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COLOURING PLANAR GRAPHS WITH BOUNDED
MONOCHROMATIC COMPONENTS

A.N. GLEBOV

ABSTRACT. Borodin and Ivanova proved that every planar graph of girth at least 7 is 2-choosable with the property that each monochromatic component is a path with at most 3 vertices. Axenovich et al. proved that every planar graph of girth 6 is 2-choosable so that each monochromatic component is a path with at most 15 vertices. We improve both these results by showing that planar graphs of girth at least 6 are 2-choosable so that each monochromatic component is a path with at most 3 vertices. Our second result states that every planar graph of girth 5 is 2-choosable so that each monochromatic component is a tree with at most 515 vertices. Finally, we prove that every graph with fractional arboricity at most $\frac{2d+2}{d+2}$ is 2-choosable with the property that each monochromatic component is a tree with maximum degree at most d . This implies that planar graphs of girth 5, 6, and 8 are 2-choosable so that each monochromatic component is a tree with maximum degree at most 4, 2, and 1, respectively. All our results are obtained by applying the Nine Dragon Tree Theorem, which was recently proved by Jiang and Yang, and the Strong Nine Dragon Tree Conjecture partially confirmed by Kim et al. and Moore.

Keywords: planar graph, defective colouring, clustered colouring, list colouring, girth, maximum average degree, forest decomposition, fractional arboricity, Nine Dragon Tree Theorem.

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1. INTRODUCTION

The goal of this paper is to show how some strong results about defective and clustered colourings of planar and sparse graphs can be derived from the recently proved Nine Dragon Tree Theorem (NDTT) [15] and its stronger version known as Strong Nine Dragon Tree Conjecture (SNDTC), which was posed in [26] and partially proved in [21, 27].

A vertex k -colouring of a graph $G = (V, E)$ is a mapping $f : V \rightarrow \{1, 2, \dots, k\}$, where $\{1, 2, \dots, k\}$ is the set of colours. A *monochromatic component* is a connected component of any subgraph induced by all vertices of one colour. A colouring f is *proper* if any two adjacent vertices are coloured differently. A graph G is (properly) k -colourable if it has a proper k -colouring.

Along with the proper colouring, the various types of improper vertex colouring of graphs are intensively studied. The most popular colourings are a defective colouring, a clustered colouring and a P_n -free colouring. Cowen et al. [12] defined a *defective k -colouring with defect d* as a vertex k -colouring such that any monochromatic component has maximum degree at most d . A slightly more general concept is a *defective (d_1, \dots, d_k) -colouring* defined by the property that each vertex of any colour $i \in \{1, \dots, k\}$ has at most d_i neighbours coloured i . This implies that the subgraph induced by all vertices of colour i has maximum degree at most d_i . Note that if $d_1 = d_2 = \dots = d_k = d$, then we have the definition of a defective k -colouring with defect d .

Observe that defective colouring impose no restrictions on the size of monochromatic components, their structure or the length of monochromatic paths in a graph. Such types of restrictions give rise to other improper colourings. A vertex colouring of a graph G is *acyclic* if G contains no monochromatic cycle, and hence any monochromatic component of G is a tree. A colouring is P_n -free if G contains no monochromatic path P_n with n vertices. The concept of a *clustered colouring* was initially studied in [23] under the name of *fragmented colouring*. A k -colouring of a graph G is a *clustered k -colouring with clustering c* (or *fragmented (k, c) -colouring*) if any monochromatic component of G contains at most c vertices. Clearly, every clustered colouring with clustering c is P_c -free and is defective with defect $c - 1$; moreover, if f is acyclic, then any monochromatic component of G is a tree with at most c vertices. On the other hand, it is easy to check that if f is a P_n -free defective colouring with defect d , then f is a clustered colouring with clustering at most $\max\{n, (d - 1)^{(n+1)/2}\}$.

Note that a defective colouring with defect 0, a clustered colouring with clustering 1, and a P_2 -free colouring are all equivalent to a proper colouring of G . Furthermore, a defective colouring with defect 1 is equivalent to a clustered colouring with clustering 2 and a P_3 -free colouring: in each case every monochromatic component of G is a vertex or an edge.

For all colourings mentioned above we can consider their list versions. Suppose L is a list assignment for a graph G , which assigns a list $L(v)$ of admissible colours to every vertex $v \in V$. The assignment L has *size k* if $|L(v)| = k$ for each $v \in V$. An L -colouring of G is a vertex colouring f such that $f(v) \in L(v)$ for every $v \in V$. A graph G is k -choosable if it has a proper L -colouring for any list assignment L of size k . Along with the proper list colouring, the defective, clustered, P_n -free, and acyclic list colourings are studied (for defective list colouring it is assumed that defect d is

the same for all colours). The defectively k -choosable, clustered k -choosable, and P_n -free k -choosable graphs are defined similarly to the properly k -choosable graphs.

The famous Four Colour Theorem [2, 3] states that every planar graph is (properly) 4-colourable. The well-known result of Grötzsch [18] says that every triangle-free planar graph is 3-colourable. However, the situation with list colouring of planar graphs is significantly different. Voigt constructed an example of a planar graph that is not 4-choosable [33] and an example of a triangle-free planar graph that is not 3-choosable [34]. Thomassen [32] proved that all planar graphs are 5-choosable.

Cushing and Kierstead [13] proved that every planar graph is defectively 4-choosable with defect 1 (and thus with clustering 2). Choi and Esperet [10] showed that every graph with Euler genus $\gamma > 0$ is defectively $(0, 0, 0, 9\gamma - 4)$ -colourable. Poh [30] proved that every planar graph is 3-colourable such that each monochromatic component is a path (and hence is acyclically 3-colourable with defect 2). Cowen, Cowen, and Woodall [12] proved that every planar graph is defectively 3-choosable with defect 2. Woodall [35] generalized this result by showing that every graph with Euler genus γ is 3-choosable with defect $\max\{9, 2 + \sqrt{4\gamma + 6}\}$. Montassier and Ochem [25], for all $d > 0$, presented examples of 2-degenerate planar graphs of girth 4 (respectively, 5 and 7) that are not defectively 2-colourable with defect d (respectively, not defectively $(3, 1)$ -colourable and not defectively $(2, 0)$ -colourable). Borodin et al. [6], for every $d > 0$, constructed 2-degenerate planar graphs of girth 6 that are not defectively $(d, 0)$ -colourable.

On the other hand, it is known that if a graph G is sparse in the sense that its *maximum average degree*, $mad(G) = \max_{H \subseteq G} 2 \frac{|E(H)|}{|V(H)|}$, is low, then G is defectively (d_1, d_2) -colourable (or (d_1, d_2) -choosable) for small constants d_1 and d_2 . Borodin, Kostochka, and Yancey [8] proved that every graph G with $mad(G) \leq \frac{14}{5}$ is defectively 2-colourable with defect 1 (and thus P_3 -free 2-colourable with clustering 2). Borodin and Ivanova [5] proved that every graph G with girth $g(G) \geq 7$ and $mad(G) < \frac{14}{5}$ admits a list 2-colouring where each monochromatic component is a path with at most three vertices. Since every planar graph G with girth at least g has $mad(G) < \frac{2g}{g-2}$, the results in [5, 8] are valid for planar graphs of girth at least 7. Kim, Kostochka, and Zhu [22] proved that every triangle-free graph G with $|E(H)| < \frac{11|V(H)|+5}{9}$ for every subgraph $H \subseteq G$ has a defective $(0, 1)$ -colouring. This implies that planar graphs with girth at least 11 are defectively $(0, 1)$ -colourable. It follows from the results of Borodin and Kostochka [7] that every planar graph G with $g(G) \geq 8$ (respectively, $g(G) \geq 7$, $g(G) \geq 6$, and $g(G) \geq 5$) is defectively $(0, 2)$ -colourable (respectively, $(0, 4)$ -colourable, $(1, 4)$ -colourable, and $(2, 6)$ -colourable). Choi and Raspaud [11] proved that planar graphs with girth at least 5 are defectively $(3, 5)$ -colourable. Havet and Sereni [19] obtained, for every $d \geq 0$, that each graph G with $mad(G) < \frac{4d+4}{d+2}$ is defectively 2-choosable with defect d . This implies that every planar graph G is defectively 2-choosable with defect 1 if $g(G) \geq 8$ and with defect 2 if $g(G) \geq 6$. Škrekovski [31] proved that planar graphs with girth at least 5 are defectively 2-choosable with defect 4.

The concept of a P_n -free colouring was introduced by Chartrand, Geller, and Hedetniemi [9], who showed that for all $n > 0$, there exist planar graphs that are not P_n -free 3-colourable. On the other hand, by Grötzsch Theorem [18], every triangle-free planar graph is P_2 -free 3-colourable. Axenovich, Ueckerdt, and Weiner

[4] constructed examples of triangle-free planar graphs (of girth 4) that are not P_n -free 2-colourable for all $n > 0$. It follows from the result in [25] that not all planar graphs of girth 5 are P_3 -free 2-colourable. Glebov [16] proved that every planar graph of girth 5 is acyclically P_8 -free 2-choosable. Glebov and Zambalaeva [17] proved that planar graphs with girth at least 6 are acyclically P_6 -free 2-colourable while it was proved in [4] that any such a graph admits a list 2-colouring where each monochromatic component is a path with at most 15 vertices. By the above mentioned results in [5, 8, 19], planar graphs of girth 7 are (acyclically) P_3 -free colourable [8] and P_4 -free 2-choosable [5] while planar graphs of girth at least 8 are P_3 -free 2-choosable [19].

The result in [13] implies that every planar graph is clustered 4-choosable with clustering 2. The examples of planar graphs presented in [4, 9, 25] show that for any constant $c > 0$, there exist planar graphs that are not 3-colourable with clustering c and there exist triangle-free planar graphs that are not 2-colourable with clustering c . By the results in [4, 5, 8, 19], planar graphs of girth 6 are 2-choosable with clustering 15, planar graphs of girth 7 are 2-colourable with clustering 2 and 2-choosable with clustering 3, and planar graphs of girth at least 8 are 2-choosable with clustering 2. Dvořák and Norin [14] proved that every graph of Euler genus γ and of girth g is 4-choosable with clustering $1500(\gamma + 2)$, is 3-choosable with clustering $O(\gamma)$ if $g \geq 4$, and is 2-choosable with clustering $O(\gamma)$ if $g \geq 5$. Linial et al. [24] proved that if F is any non-trivial minor-closed family of graphs, then every n -vertex graph $G \in F$ is k -colourable with clustering $O(n^{2/(k+1)})$ and that for $k = 2$ the bound $O(n^{2/3})$ is asymptotically optimal and is attained by planar graphs. Haxell, Szabó, and Tardos [20] proved that every graph G with maximum degree Δ is 2-colourable with clustering 6 if $\Delta = 4$, is 2-colourable with clustering 20000 if $\Delta = 5$, and is 3-colourable with constant clustering if $\Delta \leq 8$. For larger values of Δ they established that G is $\lceil \frac{\Delta+1}{3} \rceil$ -colourable with constant clustering independent of Δ . Alon et al. [1], for every $c > 0$, constructed a family of 6-regular graphs that are not 2-colourable with clustering c and a family of 10-regular graphs that are not 3-colourable with clustering c .

2. OUR RESULTS

In this paper we present some new results about clustered and defective choosability of planar and sparse graphs. Our proof method essentially involves the idea of edge decomposition of a graph into forests. So, the natural measure of sparseness of a graph here would be its fractional arboricity rather than the maximum average degree. Let $H = (V_H, E_H)$ be an arbitrary subgraph of a graph G . The *fractional arboricity* of G is

$$Arb_f(G) = \max_{H \subseteq G, |V_H| > 1} \frac{|E_H|}{|V_H| - 1}.$$

Our first result is about acyclic defective choosability of a graph.

Theorem 1. *Let d be a positive integer. If G is a graph with $Arb_f(G) \leq \frac{2d+2}{d+2}$, then G is acyclically 2-choosable with defect d . This implies that every monochromatic component of G is a tree of maximum degree at most d .*

Observe that for every graph G , $Arb_f(G)$ is close to $\frac{1}{2}mad(G)$ and is greater than $\frac{1}{2}mad(G)$. Thus, our Theorem 1 can be treated as an "asymptotical improvement"

of the result by Havet and Sereni [19] that every graph G with $\text{mad}(G) < \frac{4d+4}{d+2}$ is 2-choosable with defect d .

By Euler's formula and the definition of $\text{mad}(G)$ and $\text{Arb}_f(G)$ it follows that every planar graph G of girth g satisfies $\text{mad}(G) < 2\text{Arb}_f(G) < \frac{2g}{g-2}$. Thus, we get the following

Corollary 1. *Let G be a planar graph of girth g .*

- 1) *If $g \geq 5$, then G is acyclically 2-choosable with defect 4.*
- 2) *If $g \geq 6$, then G is acyclically 2-choosable with defect 2. This implies that each monochromatic component of G is a path.*
- 3) *If $g \geq 8$, then G is 2-choosable with defect 1 (and hence with clustering 2).*

The first statement of Corollary 1 improves the result of Škrekovski [31] that every planar graph of girth 5 is 2-choosable with defect 4 while the second statement is the relaxation of the result of Axenovich et al. [4] who gave the upper bound 15 for the number of vertices in each monochromatic path. Our next theorem decreases this bound from 15 to 3 and improves the result in [17] that every planar graph of girth 6 is acyclically P_6 -free 2-colourable and the result in [5] that every planar graph of girth 7 is 2-choosable with clustering 3.

Theorem 2. *Every graph G with $\text{mad}(G) < 3$ (and hence every planar graph of girth at least 6) is acyclically 2-choosable with clustering 3. This implies that each monochromatic component of G is a path with at most 3 vertices.*

Our last result can be considered as a refinement (in the case of planar graphs) of the more general result by Dvořák and Norin [14] that every graph of girth 5 and Euler genus γ is 2-choosable with clustering $O(\gamma)$.

Theorem 3. *Every graph G with $\text{Arb}_f(G) \leq 5/3$ (and hence every planar graph of girth at least 5) is acyclically 2-choosable with clustering 515. This implies that each monochromatic component of G is a tree with at most 515 vertices.*

3. APPLICATION OF NINE DRAGON TREE THEOREM

In order to prove Theorems 1–3 we apply some results about forest decomposition of graphs. The celebrated Nash-Williams Theorem characterizes graphs that can be decomposed into k forests.

Theorem 4. *(Nash-Williams Theorem [28, 29]) The edge set of a graph G can be decomposed into k forests if and only if $\text{Arb}_f(G) \leq k$.*

If a graph G has $\text{Arb}_f(G) = k + \varepsilon$ for some small $\varepsilon > 0$, then Nash-Williams Theorem says that G decomposes into $k + 1$ forests but cannot be decomposed into k forests. However, intuitively, you can hope that there exists a decomposition of G into $k + 1$ forests such that "almost all" edges belong to some k forests while the last forest is sparse or restricted in some way. This intuition is confirmed by the following theorem of Jiang and Yang [15], which was initially formulated as the Nine Dragon Tree Conjecture in [26].

Theorem 5. *(Nine Dragon Tree Theorem [15]) Let k and d be positive integers. Every graph G with $\text{Arb}_f(G) \leq k + \frac{d}{k+d+1}$ decomposes into $k + 1$ forests such that one of the forests has maximum degree at most d .*

It was shown by Montassier et al. [26] that the fractional arboricity bound in the Nine Dragon Tree Theorem is best possible. Despite this, they posed the following significant strengthening of the Nine Dragon Tree Theorem.

Conjecture 1. (*Strong Nine Dragon Tree Conjecture* [26]) *Let k and d be positive integers. Every graph G with $\text{Arb}_f(G) \leq k + \frac{d}{k+d+1}$ decomposes into $k+1$ forests such that for one of the forests, each connected component has at most d edges.*

Kim et al. [21] verified SNDTC for $k = 1$ and $d = 2$. Actually, they proved a stronger result that every graph G with $\text{mad}(G) < 3$ decomposes into two forests such that for one of the forests, each component has at most two edges.

Moore [27] made an impressive step towards the SNDTC by showing the conjecture is true for $d \leq k+1$ and by giving a relaxed upper bound for the size of components in other cases.

Theorem 6. [27] *Let k and d be positive integers. If $d \leq k+1$, set $c(k, d) = 0$. Otherwise, set $R = \lceil \frac{d}{k+1} + 2 \rceil$ and set $c(k, d) = \frac{dk(R^R-1)}{k+1}$. Every graph G with $\text{Arb}_f(G) \leq k + \frac{d}{k+d+1}$ decomposes into $k+1$ forests such that for one of the forests, every connected component has at most $d + c(k, d)$ edges.*

Our Theorems 1–3 are easy corollaries of Theorems 5, 6, the result in [21], and the following trivial observation.

Lemma 1. *Let $G = (V, E)$ be a graph and $E = E_1 \cup E_2$ be its edge decomposition into subgraphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$. If G_1 is properly k -colourable (properly k -choosable), then G has a (list) k -colouring of its vertices such that each monochromatic component of G is contained in some connected component of G_2 .*

Indeed, any proper (list) k -colouring of G_1 gives a desired k -colouring of G . Since any forest is properly 2-choosable, Lemma 1 implies that if G has a decomposition into a forest F_1 and a forest F_2 with restricted connected components, then G is acyclically 2-choosable so that monochromatic components have the same restrictions as the components of F_2 .

Now combining Lemma 1 with Theorem 5, where $k = 1$, yields Theorem 1. Theorem 2 can be derived from Lemma 1 and the result in [21]. Finally, Theorem 3 follows from Lemma 1 and Theorem 6 with $k = 1$, $d = 4$, since $c(1, 4) = 510$ in Theorem 6.

Observe that by Lemma 1, any further improvements in approaching SNDTC will give better bounds for clustered and acyclic colourings of planar and sparse graphs. In particular, if SNDTC is true, then the bound 515 in Theorem 3 can be replaced by 5.

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ALEKSEY NIKOLAEVICH GLEBOV
SOBOLEV INSTITUTE OF MATHEMATICS,
4, KOPTYUGA AVE.,
NOVOSIBIRSK, 630090, RUSSIA
E-mail address: angle@math.nsc.ru