6\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)}, \qquad W_{2424} = -\frac{\alpha_{22}p(2p-1)}{6}.$$

The square of length of the Weyl tensor has the following form:

$$\|W\|^2 = \frac{\left(\alpha_{22}p\,(2p-1)\right)^2}{3\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)^2}.$$

It's equal to zero when either $\alpha_{22}=0$, or $p=0$, or $p=\frac{1}{2}$. But the Weyl tensor is trivial in all these cases.

Because the proof algorithm is uniform in all cases, then further we give only the number of type the metric tensor g (from table 7), the nontrivial components of Weyl tensor W and the square of length of the Weyl tensor $\|W\|^2$ for each case.

Case $1.1^1.3$.

g	W	$\|W\|^2$
1	$W_{1234} = W_{2314} = \frac{\alpha_{24}(\alpha_{13}-\alpha_{22})}{6\alpha_{13}}$, $W_{1223} = -\frac{\alpha_{22}(\alpha_{13}-\alpha_{22})}{6\alpha_{13}}$, $W_{1434} = \frac{\alpha_{44}(\alpha_{13}-\alpha_{22})}{6\alpha_{13}}$, $W_{1313} = \frac{(\alpha_{13}-\alpha_{22})}{3}$, $W_{2424} = -\frac{(\alpha_{22}\alpha_{44}-\alpha_{24}^2)(\alpha_{13}-\alpha_{22})}{3\alpha_{13}^2}$	$\frac{4(\alpha_{13}-\alpha_{22})^2}{3\alpha_{13}^4}$

Case $1.1^1.4$.

g	W	$\|W\|^2$
1	$W_{1234} = -W_{2314} = W_{1434} = -\frac{\alpha_{24}\alpha_{22}}{6\alpha_{13}}$, $W_{1313} = -\frac{\alpha_{22}}{3}$, $W_{2424} = -\frac{(\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{22}}{3\alpha_{13}^2}$, $W_{1223} = \frac{\alpha_{22}^2}{6\alpha_{13}}$	$\frac{4\alpha_{22}^2}{3\alpha_{13}^4}$

Case $1.1^1.5$.

g	W	$\|W\|^2$
1	$W_{1234} = -W_{2314} = \frac{(\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2)\alpha_{24}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1223} = -\frac{(\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2)\alpha_{22}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1434} = \frac{(\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2)\alpha_{44}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1313} = \frac{(\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2)\alpha_{13}}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{2424} = -\frac{\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2}{3\alpha_{13}}$	$\frac{4(\alpha_{22}(\alpha_{13}+\alpha_{44})-\alpha_{24}^2)^2}{3\alpha_{13}(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $1.1^1.6$.

g	W	$\|W\|^2$
1	$W_{1234} = -W_{2314} = \frac{\alpha_{24}\alpha_{22}\alpha_{13}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{2424} = -\frac{\alpha_{22}}{3}$, $W_{1223} = -\frac{\alpha_{22}^2\alpha_{13}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1434} = \frac{\alpha_{22}\alpha_{44}\alpha_{13}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1313} = \frac{\alpha_{22}\alpha_{13}^2}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$	$\frac{4\alpha_{22}^2}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $1.1^2.2$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=-\frac{\alpha_{22}\alpha_{33}\alpha_{24}p(p-1)}{6\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)}$, $W_{1212}=W_{2323}=\frac{\alpha_{22}^2\alpha_{33}p(p-1)}{6\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)}$, $W_{1414}=W_{3434}=\frac{\alpha_{22}\alpha_{33}\alpha_{44}p(p-1)}{6\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)}$, $W_{1313}=-\frac{\alpha_{22}\alpha_{33}^2p(p-1)}{3\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)}$, $W_{2424}=-\frac{\alpha_{22}p(p-1)}{3}$	$\frac{4\alpha_{22}^2p^2(p-1)^2}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $1.1^2.3$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=-\frac{(\alpha_{22}-\alpha_{33})\alpha_{24}}{6\alpha_{33}}$, $W_{1212}=W_{2323}=\frac{(\alpha_{22}-\alpha_{33})\alpha_{22}}{6\alpha_{33}}$, $W_{1414}=W_{3434}=\frac{(\alpha_{22}-\alpha_{33})\alpha_{44}}{6\alpha_{33}}$, $W_{1313}=-\frac{\alpha_{22}-\alpha_{33}}{3\alpha_{33}}$, $W_{2424}=-\frac{(\alpha_{22}-\alpha_{33})(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}{3\alpha_{33}^2}$	$\frac{4(\alpha_{22}-\alpha_{33})^2}{3\alpha_{33}^4}$

Case $1.1^2.4$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=-\frac{(\alpha_{22}+\alpha_{33})\alpha_{24}}{6\alpha_{33}}$, $W_{1212}=W_{2323}=\frac{(\alpha_{22}+\alpha_{33})\alpha_{22}}{6\alpha_{33}}$, $W_{1414}=W_{3434}=\frac{(\alpha_{22}+\alpha_{33})\alpha_{44}}{6\alpha_{33}}$, $W_{1313}=-\frac{\alpha_{22}+\alpha_{33}}{3}$, $W_{2424}=-\frac{(\alpha_{22}+\alpha_{33})(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}{3\alpha_{33}^2}$	$\frac{4(\alpha_{22}+\alpha_{33})^2}{3\alpha_{33}^4}$

Case $1.1^2.5$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=-\frac{\alpha_{22}\alpha_{24}}{6\alpha_{33}}$, $W_{1212}=W_{2323}=\frac{\alpha_{22}^2}{6\alpha_{33}}$, $W_{1414}=W_{3434}=\frac{\alpha_{22}\alpha_{44}}{6\alpha_{33}}$, $W_{1313}=-\frac{\alpha_{22}}{3}$, $W_{2424}=-\frac{(\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{22}}{3\alpha_{33}^2}$	$\frac{4\alpha_{22}^2}{3\alpha_{33}^4}$

Case $1.1^2.6$.

g	W	$\|W\|^2$
3	$W_{1214}=-W_{2334}=\frac{(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)\alpha_{24}}{6(\alpha_{24}^2-\alpha_{22}\alpha_{44})}$, $W_{1212}=W_{2323}=\frac{(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)\alpha_{22}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1414}=W_{3434}=\frac{(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)\alpha_{44}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1313}=-\frac{(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)\alpha_{33}}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{2424}=-\frac{(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)}{3\alpha_{33}}$	$\frac{4(\alpha_{22}\alpha_{33}-\alpha_{22}\alpha_{44}+\alpha_{24}^2)^2}{3\alpha_{33}^2(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $1.1^2.7$.

g	W	$\|W\|^2$
3	$W_{1214} = -W_{2334} = \frac{(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{24}}{6(\alpha_{24}^2-\alpha_{22}\alpha_{44})}$, $W_{1212} = W_{2323} = \frac{(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{22}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1414} = W_{3434} = \frac{(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{44}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1313} = -\frac{(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)\alpha_{33}}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{2424} = -\frac{(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)}{3\alpha_{33}}$	$\frac{4(\alpha_{22}\alpha_{33}+\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}{3\alpha_{33}^2(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $1.1^2.8$.

g	W	$\|W\|^2$
3	$W_{1214} = -W_{2334} = -\frac{\alpha_{22}\alpha_{24}\alpha_{33}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1212} = W_{2323} = \frac{\alpha_{22}^2\alpha_{33}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1414} = W_{3434} = \frac{\alpha_{22}\alpha_{33}\alpha_{44}}{6(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{1313} = -\frac{\alpha_{22}\alpha_{33}^2}{3(\alpha_{22}\alpha_{44}-\alpha_{24}^2)}$, $W_{2424} = -\frac{\alpha_{22}}{3}$	$\frac{4\alpha_{22}^2}{3\alpha_{33}^2(\alpha_{22}\alpha_{44}-\alpha_{24}^2)^2}$

Case $2.1^1.1$.

g	W	$\|W\|^2$
2	$W_{1234} = -W_{2314} = -\frac{\alpha_{13}+\alpha_{24}}{6}$, $W_{1313} = W_{2424} = -\frac{\alpha_{13}(\alpha_{13}+\alpha_{24})}{3\alpha_{24}}$	$\frac{4(\alpha_{13}+\alpha_{24})^2}{3\alpha_{13}^2\alpha_{24}^2}$

Case $2.1^2.1$.

g	W	$\|W\|^2$
7	$W_{1223} = -W_{1434} = -\frac{\alpha_{13}-\alpha_{44}}{6}$, $W_{1313} = -\frac{\alpha_{13}(\alpha_{13}-\alpha_{44})}{3\alpha_{44}}$, $W_{2424} = \frac{\alpha_{44}(\alpha_{13}-\alpha_{44})}{3\alpha_{13}}$	$\frac{4(\alpha_{13}-\alpha_{44})^2}{3\alpha_{13}^2\alpha_{44}^2}$

Case $2.1^2.2$.

g	W	$\|W\|^2$
7	$W_{1223} = -W_{1434} = -\frac{\alpha_{13}+\alpha_{44}}{6}$, $W_{1313} = \frac{\alpha_{13}(\alpha_{13}+\alpha_{44})}{3\alpha_{44}}$, $W_{2424} = -\frac{\alpha_{44}(\alpha_{13}+\alpha_{44})}{3\alpha_{13}}$	$\frac{4(\alpha_{13}+\alpha_{44})^2}{3\alpha_{13}^2\alpha_{44}^2}$

Case $2.1^3.1$.

g	W	$\|W\|^2$
4	$W_{1212} = W_{2323} = -\frac{\alpha_{33}+\alpha_{44}}{6}$, $W_{1414} = W_{3434} = -\frac{\alpha_{33}+\alpha_{44}}{6}$, $W_{1313} = \frac{\alpha_{33}(\alpha_{33}+\alpha_{44})}{3\alpha_{44}}$, $W_{2424} = -\frac{\alpha_{44}(\alpha_{13}+\alpha_{44})}{3\alpha_{33}}$	$\frac{4(\alpha_{33}+\alpha_{44})^2}{3\alpha_{33}^2\alpha_{44}^2}$

Case $2.1^3.2$.

g	W	$\|W\|^2$
4	$W_{1212} = W_{2323} = \frac{\alpha_{33}-\alpha_{44}}{6}$, $W_{1414} = W_{3434} = \frac{\alpha_{33}-\alpha_{44}}{6}$, $W_{1313} = -\frac{\alpha_{33}(\alpha_{33}-\alpha_{44})}{3\alpha_{44}}$, $W_{2424} = -\frac{\alpha_{44}(\alpha_{33}-\alpha_{44})}{3\alpha_{33}}$	$\frac{4(\alpha_{33}-\alpha_{44})^2}{3\alpha_{33}^2\alpha_{44}^2}$

Case $2.1^3.3$.

g	W	$\|W\|^2$
4	$W_{1212} = W_{2323} = \frac{\alpha_{33}+\alpha_{44}}{6}$, $W_{1414} = W_{3434} = \frac{\alpha_{33}+\alpha_{44}}{6}$, $W_{1313} = -\frac{\alpha_{33}(\alpha_{33}+\alpha_{44})}{3\alpha_{44}}$, $W_{2424} = -\frac{\alpha_{44}(\alpha_{33}+\alpha_{44})}{3\alpha_{33}}$	$\frac{4(\alpha_{33}+\alpha_{44})^2}{3\alpha_{33}^2\alpha_{44}^2}$

Case $2.1^4.1$.

g	W	$\|W\|^2$
8	$W_{1313} = W_{1324} = W_{2323} = W_{1414} = W_{2424} = -W_{2314} = \frac{\alpha_{24}(3\alpha_{23}^2-\alpha_{24}^2)}{\alpha_{23}^2+\alpha_{24}^2}$, $W_{1323} = W_{1314} = -W_{2324} = -W_{1424} = \frac{4\alpha_{24}^2\alpha_{23}}{3(\alpha_{23}^2+\alpha_{24}^2)^2}$, $W_{1234} = \frac{2\alpha_{24}}{3}$	$\frac{64\alpha_{24}^2}{(\alpha_{23}^2+\alpha_{24}^2)^2}$

Case $2.2^1.1$.

g	W	$\|W\|^2$
9	$W_{1234} = W_{1313} = W_{1324} = W_{2424} = -\frac{\alpha_{22}}{2}$, $W_{1223} = \frac{\alpha_{22}^2}{4\alpha_{24}}$	$\frac{6\alpha_{22}^2}{\alpha_{24}^4}$

□

Proposition 4. *If $(M = G/H, g)$ is locally homogeneous pseudo-Riemannan manifold of dimension 4 with a nontrivial isotropy subgroup and G/H contains in the below list, then the square of length of the Weyl tensor $\|W\|^2$ is equal to zero and the Weyl tensor W isn't equal to zero for any invariant metric g:*

$$1.3^1.(3, 6, 10, 11, 13, 14, 17, 22, 23, 26, 27, 31)$$
$$1.4^1.(6, 7, 11, 13, 17, 20\text{-}22, 24, 25)$$
$$2.2^1.5 \quad 2.2^2.3 \quad 2.5^1.(3, 5, 7, 8, 11\text{-}13)$$
$$2.5^2.6 \quad 3.2^1.3 \quad 4.3^1.1$$

Proof. Sequentially consider all the cases that are given above.

Case $1.3^1.3$. In this case the Lie brackets are as follows:

$$[e_1, u_3] = u_1, \quad [e_1, u_4] = u_2, \quad [u_1, u_3] = u_1, \quad [u_2, u_4] = u_2, \quad [u_3, u_4] = e_1,$$

where $\mathfrak{h} = \operatorname{span}(e_1)$, $\mathfrak{m} = \operatorname{span}(u_1, u_2, u_3, u_4)$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{11} = 0, \quad \alpha_{12} = 0, \quad \alpha_{22} = 0, \quad \alpha_{13} = 0, \quad \alpha_{24} = 0, \quad \alpha_{23} + \alpha_{14} = 0.$$

Thus, the invariant metric tensor has the form 5.

The nontrivial component of the Weyl tensor is:

$$W_{3434} = -\alpha_{23}.$$

It can't be equal to zero, because for $\alpha_{23} = 0$ the metric tensor would be degenerate.

By direct calculations, we see that the square of length of the Weyl tensor is trivial.

Because the proof algorithm is uniform in all cases, then further we give only the number of type the metric tensor g (from table 7), the nontrivial components of Weyl tensor W and the square of length of the Weyl tensor $\|W\|^2$ for each case. The square of length of the Weyl tensor is trivial and the Weyl tensor is nontrivialdue to restrictions on the components of the metric tensor, which given in the table 7 in all cases.

Case $1.3^1.6$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.10$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.11$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.13$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.14$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.17$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.22$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.23$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.26$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.27$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.3^1.31$.

g	W
5	$W_{3434} = -\alpha_{23}$

Case $1.4^1.6$.

g	W
6	$W_{2323} = -\frac{\alpha_{22}^2}{2\alpha_{44}}$, $W_{3434} = \frac{\alpha_{22}}{2}$

Case $1.4^1.7$.

g	W
6	$W_{2323} = \frac{\alpha_{22}^2}{2\alpha_{44}}$, $W_{3434} = -\frac{\alpha_{22}}{2}$

Case $1.4^1.11$.

g	W
6	$W_{2334} = \frac{\alpha_{44}}{2}$, $W_{2323} = -\frac{r\alpha_{22}+\alpha_{44}}{2}$, $W_{3434} = -\frac{\alpha_{44}(r\alpha_{22}+\alpha_{44})}{2\alpha_{22}}$

Case $1.4^1.13$.

g	W
6	$W_{2334} = -\alpha_{44}$, $W_{2323} = \frac{\alpha_{22}(1-r)-\alpha_{44}}{2}$, $W_{3434} = \frac{\alpha_{44}(\alpha_{22}(r-1)+\alpha_{44})}{2\alpha_{22}}$

Case $1.4^1.17$.

g	W
6	$W_{2323} = -\frac{\alpha_{44}}{2}$, $W_{3434} = \frac{\alpha_{44}^2}{2\alpha_{22}}$

Case $1.4^1.20$.

g	W
6	$W_{2323} = -\frac{\alpha_{44}}{2}$, $W_{3434} = \frac{\alpha_{44}^2}{2\alpha_{22}}$

Case $1.4^1.21$.

g	W
6	$W_{2323} = -\frac{\alpha_{22}}{2}$, $W_{3434} = \frac{\alpha_{44}}{2}$

Case $1.4^1.22$.

g	W
6	$W_{2323} = \frac{\alpha_{22}}{2}$, $W_{3434} = -\frac{\alpha_{44}}{2}$

Case $1.4^1.24$.

g	W
6	$W_{2323} = -\frac{\alpha_{22}}{2}$, $W_{3434} = \frac{\alpha_{44}}{2}$

Case $1.4^1.25$.

g	W
6	$W_{2323} = \frac{\alpha_{22}}{2}$, $W_{3434} = -\frac{\alpha_{44}}{2}$

Case $2.2^1.5$.

g	W
10	$W_{2323} = \alpha_{24}$

Case $2.2^2.3$.

g	W
11	$W_{3434} = -\alpha_{23}$

Case $2.5^1.3$.

g	W
12	$W_{2323} = \alpha_{24}$, $W_{3434} = -\lambda\alpha_{24}$

Case $2.5^1.5$.

g	W
12	$W_{2323} = \alpha_{24}$, $W_{3434} = \lambda\alpha_{24}$

Case $2.5^1.7$.

g	W
12	$W_{2323} = \alpha_{24}$, $W_{3434} = -\lambda\alpha_{24}$

Case $2.5^1.8$.

g	W
12	$W_{2323} = \alpha_{24}$, $W_{3434} = -\lambda\alpha_{24}$

Case $2.5^1.11$.

g	W
12	$W_{2323} = -W_{3434} = \alpha_{24}$

Case $2.5^1.12$.

g	W
12	$W_{2323} = W_{3434} = \alpha_{24}$

Case $2.5^1.13$.

g	W
12	$W_{2323} = \alpha_{24}$

Case $2.5^2.6$.

g	W
13	$W_{2323} = W_{3434} = \alpha_{44}$

Case $3.2^1.3$.

g	W
14	$W_{2323} = \alpha_{24}$

Case $4.3^1.1$.

g	W
14	$W_{3434} = -\alpha_{24}$

□

Proposition 5. *If $(M = G/H, g)$ is locally homogeneous pseudo-Riemannian manifold of dimension 4 with a nontrivial isotropy subgroup and G/H contains in the below list, then the square of length of the Weyl tensor $\|W\|^2$ is equal to zero for any invariant metric g, but the Weyl tensor can be equal to zero for some invariant metric:*

$$1.3^1.(2, 4, 5, 7\text{-}9, 12, 15, 16, 19\text{-}21, 24, 25, 28\text{-}30)$$
$$1.4^1.(1\text{-}5, 9, 10, 12, 14\text{-}16, 18, 19)$$
$$2.2^1.4 \quad 2.2^2.(1, 2) \quad 2.5^1.(1, 2) \quad 2.5^2.(1\text{-}5)$$

Proof. Sequentially consider all the cases that are given above.

Case $1.3^1.2$. In this case the Lie brackets are as follows:

$$[e_1, u_3] = u_1, \qquad [e_1, u_4] = u_2,$$
$$[u_1, u_3] = -\lambda e_1 + (\lambda + 1)u_1 + \lambda u_2, \qquad [u_2, u_4] = u_2,$$

where $\mathfrak{h} = \mathrm{span}\,(e_1)$, $\mathfrak{m} = \mathrm{span}\,(u_1, u_2, u_3, u_4)$, $|\lambda| \leqslant 1$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{11} = 0, \quad \alpha_{12} = 0, \quad \alpha_{22} = 0, \quad \alpha_{13} = 0, \quad \alpha_{24} = 0, \quad \alpha_{23} + \alpha_{14} = 0.$$

Thus, the invariant metric tensor has the form 5.

The nontrivial component of the Weyl tensor is:

$$W_{3434} = -\frac{\lambda\alpha_{44}}{2}.$$

In can be equal to zero either $\lambda = 0$, or $\alpha_{44} = 0$.

By direct calculations, we see that the square of length of the Weyl tensor is trivial.

Because the proof algorithm is uniform in all cases, then further we give only the number of type the metric tensor g (from table 7), the nontrivial components of Weyl tensor W and the square of length of the Weyl tensor $\|W\|^2$ for each case. The square of length of the Weyl tensor is trivial in all cases.

Case $1.3^1.4$**.**

g	W
5	$W_{3434} = -\frac{(\lambda^2+1)\alpha_{44}}{2}$

Case $1.3^1.5$**.**

g	W
5	$W_{3434} = \frac{\lambda(2\alpha_{34}(\mu-1)-\lambda\alpha_{44})-\mu(\alpha_{33}(\mu-1)+\alpha_{44})}{2(\mu-1)}$

Case $1.3^1.7$**.**

g	W
5	$W_{3434} = \frac{2\alpha_{34}+\alpha_{4,4}-\lambda\alpha_{33}}{2(1+\lambda)}$

Case $1.3^1.8$**.**

g	W
5	$W_{3434} = -\alpha_{33}$

Case $1.3^1.9$**.**

g	W
5	$W_{3434} = -\alpha_{33}\lambda(\lambda+1)$

Case $1.3^1.12$**.**

g	W
5	$W_{3434} = \frac{\alpha_{33}(1-2\mu)(\mu+\lambda-1)}{2}$

Case $1.3^1.15$**.**

g	W
5	$W_{3434} = -\frac{\alpha_{33}+\alpha_{44}}{2}$

Case $1.3^1.16$**.**

g	W
5	$W_{3434} = \frac{\alpha_{33}-\alpha_{44}}{2}$

Case $1.3^1.19$**.**

g	W
5	$W_{3434} = -\frac{\alpha_{33}}{2}$

Case $1.3^1.20$**.**

g	W
5	$W_{3434} = -\alpha_{33}$

Case $1.3^1.21$**.**

g	W
5	$W_{3434} = \frac{\alpha_{33}\lambda(1-2\lambda)}{2}$

Case $1.3^1.24$**.**

g	W
5	$W_{3434} = \frac{(2-3\lambda)(2\lambda\alpha_{44}(\lambda-1)-\alpha_{33})}{4(\lambda-1)}$

Case $1.3^1.25$.

g	W
5	$W_{3434} = \frac{(2-3\lambda)(2\lambda\alpha_{44}(\lambda-1)+\alpha_{33})}{4(\lambda-1)}$

Case $1.3^1.28$.

g	W
5	$W_{3434} = \frac{3(\alpha_{33}-2\alpha_{44})}{4}$

Case $1.3^1.29$.

g	W
5	$W_{3434} = -\frac{3(\alpha_{33}+2\alpha_{44})}{4}$

Case $1.3^1.30$.

g	W
5	$W_{3434} = \frac{\alpha_{33}(\mu-1)+2\alpha_{34}+\alpha_{44}(\lambda-1)}{2(\mu(1-\lambda)+\lambda)}$

Case $1.4^1.1$.

g	W
6	$W_{2323} = -W_{3434} = \frac{\alpha_{33}(2\alpha_{22}-\alpha_{44})}{2\alpha_{44}}$, $W_{2334} = \frac{3\alpha_{33}}{2}$

Case $1.4^1.2$.

g	W
6	$W_{2323} = -\frac{\alpha_{22}\alpha_{33}(p-3)}{2\alpha_{44}}$, $W_{3434} = \frac{\alpha_{33}(p-3)}{2}$

Case $1.4^1.3$.

g	W
6	$W_{2323} = \frac{\alpha_{22}(\alpha_{33}-\alpha_{44})}{2\alpha_{44}}$, $W_{3434} = \frac{\alpha_{44}-\alpha_{33}}{2}$

Case $1.4^1.4$.

g	W
6	$W_{2323} = \frac{\alpha_{22}(\alpha_{33}+\alpha_{44})}{2\alpha_{44}}$, $W_{3434} = -\frac{\alpha_{44}+\alpha_{33}}{2}$

Case $1.4^1.5$.

g	W
6	$W_{2323} = -\frac{\alpha_{33}}{2}$, $W_{3434} = \frac{\alpha_{33}\alpha_{44}}{2\alpha_{22}}$

Case $1.4^1.9$.

g	W
6	$W_{2323} = W_{3434} = \frac{\alpha_{22}(p(p+1)-r)-\alpha_{44}}{2\alpha_{22}}$, $W_{2334} = -\frac{\alpha_{44}(2p+1)}{2}$

Case $1.4^1.10$.

g	W
6	$W_{2323} = \frac{\alpha_{22}(p(p+1)-r)}{2}$, $W_{3434} = \frac{\alpha_{44}(-p(p+1)+r)}{2}$

Case $1.4^1.12$.

g	W
6	$W_{2323} = -\frac{r\alpha_{22}}{2}$, $W_{3434} = \frac{r\alpha_{44}}{2}$

Case 1.4[1].14.

g	W
6	$W_{2323} = \frac{\alpha_{22}(r-1)}{2}$, $W_{3434} = \frac{\alpha_{44}(r-1)}{2}$

Case 1.4[1].15.

g	W
6	$W_{2323} = -\frac{\alpha_{22}+\alpha_{44}}{2}$, $W_{3434} = \frac{\alpha_{44}(\alpha_{22}+\alpha_{44})}{2\alpha_{22}}$

Case 1.4[1].16.

g	W
6	$W_{2323} = \frac{\alpha_{22}-\alpha_{44}}{2}$, $W_{3434} = -\frac{\alpha_{44}(\alpha_{22}-\alpha_{44})}{2\alpha_{22}}$

Case 1.4[1].18.

g	W
6	$W_{2323} = -\frac{\alpha_{22}+\alpha_{44}}{2}$, $W_{3434} = \frac{\alpha_{44}(\alpha_{22}+\alpha_{44})}{2\alpha_{22}}$

Case 1.4[1].19.

g	W
6	$W_{2323} = \frac{\alpha_{22}-\alpha_{44}}{2}$, $W_{3434} = -\frac{\alpha_{44}(\alpha_{22}-\alpha_{44})}{2\alpha_{22}}$

Case 2.2[1].4.

g	W
10	$W_{2323} = -\alpha_{23}$

Case 2.2[2].1.

g	W
11	$W_{3434} = -\alpha_{44}$

Case 2.2[2].2.

g	W
11	$W_{3434} = -\alpha_{44}$

Case 2.5[1].1.

g	W
12	$W_{2323} = W_{3434} = -3\alpha_{33}$

Case 2.5[1].2.

g	W
12	$W_{2323} = -3\alpha_{33}$

Case 2.5[2].1.

g	W
6	$W_{2323} = -\frac{\alpha_{33}}{2}$, $W_{3434} = \frac{\alpha_{33}\alpha_{44}}{2\alpha_{22}}$

Case $2.5^2.2$**.**

g	W
13	$W_{2323} = -W_{3434} = \alpha_{44}s$

Case $2.5^2.3$**.**

g	W
13	$W_{2323} = -W_{3434} = -\alpha_{44}s$

Case $2.5^2.4$**.**

g	W
13	$W_{2323} = -W_{3434} = \alpha_{44}s$

Case $2.5^2.5$**.**

g	W
13	$W_{2323} = W_{3434} = -\alpha_{44}s$

□

The four-dimensional locally homogeneous (pseudo)Riemannian manifolds, which can't have the isotropic Weyl tensor are listed in Propositions 1, 2, 3. The four-dimensional locally homogeneous (pseudo)Riemannian manifolds, for which the Weyl tensor is isotropic for any invariant metric, are listed in Proposition 4. The four-dimensional locally homogeneous (pseudo)Riemannian manifolds, for which the Weyl tensor is isotropic under certain conditions of the ''inequality" type, are contained in Proposition 5.

This manifolds together with corresponding conditions are contained in tables 1–6. The three remaining cases from the classification [10] are considered below.

Case $1.1^1.1$**.** In this case the Lie brackets are as follows:

$$[e_1, u_1] = u_1, \quad [e_1, u_3] = -u_3, \quad [u_1, u_3] = u_2, \quad [u_2, u_4] = u_2, \quad [u_3, u_4] = u_3,$$

where $\mathfrak{h} = \operatorname{span}(e_1)$, $\mathfrak{m} = \operatorname{span}(u_1, u_2, u_3, u_4)$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{12} = 0, \quad \alpha_{14} = 0, \quad \alpha_{11} = 0, \quad \alpha_{23} = 0, \quad \alpha_{33} = 0, \quad \alpha_{34} = 0.$$

Thus, the invariant metric tensor has the form 1.

The nontrivial components of the Weyl tensor are:

$$W_{1234} = -W_{2314} = \frac{(\alpha_{24}(\alpha_{13}+2\alpha_{24})(\alpha_{13}+\alpha_{24}) - \alpha_{22}\alpha_{44}(3\alpha_{13}+2\alpha_{24}))\,\alpha_{22}}{12\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)\alpha_{13}},$$

$$W_{1223} = -\frac{\left(\alpha_{13}^2 - 2\alpha_{22}\alpha_{44} + 2\alpha_{24}^2\right)\alpha_{22}^2}{12\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)\alpha_{13}},$$

$$W_{1434} = \frac{\left(\alpha_{13}^2 - 2\alpha_{22}\alpha_{44} + 2\alpha_{24}^2\right)\alpha_{44}\alpha_{22}}{2\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)\alpha_{13}},$$

$$W_{1324} = -\frac{\alpha_{22}}{2},$$

$$W_{1313} = \frac{\left(\alpha_{13}^2 - 2\alpha_{22}\alpha_{44} + 2\alpha_{24}^2\right)\alpha_{22}}{6\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)},$$

$$W_{2424} = -\frac{\left(\alpha_{13}^2 - 2\alpha_{22}\alpha_{44} + 2\alpha_{24}^2\right)\alpha_{22}}{6\alpha_{13}^2}.$$

The square of length of the Weyl tensor has the following form:

$$\|W\|^2 = \frac{\alpha_{22}^2\left(2\alpha_{13}^4 + \left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)\left(4\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right) - 13\alpha_{13}^2\right)\right)}{3\left(\alpha_{22}\alpha_{44}-\alpha_{24}^2\right)^2\alpha_{13}^4}.$$

It is equal to zero in two cases:

(1) for $\alpha_{22} = 0$. However, the components of the Weyl tensor become zero. Hence, the Weyl tensor isn't isotropic in this case;

(2) for $\alpha_{22} = \frac{\alpha_{13}^2\left(13\pm3\sqrt{17}\right)+8\alpha_{24}^2}{8\alpha_{44}}$, the Weyl tensor is isotropic.

Case 1.1^2.1**.** In this case the Lie brackets are as follows:

$$[e_1,u_1] = u_3, \qquad [e_1,u_3] = -u_1, \qquad [u_1,u_3] = -u_2,$$
$$[u_1,u_4] = u_1, \qquad [u_2,u_4] = 2u_2, \qquad [u_3,u_4] = u_3,$$

where $\mathfrak{h} = \operatorname{span}(e_1)$, $\mathfrak{m} = \operatorname{span}(u_1,u_2,u_3,u_4)$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{23} = 0, \quad \alpha_{34} = 0, \quad \alpha_{12} = 0, \quad \alpha_{13} = 0, \quad \alpha_{14} = 0, \quad \alpha_{33} - \alpha_{11} = 0.$$

Thus, the invariant metric tensor has the form 3.

The nontrivial components of the Weyl tensor are:

$$W_{1214} = W_{1414} = -W_{2334} = -W_{3434} = \frac{\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2 + 2\alpha_{33}^2\right)\alpha_{22}\alpha_{44}}{6\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right)\alpha_{33}},$$
$$W_{1234} = 2W_{1324} = W_{2314} = \frac{\alpha_{22}}{2},$$
$$W_{1212} = W_{2323} = \frac{\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2 + 2\alpha_{33}^2\right)\alpha_{22}^2}{6\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right)\alpha_{33}},$$
$$W_{1313} = W_{2424} = -\frac{\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2 + 2\alpha_{33}^2\right)\alpha_{22}}{3\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right)}.$$

The square of length of the Weyl tensor has the following form:

$$\|W\|^2 = \frac{4\alpha_{22}^2(8\alpha_{33}^4 + \left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right)(\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right) + 13\alpha_{33}^2))}{3\left(\alpha_{22}\alpha_{44} - \alpha_{24}^2\right)^2\alpha_{33}^4}.$$

It is equal to zero in two cases:

(1) for $\alpha_{22} = 0$. However, the components of the Weyl tensor become zero. Hence, the Weyl tensor isn't isotropic in this case;
(2) for $\alpha_{22} = \frac{2\alpha_{24}^2 - \alpha_{33}^2(13 \pm 3\sqrt{17})}{2\alpha_{44}}$, the Weyl tensor is isotropic.

Case $1.3^1.1$**.** In this case the Lie brackets are as follows:

$$[e_1, u_1] = e_1, \qquad [e_1, u_3] = u_1, \qquad [e_1, u_4] = u_2, \qquad [u_1, u_2] = -\frac{u_2}{2},$$
$$[u_1, u_3] = u_3, \qquad [u_1, u_4] = \frac{u_4}{2}, \qquad [u_2, u_3] = \frac{u_4}{2},$$

where $\mathfrak{h} = \operatorname{span}(e_1)$, $\mathfrak{m} = \operatorname{span}(u_1, u_2, u_3, u_4)$.

We calculate the isotropy representation (1):

$$\psi_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let the metric tensor has the form:

$$g = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{pmatrix},$$

then the invariance condition of the metric tensor (2) will be of the form:

$$\alpha_{11} = 0, \quad \alpha_{12} = 0, \quad \alpha_{22} = 0, \quad \alpha_{13} = 0, \quad \alpha_{24} = 0, \quad \alpha_{23} + \alpha_{14} = 0.$$

Thus, the invariant metric tensor has the form 5.

The nontrivial components of the Weyl tensor are:

$$W_{1323} = W_{1314} = -\frac{3\alpha_{34}}{8}, \qquad W_{2334} = W_{1434} = -\frac{\alpha_{34}\alpha_{44}}{16\alpha_{23}},$$
$$W_{1334} = -\frac{7\alpha_{33}\alpha_{44} - 6\alpha_{34}^2}{16\alpha_{23}}, \quad W_{2434} = -\frac{\alpha_{44}^2}{16\alpha_{23}},$$
$$W_{1313} = -\frac{3\alpha_{33}}{4}, \qquad W_{3434} = -\frac{5\alpha_{44}(\alpha_{33}\alpha_{44} - \alpha_{34}^2)}{8\alpha_{23}^2},$$
$$W_{1324} = W_{2323} = W_{2314} = W_{1414} = -\frac{\alpha_{44}}{8}.$$

The square of length of the Weyl tensor has the following form:

$$\|W\|^2 = \frac{3\alpha_{44}^2}{8\alpha_{23}^4}.$$

It's equal to zero, when $\alpha_{44} = 0$. If $\alpha_{44} = 0$, then components W_{1323}, W_{1314}, W_{2313}, W_{1413}, W_{1334}, W_{3413} aren't equal to zero for $\alpha_{34} \neq 0$ and $W_{1313} \neq 0$ for $\alpha_{33} \neq 0$.

Thus, the Weyl tensor is isotropic in case $1.3^1.1$, if one of the following conditions is true:

(1) $\alpha_{44} = 0, \alpha_{34} \neq 0$,
(2) $\alpha_{44} = 0, \alpha_{33} \neq 0$.

In conclusion we note, that for 186 manifolds from the classification [10] the following holds:

- 77 manifolds can have the isotropic Weyl tensor W, moreover:
  - the Weyl tensor W is isotropic for any invariant metric for 34 manifolds;
  - the Weyl tensor W is isotropic only under certain conditions on an invariant metric for the remaining 43 cases, moreover:
    - conditions on an invariant metric are conditions of an "equality" type for 3 manifolds;
    - conditions on an invariant metric are conditions of an "inequality" type for 40 manifolds;
- 109 manifolds can't have the isotropic Weyl tensor W, moreover:
  - the square of length of the Weyl tensor $||W||^2$ isn't equal to zero for any invariant metric in 15 cases;
  - all of components of the Weyl tensor are equal to zero for any invariant metric in 74 cases;
  - the triviality of the square of length of the Weyl tensor $\|W\|^2$ for some invariant metric entails the triviality of the Weyl tensor W in 20 cases.

TABLE 8. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds

№	Lie brackets
$1.1^1.1$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_1,u_3]=u_2,\ [u_2,u_4]=u_2,\ [u_3,u_4]=u_3$
$1.1^1.2$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_2,u_4]=pu_2,\ [u_3,u_4]=u_3$
$1.1^1.3$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_1,u_3]=e_1+u_2$
$1.1^1.4$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_1,u_3]=u_2$
$1.1^1.5$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_1,u_3]=e_1,\ [u_2,u_4]=u_2$
$1.1^1.6$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_2,u_4]=u_2$
$1.1^1.7$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [u_1,u_3]=e_1$
$1.1^1.8$	$[e_1,u_1]=u_1,\ [e_1,u_2]=\frac{1}{2}u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\frac{1}{2}u_4,\ [u_1,u_3]=-2e_1,$ $[u_1,u_4]=u_2,\ [u_2,u_3]=u_4$
$1.1^1.9$	$[e_1,u_1]=u_1,\ [e_1,u_2]=\frac{1}{2}u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\frac{1}{2}u_4,\ [u_1,u_4]=u_2$
$1.1^1.10$	$[e_1,u_1]=u_1,\ [e_1,u_2]=\lambda u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\lambda u_4,\ \lambda\in[0,1]$
$1.1^2.1$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=-u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=2u_2,\ [u_3,u_4]=u_3$
$1.1^2.2$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_4]=u_1,\ [u_2,u_4]=pu_2,\ [u_3,u_4]=u_3$
$1.1^2.3$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=e_1+u_2$
$1.1^2.4$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=-e_1+u_2$
$1.1^2.5$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=u_2$
$1.1^2.6$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=e_1,\ [u_2,u_4]=u_2$
$1.1^2.7$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=-e_1,\ [u_2,u_4]=u_2$
$1.1^2.8$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_2,u_4]=u_2$
$1.1^2.9$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=e_1$
$1.1^2.10$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [u_1,u_3]=-e_1$
$1.1^2.11$	$[e_1,u_1]=u_3,\ [e_1,u_2]=\frac{1}{2}u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=-\frac{1}{2}u_2,\ [u_1,u_2]=u_2,$ $[u_1,u_3]=-4e_1,\ [u_1,u_4]=-u_4,\ [u_2,u_3]=-u_4,\ [u_3,u_4]=u_2$
$1.1^2.12$	$[e_1,u_1]=u_3,\ [e_1,u_2]=\lambda u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=-\lambda u_2,\ \lambda\in[0,1]$
$1.1^3.1$	$[e_1,u_1]=u_1,\ [e_1,u_2]=\lambda u_4,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\lambda u_2,\ \lambda\in(0,1]$
$1.1^4.1$	$[e_1,u_1]=u_3,\ [e_1,u_2]=-\lambda u_2,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=\lambda u_4,\ \lambda\in(0,1)$
$1.1^5.1$	$[e_1,u_1]=\cos(\varphi/2)u_1-\sin(\varphi/2)u_2,\ [e_1,u_2]=\cos(\varphi/2)u_2+\sin(\varphi/2)u_1,$ $[e_1,u_3]=-\cos(\varphi/2)u_3+\sin(\varphi/2)u_4,\ [e_1,u_4]=-\cos(\varphi/2)u_4-\sin(\varphi/2)u_3,\ \varphi\in(0,\pi/2]$
$1.1^6.1$	$[e_1,u_1]=-\cos(\varphi/2)u_2-\sin(\varphi/2)u_1,\ [e_1,u_2]=\cos(\varphi/2)u_1-\sin(\varphi/2)u_2,$ $[e_1,u_3]=-\cos(\varphi/2)u_4+\sin(\varphi/2)u_3,\ [e_1,u_4]=\cos(\varphi/2)u_3+\sin(\varphi/2)u_4,\ \varphi\in(0,\pi/2)$
$1.2^1.1$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_1+u_2,\ [e_1,u_3]=-u_3-u_4,\ [e_1,u_4]=-u_4$
$1.2^2.1$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_2+u_4,\ [e_1,u_4]=-u_1-u_3$
$1.3^1.1$	$[e_1,u_1]=e_1,\ [e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_2]=-\frac{1}{2}u_2,\ [u_1,u_3]=u_3,\ [u_1,u_4]=\frac{1}{2}u_4,$ $[u_2,u_3]=\frac{1}{2}u_4$
$1.3^1.2$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\lambda e_1+(\lambda+1)u_1+\lambda u_2,\ [u_2,u_4]=u_2,\ \lambda\in[-1,1]$
$1.3^1.3$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=e_1$
$1.3^1.4$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-(1+\lambda^2)e_1+2\lambda u_1+(1+\lambda^2)u_2,\ [u_2,u_4]=u_2,$ $\lambda\geqslant 0$
$1.3^1.5$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\frac{\lambda^2+\mu}{\mu-1}e_1+\frac{1+\lambda^2}{\mu-1}u_2,\ [u_1,u_4]=-\lambda e_1+u_1+\lambda u_2,$ $[u_2,u_3]=-\lambda e_1+u_1+\lambda u_2,\ [u_2,u_4]=-\mu e_1+(\mu+1)u_2,\ \lambda\geqslant 0,\ \mu\neq 1$
$1.3^1.6$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=u_1,\ [u_2,u_4]=u_2,$ $[u_3,u_4]=e_1$
$1.3^1.7$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=\frac{1}{1+\lambda}e_1+\frac{\lambda}{1+\lambda}u_1-\frac{1}{1+\lambda}u_2,$ $[u_1,u_4]=-\frac{1}{1+\lambda}e_1+\frac{1}{1+\lambda}u_1+\frac{1}{1+\lambda}u_2,\ [u_2,u_3]=-\frac{1}{1+\lambda}e_1+\frac{1}{1+\lambda}u_1+\frac{1}{1+\lambda}u_2,$ $[u_2,u_4]=-\frac{\lambda}{1+\lambda}e_1+\frac{\lambda}{1+\lambda}u_1+\frac{1+2\lambda}{1+\lambda}u_2,\ \lambda\neq -1$
$1.3^1.8$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_3]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=-u_3$
$1.3^1.9$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_3]=\lambda u_1,\ [u_2,u_4]=-\lambda e_1+(\lambda+1)u_2,\ [u_3,u_4]=-\lambda u_3$
$1.3^1.10$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_4]=u_2,\ [u_3,u_4]=e_1$
$1.3^1.11$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_3]=-u_1,\ [u_2,u_4]=e_1,\ [u_3,u_4]=e_1+u_3$

TABLE 9. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
$1.3^1.12$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\mu u_1,\ [u_2,u_4]=-\lambda\mu e_1+(\lambda+\mu)u_2,$ $[u_3,u_4]=(1-\mu)u_3$
$1.3^1.13$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\frac{1}{2}u_1,\ [u_2,u_4]=-\frac{\lambda}{2}e_1+\left(\lambda+\frac{1}{2}\right)u_2,$ $[u_3,u_4]=e_1+\frac{1}{2}u_3$
$1.3^1.14$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=(1-\lambda)u_1,\ [u_2,u_4]=\lambda(\lambda-1)e_1+u_2,$ $[u_3,u_4]=e_1+\lambda u_3,\ \lambda\neq\frac{1}{2}$
$1.3^1.15$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-e_1+2u_1,\ [u_1,u_4]=u_2,\ [u_2,u_3]=u_2,$ $[u_2,u_4]=-e_1+u_1$
$1.3^1.16$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-e_1+2u_1,\ [u_1,u_4]=u_2,\ [u_2,u_3]=u_2,$ $[u_2,u_4]=e_1-u_1$
$1.3^1.17$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_4]=u_1,\ [u_3,u_4]=e_1$
$1.3^1.18$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_4]=u_1$
$1.3^1.19$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=u_1,\ [u_2,u_4]=-e_1+u_1+2u_2$
$1.3^1.20$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_2,u_3]=u_1,\ [u_2,u_4]=u_2-u_1,\ [u_3,u_4]=-u_3$
$1.3^1.21$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\lambda u_1,$ $[u_2,u_4]=-\lambda e_1+(1-\lambda)u_1+(1+\lambda)u_2,\ [u_3,u_4]=(1-\lambda)u_3,\ \lambda\neq 1$
$1.3^1.22$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_3]=\frac{1}{2}u_1,\ [u_2,u_4]=-\frac{1}{2}e_1+\frac{1}{2}u_1+\frac{3}{2}u_2,$ $[u_3,u_4]=e_1+\frac{1}{2}u_3$
$1.3^1.23$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_1+u_2,\ [u_3,u_4]=e_1+u_3$
$1.3^1.24$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=(1-2\lambda)e_1+2\lambda u_1,\ [u_1,u_4]=(2\lambda-1)u_2,$ $[u_2,u_3]=\lambda u_2,\ [u_2,u_4]=\frac{2\lambda-1}{2\lambda-2}e_1-\frac{1}{2\lambda-2}u_1,\ [u_3,u_4]=(\lambda-1)u_4,\ \lambda\neq 1$
$1.3^1.25$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=(1-2\lambda)e_1+2\lambda u_1,\ [u_1,u_4]=(2\lambda-1)u_2,$ $[u_2,u_3]=\lambda u_2,\ [u_2,u_4]=\frac{1-2\lambda}{2\lambda-2}e_1+\frac{1}{2\lambda-2}u_1,\ [u_3,u_4]=(\lambda-1)u_4,\ \lambda\neq 1$
$1.3^1.26$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\frac{1}{3}e_1+\frac{4}{3}u_1,\ [u_1,u_4]=\frac{1}{3}u_2,\ [u_2,u_3]=\frac{2}{3}u_2,$ $[u_2,u_4]=-\frac{1}{2}e_1+\frac{3}{2}u_1,\ [u_3,u_4]=e_1-\frac{1}{3}u_4$
$1.3^1.27$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=-\frac{1}{3}e_1+\frac{4}{3}u_1,\ [u_1,u_4]=\frac{1}{3}u_2,\ [u_2,u_3]=\frac{2}{3}u_2,$ $[u_2,u_4]=\frac{1}{2}e_1-\frac{3}{2}2u_1,\ [u_3,u_4]=e_1-\frac{1}{3}u_4$
$1.3^1.28$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=2u_1,\ [u_1,u_4]=2u_2,\ [u_2,u_3]=u_2,$ $[u_2,u_4]=e_1-\frac{1}{2}u_1,\ [u_3,u_4]=u_4$
$1.3^1.29$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=2u_1,\ [u_1,u_4]=2u_2,\ [u_2,u_3]=u_2,$ $[u_2,u_4]=-e_1+\frac{1}{2}u_1,\ [u_3,u_4]=u_4$
$1.3^1.30$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_1,u_3]=\frac{\lambda\mu(\lambda-1)}{\lambda+\mu-\lambda\mu}e_1+\frac{\lambda^2+\mu-\lambda^2\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda(1-\lambda)}{\lambda+\mu-\lambda\mu}u_2,$ $[u_1,u_4]=-\frac{\lambda\mu}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda}{\lambda+\mu-\lambda\mu}u_2,$ $[u_2,u_3]=-\frac{\lambda\mu}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda}{\lambda+\mu-\lambda\mu}u_2,$ $[u_2,u_4]=\frac{\lambda\mu(\mu-1)}{\lambda+\mu-\lambda\mu}e_1+\frac{\mu(1-\mu)}{\lambda+\mu-\lambda\mu}u_1+\frac{\lambda+\mu^2-\mu^2\lambda}{\lambda+\mu-\lambda\mu}u_2,\ \lambda+\mu-\lambda\mu\neq 0,\ 1\leqslant\mu\leqslant\lambda,\ \lambda\mu>0$
$1.3^1.31$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2,\ [u_3,u_4]=e_1$
$1.3^1.32$	$[e_1,u_3]=u_1,\ [e_1,u_4]=u_2$
$1.4^1.1$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_2]=u_1,\ [u_1,u_3]=u_2,\ [u_1,u_4]=u_1,$ $[u_2,u_3]=u_3,\ [u_3,u_4]=-u_3$
$1.4^1.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=pu_1,\ [u_2,u_4]=(p-1)u_2,$ $[u_3,u_4]=(p-2)u_3$
$1.4^1.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=2u_1,\ [u_2,u_3]=e_1,\ [u_2,u_4]=u_2$
$1.4^1.4$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [e_1,u_4]=e_1,\ [u_1,u_4]=2u_1,\ [u_2,u_3]=-e_1,\ [u_2,u_4]=u_2$
$1.4^1.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_2]=u_1,\ [u_1,u_3]=u_2,\ [u_2,u_3]=u_3$
$1.4^1.6$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=u_1+u_3$
$1.4^1.7$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=-u_1+u_3$
$1.4^1.8$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=u_3$
$1.4^1.9$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2+u_4,\ [u_3,u_4]=pu_4$
$1.4^1.10$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2,\ [u_3,u_4]=pu_4$

TABLE 10. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
$1.4^1.11$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2+u_4,\ [u_3,u_4]=u_1-u_4$
$1.4^1.12$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_1,u_3]=u_1,\ [u_2,u_3]=re_1+u_2,\ [u_3,u_4]=u_1-u_4$
$1.4^1.13$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=re_1+u_4,\ [u_3,u_4]=u_4$
$1.4^1.14$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=re_1,\ [u_3,u_4]=u_4$
$1.4^1.15$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1+u_4,\ [u_3,u_4]=u_1$
$1.4^1.16$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1+u_4,\ [u_3,u_4]=u_1$
$1.4^1.17$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=u_4,\ [u_3,u_4]=u_1$
$1.4^1.18$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1+u_4$
$1.4^1.19$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1+u_4$
$1.4^1.20$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=u_4$
$1.4^1.21$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1,\ [u_3,u_4]=u_1$
$1.4^1.22$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1,\ [u_3,u_4]=u_1$
$1.4^1.23$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_3,u_4]=u_1$
$1.4^1.24$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=e_1$
$1.4^1.25$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2,\ [u_2,u_3]=-e_1$
$1.4^1.26$	$[e_1,u_2]=u_1,\ [e_1,u_3]=u_2$
$2.1^1.1$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_2,\ [e_2,u_4]=-u_4,\ [u_1,u_3]=e_1,\ [u_2,u_4]=e_2$
$2.1^1.2$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_2,\ [e_2,u_4]=-u_4,\ [u_1,u_3]=e_1$
$2.1^1.3$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_2,\ [e_2,u_4]=-u_4$
$2.1^2.1$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1,\ [u_2,u_4]=e_2$
$2.1^2.2$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1,\ [u_2,u_4]=-e_2$
$2.1^2.3$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1$
$2.1^2.4$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_2,u_4]=e_2$
$2.1^2.5$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_2,u_4]=-e_2$
$2.1^2.6$	$[e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2$
$2.1^3.1$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1,\ [u_2,u_4]=e_2$
$2.1^3.2$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1,\ [u_2,u_4]=-e_2$
$2.1^3.3$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=-e_1,\ [u_2,u_4]=-e_2$
$2.1^3.4$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=e_1$
$2.1^3.5$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [u_1,u_3]=-e_1$
$2.1^3.6$	$[e_1,u_1]=u_3,\ [e_1,u_3]=-u_1,\ [e_2,u_2]=u_4,\ [e_2,u_4]=-u_2$
$2.1^4.1$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_1]=u_2,\ [e_2,u_2]=-u_1,$ $[e_2,u_3]=-u_4,\ [e_2,u_4]=u_3,\ [u_1,u_3]=e_1,\ [u_1,u_4]=e_2,\ [u_2,u_3]=e_2,\ [u_2,u_4]=-e_1$
$2.1^4.2$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_1]=u_2,\ [e_2,u_2]=-u_1,$ $[e_2,u_3]=-u_4,\ [e_2,u_4]=u_3$
$2.2^1.1$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_2,u_4]=-2e_2,$ $[u_1,u_3]=u_2,\ [u_1,u_4]=-u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=2u_3$
$2.2^1.2$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [u_1,u_2]=e_2,$ $[u_1,u_3]=u_4,\ [u_2,u_3]=(p-1)u_3,\ [u_2,u_4]=pu_4$
$2.2^1.3$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [u_2,u_3]=u_3,$ $[u_2,u_4]=u_4$
$2.2^1.4$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,$ $[u_1,u_3]=e_2,\ [u_2,u_3]=e_1,\ [u_2,u_4]=e_2$
$2.2^1.5$	$[e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,$ $[u_2,u_3]=e_2$
$2.2^1.6$	$[e_1,e_2]=\frac{3}{2}e_2,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-\frac{1}{2}u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=\frac{1}{2}u_4,$ $[e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [u_1,u_2]=u_4$
$2.2^1.7$	$[e_1,e_2]=(1-\lambda)e_2,\ [e_1,u_1]=u_1,\ [e_1,u_2]=\lambda u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\lambda u_4,$ $[e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ \lambda\in[-1,1]$
$2.2^2.1$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,\ [e_2,u_3]=u_1,\ [e_2,u_4]=u_2,$ $[u_1,u_3]=e_2,\ [u_2,u_4]=e_2,\ [u_3,u_4]=-e_1$

Table 11. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
$2.2^2.2$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,\ [e_2,u_3]=u_1,\ [e_2,u_4]=u_2,$ $[u_1,u_3]=-e_2,\ [u_2,u_4]=-e_2,\ [u_3,u_4]=e_1$
$2.2^2.3$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,\ [e_2,u_3]=u_1,\ [e_2,u_4]=u_2,$ $[u_3,u_4]=e_2$
$2.2^2.4$	$[e_1,u_1]=u_2,\ [e_1,u_2]=-u_1,\ [e_1,u_3]=u_4,\ [e_1,u_4]=-u_3,\ [e_2,u_3]=u_1,\ [e_2,u_4]=u_2$
$2.2^3.1$	$[e_1,e_2]=-2\sin(\varphi/2)e_2,\ [e_1,u_1]=-\sin(\varphi/2)u_1-\cos(\varphi/2)u_2,$ $[e_1,u_2]=-\sin(\varphi/2)u_2+\cos(\varphi/2)u_1,\ [e_1,u_3]=\sin(\varphi/2)u_3-\cos(\varphi/2)u_4,$ $[e_1,u_4]=\sin(\varphi/2)u_4+\cos(\varphi/2)u_3,\ [e_2,u_3]=u_1,\ [e_2,u_4]=u_2,\ \varphi\in(0,\pi)$
$2.3^1.1$	$[e_1,e_2]=2e_2,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_2-u_3,\ [e_1,u_4]=u_1+u_4,$ $[e_2,u_2]=u_1,\ [e_2,u_3]=-u_4$
$2.4^1.1$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=u_2,\ [u_1,u_2]=u_1,$ $[u_1,u_3]=u_2,\ [u_2,u_3]=u_3$
$2.4^1.2$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=u_2,\ [u_1,u_4]=u_1,$ $[u_2,u_4]=u_2,\ [u_3,u_4]=u_3$
$2.4^1.3$	$[e_1,e_2]=e_2,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=u_2$
$2.5^1.1$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_1,u_4]=-2e_1,\ [e_2,u_2]=-2e_2,\ [e_2,u_3]=-u_2,$ $[e_2,u_4]=u_1,\ [u_1,u_2]=2e_2-u_1,\ [u_1,u_3]=u_2+u_4,\ [u_1,u_4]=2e_1-u_1,$ $[u_2,u_3]=-2u_3,\ [u_2,u_4]=u_2-u_4,\ [u_3,u_4]=2u_3$
$2.5^1.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_2]=-2e_2,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,$ $[u_1,u_2]=-u_1,\ [u_1,u_3]=u_4,\ [u_2,u_3]=-2u_3,\ [u_2,u_4]=-u_4$
$2.5^1.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_1,u_3]=u_1,$ $[u_2,u_3]=e_1+pe_2+(1-q)u_2,\ [u_2,u_4]=qu_1,\ [u_3,u_4]=-(p+q)e_1+\lambda e_2-(1+q)u_4,$ $q\geqslant 0\ (if\ \lambda\neq 0),\ q\in\mathbb{R}\ (if\ \lambda=0)$
$2.5^1.4$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_1,u_3]=u_1,$ $[u_2,u_3]=qe_2+(1-p)u_2,\ [u_2,u_4]=pu_1,\ [u_3,u_4]=-(p+q)e_1-(1+p)u_4,\ p\geqslant 0$
$2.5^1.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1+qe_2-u_2,$ $[u_2,u_4]=u_1,\ [u_3,u_4]=-qe_1-\lambda e_2-u_4$
$2.5^1.6$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=qe_2-u_2,$ $[u_2,u_4]=u_1,\ [u_3,u_4]=-qe_1-u_4$
$2.5^1.7$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1+e_2,$ $[u_3,u_4]=-e_1+\lambda e_2$
$2.5^1.8$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1-e_2,$ $[u_3,u_4]=e_1+\lambda e_2$
$2.5^1.9$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_2,\ [u_3,u_4]=-e_1$
$2.5^1.10$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=-e_2,\ [u_3,u_4]=e_1$
$2.5^1.11$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1,\ [u_3,u_4]=e_2$
$2.5^1.12$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1,\ [u_3,u_4]=-e_2$
$2.5^1.13$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [u_2,u_3]=e_1$
$2.5^1.14$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_4,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1$
$2.5^2.1$	$[e_1,u_2]=-e_1+u_1,\ [e_1,u_3]=-u_2,\ [e_1,u_4]=e_2,\ [e_2,u_2]=-e_2,\ [e_2,u_3]=u_4,$ $[e_2,u_4]=-e_1-u_1,\ [u_1,u_2]=e_1-u_1,\ [u_1,u_3]=u_2,\ [u_1,u_4]=-e_2,\ [u_2,u_3]=-2u_3,$ $[u_2,u_4]=-u_4$
$2.5^2.2$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,\ [u_1,u_3]=u_1,$ $[u_2,u_3]=(p+s)e_1+re_2+u_2-2ru_4,\ [u_2,u_4]=2ru_1,$ $[u_3,u_4]=-re_1+(p-s)e_2-2ru_2-u_4,\ r\geqslant 0,\ s\geqslant 0$
$2.5^2.3$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,\ [u_2,u_3]=-(r+s)e_1-u_4,$ $[u_2,u_4]=u_1,\ [u_3,u_4]=(s-r)e_2-u_2,\ s\geqslant 0$
$2.5^2.4$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,\ [u_2,u_3]=(1+s)e_1,$ $[u_3,u_4]=(1-s)e_2,\ s\geqslant 0$
$2.5^2.5$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,\ [u_2,u_3]=-(1+s)e_1,$ $[u_3,u_4]=(s-1)e_2,\ s\geqslant 0$
$2.5^2.6$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1,\ [u_2,u_3]=e_2,\ [u_3,u_4]=e_1$
$2.5^2.7$	$[e_1,u_2]=u_1,\ [e_1,u_3]=-u_2,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_1$

TABLE 12. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
$3.1^1.1$	$[e_1,e_3]=e_3,\ [e_2,e_3]=-e_3,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_2,\ [e_2,u_4]=-u_4,\ [e_3,u_2]=u_1,\ [e_3,u_3]=-u_4$
$3.1^2.1$	$[e_1,e_3]=2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_1]=u_2,\ [e_2,u_2]=-u_1,\ [e_2,u_3]=u_4,\ [e_2,u_4]=-u_3,\ [e_3,u_3]=u_1,\ [e_3,u_4]=u_2$
$3.2^1.1$	$[e_1,e_2]=e_2,\ [e_1,e_3]=e_3,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_2,u_4]=-2e_2,\ [e_3,u_2]=-2e_3,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_1,u_2]=2e_3-u_1,\ [u_1,u_3]=u_2+u_4,\ [u_1,u_4]=2e_2-u_1,\ [u_2,u_3]=-2u_3,\ [u_2,u_4]=u_2-u_4,\ [u_3,u_4]=2u_3$
$3.2^1.2$	$[e_1,e_2]=e_2,\ [e_1,e_3]=e_3,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_2,u_4]=-2e_2,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_1,u_3]=u_2,\ [u_1,u_4]=-u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=2u_3$
$3.2^1.3$	$[e_1,e_3]=2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_2,u_3]=e_2$
$3.2^1.4$	$[e_1,e_2]=(1-\lambda)e_2,\ [e_1,e_3]=(1+\lambda)e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=\lambda u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\lambda u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ \lambda\geqslant 0$
$3.2^2.1$	$[e_1,e_2]=e_2,\ [e_1,e_3]=e_3,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,u_2]=-e_2+u_1,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=e_3,\ [e_3,u_2]=-e_3,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-e_2-u_1,\ [u_1,u_2]=e_2-u_1,\ [u_1,u_3]=u_2,\ [u_1,u_4]=-e_3,\ [u_2,u_3]=-2u_3,\ [u_2,u_4]=-u_4$
$3.2^2.2$	$[e_1,e_2]=e_2-\lambda e_3,\ [e_1,e_3]=e_3+\lambda e_2,\ [e_1,u_1]=u_1,\ [e_1,u_2]=\lambda u_4,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=-\lambda u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_2,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-u_1,\ \lambda\geqslant 0$
$3.3^1.1$	$[e_1,e_2]=-e_2,\ [e_1,e_3]=e_3,\ [e_1,u_2]=u_2,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_1,u_3]=u_1,\ [u_2,u_3]=pe_3+u_2,\ [u_3,u_4]=-pe_2-u_4$
$3.3^1.2$	$[e_1,e_2]=-e_2,\ [e_1,e_3]=e_3,\ [e_1,u_2]=u_2,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_2,u_3]=e_3,\ [u_3,u_4]=-e_2$
$3.3^1.3$	$[e_1,e_2]=-e_2,\ [e_1,e_3]=e_3,\ [e_1,u_2]=u_2,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1,\ [u_2,u_3]=-e_3,\ [u_3,u_4]=e_2$
$3.3^1.4$	$[e_1,e_2]=-e_2,\ [e_1,e_3]=e_3,\ [e_1,u_2]=u_2,\ [e_1,u_4]=-u_4,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=u_1$
$3.3^2.1$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_2]=u_4,\ [e_1,u_4]=-u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_2,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-u_1,\ [u_1,u_3]=u_1,\ [u_2,u_3]=pe_2+u_2,\ [u_3,u_4]=pe_3-u_4$
$3.3^2.2$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_2]=u_4,\ [e_1,u_4]=-u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_2,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-u_1,\ [u_2,u_3]=e_2,\ [u_3,u_4]=e_3$
$3.3^2.3$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_2]=u_4,\ [e_1,u_4]=-u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_2,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-u_1,\ [u_2,u_3]=-e_2,\ [u_3,u_4]=-e_3$
$3.3^2.4$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_2]=u_4,\ [e_1,u_4]=-u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_2,\ [e_3,u_3]=u_4,\ [e_3,u_4]=-u_1$
$3.4^1.1$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_1]=u_2,\ [e_3,u_4]=-u_3$
$3.4^2.1$	$[e_1,e_2]=2e_3,\ [e_1,e_3]=-2e_2,\ [e_1,u_1]=u_3,\ [e_1,u_2]=-u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=u_2,\ [e_2,e_3]=2e_1,\ [e_2,u_1]=-u_2,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_2,u_4]=u_3,\ [e_3,u_1]=u_4,\ [e_3,u_2]=u_3,\ [e_3,u_3]=-u_2,\ [e_3,u_4]=-u_1$
$3.5^1.1$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=2u_1,\ [e_1,u_3]=-2u_3,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-2u_2,\ [e_3,u_1]=2u_2,\ [e_3,u_2]=-u_3,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=u_3$
$3.5^1.2$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=2u_1,\ [e_1,u_3]=-2u_3,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-2u_2,\ [e_3,u_1]=2u_2,\ [e_3,u_2]=-u_3,\ [u_1,u_2]=e_2,\ [u_1,u_3]=e_1,\ [u_2,u_3]=e_3$
$3.5^1.3$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=2u_1,\ [e_1,u_3]=-2u_3,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-2u_2,\ [e_3,u_1]=2u_2,\ [e_3,u_2]=-u_3,\ [u_1,u_2]=-e_2,\ [u_1,u_3]=-e_1,\ [u_2,u_3]=-e_3$
$3.5^1.4$	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=2u_1,\ [e_1,u_3]=-2u_3,\ [e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-2u_2,\ [e_3,u_1]=2u_2,\ [e_3,u_2]=-u_3$
$3.5^2.1$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_1]=-u_2,\ [e_1,u_2]=u_1,\ [e_2,e_3]=-e_1,\ [e_2,u_1]=-u_3,\ [e_2,u_3]=u_1,\ [e_3,u_2]=-u_3,\ [e_3,u_3]=u_2,\ [u_1,u_4]=u_1,\ [u_2,u_4]=u_2,\ [u_3,u_4]=u_3$
$3.5^2.2$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_1]=-u_2,\ [e_1,u_2]=u_1,\ [e_2,e_3]=-e_1,\ [e_2,u_1]=-u_3,\ [e_2,u_3]=u_1,\ [e_3,u_2]=-u_3,\ [e_3,u_3]=u_2,\ [u_1,u_2]=e_1,\ [u_1,u_3]=e_2,\ [u_2,u_3]=e_3$
$3.5^2.3$	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_1]=-u_2,\ [e_1,u_2]=u_1,\ [e_2,e_3]=-e_1,\ [e_2,u_1]=-u_3,\ [e_2,u_3]=u_1,\ [e_3,u_2]=-u_3,\ [e_3,u_3]=u_2,\ [u_1,u_2]=-e_1,\ [u_1,u_3]=-e_2,\ [u_2,u_3]=-e_3$

Table 13. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
3.5^2.4	$[e_1,e_2]=-e_3,\ [e_1,e_3]=e_2,\ [e_1,u_1]=-u_2,\ [e_1,u_2]=u_1,\ [e_2,e_3]=-e_1,\ [e_2,u_1]=-u_3,$ $[e_2,u_3]=u_1,\ [e_3,u_2]=-u_3,\ [e_3,u_3]=u_2$
4.1^1.1	$[e_1,e_3]=e_3,\ [e_1,e_4]=e_4,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,e_3]=-e_3,\ [e_2,e_4]=e_4,$ $[e_2,u_2]=u_2,\ [e_2,u_4]=-u_4,\ [e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_4,u_3]=-u_2,\ [e_4,u_4]=u_1$
4.1^2.1	$[e_1,e_3]=e_3,\ [e_1,e_4]=e_4,\ [e_1,u_1]=u_1,\ [e_1,u_3]=-u_3,\ [e_2,e_3]=-e_4,\ [e_2,e_4]=e_3,$ $[e_2,u_2]=u_4,\ [e_2,u_4]=-u_2,\ [e_3,u_2]=u_1,\ [e_3,u_3]=-u_2,\ [e_4,u_3]=u_4,\ [e_4,u_4]=-u_1$
4.2^1.1	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,u_1]=u_1,\ [e_2,u_2]=u_2,\ [e_2,u_3]=-u_3,\ [e_2,u_4]=-u_4,\ [e_3,e_4]=e_1,\ [e_3,u_2]=u_1,$ $[e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3,\ [u_1,u_3]=e_1+3e_2,\ [u_1,u_4]=2e_3,$ $[u_2,u_3]=2e_4,\ [u_2,u_4]=-e_1+3e_2$
4.2^1.2	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,u_1]=u_1,\ [e_2,u_2]=u_2,\ [e_2,u_3]=-u_3,\ [e_2,u_4]=-u_4,\ [e_3,e_4]=e_1,\ [e_3,u_2]=u_1,$ $[e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3$
4.2^2.1	$[e_1,e_3]=2e_4,\ [e_1,e_4]=-2e_3,\ [e_1,u_1]=u_3,\ [e_1,u_2]=-u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=u_2,$ $[e_2,u_1]=u_3,\ [e_2,u_2]=u_4,\ [e_2,u_3]=-u_1,\ [e_2,u_4]=-u_2,\ [e_3,e_4]=2e_1,\ [e_3,u_1]=-u_2,$ $[e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_3,u_4]=u_3,\ [e_4,u_1]=u_4,\ [e_4,u_2]=u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=-u_1,\ [u_1,u_2]=-e_3,\ [u_1,u_3]=e_1+3e_2,\ [u_1,u_4]=e_4,\ [u_2,u_3]=e_4,$ $[u_2,u_4]=-e_1+3e_2,\ [u_3,u_4]=-e_3$
4.2^2.2	$[e_1,e_3]=2e_4,\ [e_1,e_4]=-2e_3,\ [e_1,u_1]=u_3,\ [e_1,u_2]=-u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=u_2,$ $[e_2,u_1]=u_3,\ [e_2,u_2]=u_4,\ [e_2,u_3]=-u_1,\ [e_2,u_4]=-u_2,\ [e_3,e_4]=2e_1,\ [e_3,u_1]=-u_2,$ $[e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_3,u_4]=u_3,\ [e_4,u_1]=u_4,\ [e_4,u_2]=u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=-u_1,\ [u_1,u_2]=e_3,\ [u_1,u_3]=-e_1-3e_2,\ [u_1,u_4]=-e_4,\ [u_2,u_3]=-e_4,$ $[u_2,u_4]=e_1-3e_2,\ [u_3,u_4]=e_3$
4.2^2.3	$[e_1,e_3]=2e_4,\ [e_1,e_4]=-2e_3,\ [e_1,u_1]=u_3,\ [e_1,u_2]=-u_4,\ [e_1,u_3]=-u_1,\ [e_1,u_4]=u_2,$ $[e_2,u_1]=u_3,\ [e_2,u_2]=u_4,\ [e_2,u_3]=-u_1,\ [e_2,u_4]=-u_2,\ [e_3,e_4]=2e_1,\ [e_3,u_1]=-u_2,$ $[e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_3,u_4]=u_3,\ [e_4,u_1]=u_4,\ [e_4,u_2]=u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=-u_1$
4.2^3.1	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,u_1]=-u_4,\ [e_2,u_2]=u_3,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [e_3,e_4]=e_1,\ [e_3,u_2]=u_1,$ $[e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3,\ [u_1,u_2]=3e_2,\ [u_1,u_3]=e_1,\ [u_1,u_4]=2e_3,$ $[u_2,u_3]=2e_4,\ [u_2,u_4]=-e_1,\ [u_3,u_4]=3e_2$
4.2^3.2	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,u_1]=-u_4,\ [e_2,u_2]=u_3,\ [e_2,u_3]=-u_2,\ [e_2,u_4]=u_1,\ [e_3,e_4]=e_1,\ [e_3,u_2]=u_1,$ $[e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3$
4.3^1.1	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_1]=u_2,\ [e_3,u_4]=-u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=u_1,\ [u_3,u_4]=e_4$
4.3^1.2	$[e_1,e_2]=2e_2,\ [e_1,e_3]=-2e_3,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,e_3]=e_1,\ [e_2,u_2]=u_1,\ [e_2,u_3]=-u_4,\ [e_3,u_1]=u_2,\ [e_3,u_4]=-u_3,\ [e_4,u_3]=-u_2,$ $[e_4,u_4]=u_1$
5.1^1.1	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,e_5]=2e_5,\ [e_2,u_1]=u_1,\ [e_2,u_2]=u_2,\ [e_2,u_3]=-u_3,\ [e_2,u_4]=-u_4,\ [e_3,e_4]=e_1,$ $[e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3,\ [e_5,u_3]=-u_2,\ [e_5,u_4]=u_1$
6.1^1.1	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,e_5]=2e_5,\ [e_2,e_6]=-2e_6,\ [e_2,u_1]=u_1,\ [e_2,u_2]=u_2,\ [e_2,u_3]=-u_3,\ [e_2,u_4]=-u_4,$ $[e_3,e_4]=e_1,\ [e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3,\ [e_5,e_6]=-e_2,$ $[e_5,u_3]=-u_2,\ [e_5,u_4]=u_1,\ [e_6,u_1]=-u_4,\ [e_6,u_2]=u_3,\ [u_1,u_2]=2e_5,$ $[u_1,u_3]=e_1+e_2,\ [u_1,u_4]=2e_3,\ [u_2,u_3]=2e_4,\ [u_2,u_4]=-e_1+e_2,\ [u_3,u_4]=2e_6$
6.1^1.2	$[e_1,e_3]=2e_3,\ [e_1,e_4]=-2e_4,\ [e_1,u_1]=u_1,\ [e_1,u_2]=-u_2,\ [e_1,u_3]=-u_3,\ [e_1,u_4]=u_4,$ $[e_2,e_5]=2e_5,\ [e_2,e_6]=-2e_6,\ [e_2,u_1]=u_1,\ [e_2,u_2]=u_2,\ [e_2,u_3]=-u_3,\ [e_2,u_4]=-u_4,$ $[e_3,e_4]=e_1,\ [e_3,u_2]=u_1,\ [e_3,u_3]=-u_4,\ [e_4,u_1]=u_2,\ [e_4,u_4]=-u_3,\ [e_5,e_6]=-e_2,$ $[e_5,u_3]=-u_2,\ [e_5,u_4]=u_1,\ [e_6,u_1]=-u_4,\ [e_6,u_2]=u_3$

TABLE 14. The classification of four-dimensional locally homogeneous (pseudo)Riemannian manifolds. Continuation

№	Lie brackets
$6.1^2.1$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=-e_1$, $[e_3,e_6]=-e_2$, $[e_3,u_1]=-u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=-e_4$, $[e_5,u_2]=-u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=-u_4$, $[e_6,u_4]=u_3$, $[u_1,u_2]=e_1$, $[u_1,u_3]=e_2$, $[u_1,u_4]=e_3$, $[u_2,u_3]=e_4$, $[u_2,u_4]=e_5$, $[u_3,u_4]=e_6$
$6.1^2.2$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=-e_1$, $[e_3,e_6]=-e_2$, $[e_3,u_1]=-u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=-e_4$, $[e_5,u_2]=-u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=-u_4$, $[e_6,u_4]=u_3$, $[u_1,u_2]=-e_1$, $[u_1,u_3]=-e_2$, $[u_1,u_4]=-e_3$, $[u_2,u_3]=-e_4$, $[u_2,u_4]=-e_5$, $[u_3,u_4]=-e_6$
$6.1^2.3$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=-e_1$, $[e_3,e_6]=-e_2$, $[e_3,u_1]=-u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=-e_4$, $[e_5,u_2]=-u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=-u_4$, $[e_6,u_4]=u_3$
$6.1^3.1$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=e_1$, $[e_3,e_6]=e_2$, $[e_3,u_1]=u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=e_4$, $[e_5,u_2]=u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=u_4$, $[e_6,u_4]=u_3$, $[u_1,u_2]=e_1$, $[u_1,u_3]=e_2$, $[u_1,u_4]=-e_3$, $[u_2,u_3]=e_4$, $[u_2,u_4]=-e_5$, $[u_3,u_4]=-e_6$
$6.1^3.2$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=e_1$, $[e_3,e_6]=e_2$, $[e_3,u_1]=u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=e_4$, $[e_5,u_2]=u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=u_4$, $[e_6,u_4]=u_3$, $[u_1,u_2]=-e_1$, $[u_1,u_3]=-e_2$, $[u_1,u_4]=e_3$, $[u_2,u_3]=-e_4$, $[u_2,u_4]=e_5$, $[u_3,u_4]=e_6$
$6.1^3.3$	$[e_1,e_2]=-e_4$, $[e_1,e_3]=-e_5$, $[e_1,e_4]=e_2$, $[e_1,e_5]=e_3$, $[e_1,u_1]=-u_2$, $[e_1,u_2]=u_1$, $[e_2,e_3]=-e_6$, $[e_2,e_4]=-e_1$, $[e_2,e_6]=e_3$, $[e_2,u_1]=-u_3$, $[e_2,u_3]=u_1$, $[e_3,e_5]=e_1$, $[e_3,e_6]=e_2$, $[e_3,u_1]=u_4$, $[e_3,u_4]=u_1$, $[e_4,e_5]=-e_6$, $[e_4,e_6]=e_5$, $[e_4,u_2]=-u_3$, $[e_4,u_3]=u_2$, $[e_5,e_6]=e_4$, $[e_5,u_2]=u_4$, $[e_5,u_4]=u_2$, $[e_6,u_3]=u_4$, $[e_6,u_4]=u_3$

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