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СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports

http://semr.math.nsc.ru

Том 18, №1, стр. 72–85 (2021) УДК 517.956.6
DOI 10.33048/semi.2021.18.007 MSC 35M10, 35J10, 35C15

ANALOGUES OF THE CAUCHY-GOURSAT PROBLEM FOR A LOADED THIRD-ORDER HYPERBOLIC TYPE EQUATION IN AN INFINITE THREE-DIMENSIONAL DOMAIN

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Abstract. In this article it is studied the analogue of Cauchy–Goursat problem for a loaded third-order hyperbolic type differential equation in an infinite three-dimensional domain. The main research method is the Fourier transform of studying the analogous of the Cauchy–Goursat problem. Based on this Fourier transform, the given problem reduces to a flat analogue of the Cauchy–Goursat problem with a spectral parameter with boundary value conditions. The asymptotic behavior of the solution of plane analogues of the Cauchy–Goursat problem for large values of the spectral parameter is studied. Sufficient conditions are obtained, according to which all operations in this paper are legal.

Keywords: Cauchy–Goursat problem, loaded third-order hyperbolic type equation, three-dimensional domain, Fourier transform.

1. Introduction

One of the important classes of nonclassical differential equations of mathematical physics is the theory of differential equations of composite and mixed-composite types, the main parts of which contain operators of elliptic, elliptic-hyperbolic or parabolic-hyperbolic types. Correct boundary value problems for differential equations of elliptic-hyperbolic and parabolic-hyperbolic types of the third order, when the principal part of the operators contains a derivative with respect to variables x or y, were for the first time studied in the works of A. V. Bitsadze and M. S. Salakhitdinov [1], M. S. Salakhitdinov [2] and T. D. Dzhuraev

Islomov, B.I., Alikulov, E.K., Analogues of the Cauchy-Goursat problem for a loaded third-order hyperbolic type equation in an infinite three-dimensional domain.

Received November, 16, 2019, published February, 5, 2021.

[3]. In these works, in studying boundary value problems, the representation of the general solution of the mixed-composite type equation was used. The mixed-composite type differential equations have an important place in the theory of differential equations, consisting the product of permutation of differential operators. Further, this direction was developed in the works [4]-[9] for various type third-order partial differential equations.

Many problems of mathematical physics and biology, especially problems of long-term forecasting and regulation of groundwater, problems of heat and mass transfer at a finite rate, problems of optimal control of the agroecosystem and many other problems lead to solve the boundary value problems for a loaded partial differential equations [10].

In 1969 A. M. Nakhushev proposed a number of new problems, which later were included in the mathematical literature under the name "boundary value problems with displacement". It turns out that such boundary value problems of a new type are closely related to the study of loaded differential equations [11].

Definition 1. *Differential equation $\mathcal{A}\,u\,(x) = f\,(x)$, given in n-dimensional domain Ω of Euclidean space of points $x = (x_1,\, x_2, ...,\, x_n)$, is called a loaded, if it contains a trace of some operations from the desired solution $u\,(x)$ on the closure $\overline{\Omega}$ of manifolds of dimension less than n.*

Definition 2. *The equation will be called a loaded differential equation in the domain $\Omega \subset \mathbb{R}^n$, if it contains at least one derivative of the required solution $u\,(x)$ on manifolds of nonzero measure belonging to $\overline{\Omega}$.*

Important loaded differential and integro-differential equations in the domain $\Omega \in \mathbb{R}^n$ can be represented as [12]

$$\mathcal{A}\,u\,(x) \equiv \mathcal{L}\,u\,(x) + \mathcal{M}\,u\,(x) = f\,(x), \qquad (*)$$

where $\mathcal{L}$ is differential operator, and $\mathcal{M}$ is differential, integral or integro-differential operator, respectively, including the operation of taking a trace from the desired solution $u\,(x)$ on manifolds of nonzero measure belonging to $\overline{\Omega}$ (measures strictly less than n).

For example, problems of vibrations of a string loaded with lumped masses are reduced to the simplest equation of the form of a sum of operators, which are widely used in physics and technology [13]. The solution of many problems of optimal control of the agroexystem [12], the numerical solution of integro-differential equations [14], the study of inverse problems [15] and the equivalent transformation of nonlocal boundary value problems [16] are reduced to the study of differential equations of the form (*).

Note that the presence of a loaded operator M requires researcher of additional considerations on the application of the well-known theory of boundary value problems for differential equations of the form $\mathcal{L}\,u\,(x) = f\,(x),\;\; x \in \Omega$.

On the other hand, the problem of finding solutions of loaded differential equations in predetermined classes can lead to new problems for unloaded differential equations [17].

Local and nonlocal problems for second-order partial differential equations of hyperbolic and mixed types with loaded terms in special domains were considered in [18]-[22]. In the study of these problems, the problem of studying integral and integro-differential equations arises. This direction developed in the works [23]-[26].

Boundary value problems for the loaded hyperbolic, mixed and mixed-composite types of the third order partial differential equations are still poorly investigated. This is due, first of all, to the lack of representation of the general solution for such differential equations. On the other hand, such problems are reduced to little-studied integral and integro-differential equations with a shift. Here we note the papers [27, 28, 29].

The theory of boundary value problems for loaded second order equations with an integro-differential operator was studied in [30]-[33].

Since 1962, after the publication of the well-known works by A. V. Bitsadze [34, 35] many mathematicians began to study boundary value problems for equations of mixed elliptic-hyperbolic type in three-dimensional domains. A number of research papers appeared (see, for example [36]-[41], where three-dimensional analogues of the Tricomi, Gellerstedt, and Keldysh problems for second order equations of mixed type were considered.

We note, that the boundary value problems for the loaded third-order differential equations of hyperbolic, mixed and mixed-composite types in infinite three-dimensional domain studied a little (here we refer the readers to the works [42, 43]).

In this paper, we study analogues of the Cauchy–Goursat problem for a loaded third-order hyperbolic type differential equation in an infinite three-dimensional domain. The main method of studying the problem is the Fourier transform. Based on the Fourier transform, the partial differential equations are reduced to a flat analogue of the Cauchy–Goursat problems with a spectral parameter. In particular, the spectral parameter can appear in the boundary conditions. A solution of the problem is found in a convenient form for further investigation of new kind of boundary value problems.

2. Problem formulation

Let Ω is three dimensional domain bounded by surfaces:

$$\Gamma_0 : \ 0 < x < 1, \ \ y = 0, \ \ z \in \mathbb{R}; \ \ \Gamma_1 : \ x + y = 0, \ \ 0 \le x \le \frac{1}{2}, \ \ z \in \mathbb{R};$$

$$\Gamma_2 : \ x - y = 1, \ \ \frac{1}{2} \le x \le 1, \ \ z \in \mathbb{R}.$$

We introduce the designations $\sigma_j = \Gamma_j \cap \{z = 0\}$, $j = \overline{0, 2}$, $D = \Omega \cap \{z = 0\}$. In the infinite three-dimensional domain Ω we consider the following differential equation

$$\frac{\partial}{\partial x}(U_{xx} - U_{yy} + U_{zz}) - \mu U(x, 0, z) = 0, \tag{1}$$

where $\mu = \text{const} < 0$.

Problem AG_1. Find the function $U(x, y, z)$ with following properties:

1). $U(x, y, z)$ is continuous up to the boundary of the domain Ω;

2). $U_x \in C(\Omega \cup \Gamma_1)$, $U_y \in C(\Omega \cup \Gamma_0 \cup \Gamma_1)$;

3). U_{xxx}, U_{xzz}, $U_{xyy} \in C(\Omega)$ and satisfies differential equation (1) in Ω;

4). $U(x, y, z)$ satisfies the following conditions

$$\lim_{y \to -0} U_y(x, y, z) = \Phi(x, z), \ 0 \le x < 1, \ \ z \in \mathbb{R}, \tag{2}$$

$$U(x, y, z)|_{\Gamma_1} = \Psi_1(x, z), \ \frac{\partial U(x, y, z)}{\partial n}\bigg|_{\Gamma_1} = \Psi_2(x, z), 0 \le x \le \frac{1}{2}, \ \ z \in \mathbb{R}, \tag{3}$$

$$\lim_{|z|\to\infty} U = \lim_{|z|\to\infty} U_x = \lim_{|z|\to\infty} U_y = \lim_{|z|\to\infty} U_z = 0, \tag{4}$$

where n is internal normal, $\Phi(x, z), \Psi_1(x, z), \Psi_2(x, z)$ are given sufficiently smooth functions, and

$$\lim_{|z|\to\infty} \Phi(x, z) = 0, \ \lim_{|z|\to\infty} \Psi_j(x, z) = 0, \ j = 1, 2. \tag{5}$$

3. Investigation of the AG_1

According to the condition of the problem statement AG_1, its solutions can be represented in the form of the Fourier integral:

$$U(x, y, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} u(x, y; \lambda) \cdot e^{-i\lambda z} d\lambda. \tag{6}$$

So, by the aid of Fourier transform (6), the equation (1) we reduce to the following equation

$$\frac{\partial}{\partial x}(u_{xx} - u_{yy} - \lambda^2 u) - \mu u(x, 0, \lambda) = 0. \tag{7}$$

Then the problem AG_1 is replaced with the following equivalent flat problem.

Problem $AG_{1\lambda}$. Find a function $u(x, y, \lambda)$ such that:

1). $u(x, y, \lambda) \in C(\overline{D}) \cap C^2(D)$, $u_x(x, y, \lambda) \in C(D \cup \sigma_1)$, $u_y(x, y, \lambda) \in C(D \cup \sigma_0 \cup \sigma_1)$ and satisfies the equation (7) in the domain D;

2). $u(x, y, \lambda)$ satisfies to the conditions

$$\lim_{y\to -0} u_y(x, y, \lambda) = \varphi(x, \lambda), \ x \in [0, 1), \ \lambda \in \mathbb{R}, \tag{8}$$

$$u(x, y, \lambda)|_{\sigma_1} = \psi_1(x, \lambda), \ \left.\frac{\partial u(x, y, \lambda)}{\partial n}\right|_{\sigma_1} = \psi_2(x, \lambda), \ 0 \le x \le \frac{1}{2}, \ \lambda \in \mathbb{R}, \tag{9}$$

where $\varphi(x, \lambda), \ \psi_1(x, \lambda), \ \psi_2(x, \lambda)$ are given functions and

$$\begin{aligned} \varphi(x, \lambda) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Phi(x, z) e^{i\lambda z} dz, \\ \psi_j(x, \lambda) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi_j(x, z) e^{i\lambda z} dz, \ \ j = 1, 2, \end{aligned} \tag{10}$$

$$2\varphi(0, \lambda) = \sqrt{2}\psi_2(0, \lambda) - \psi_1'(0, \lambda), \ \ \varphi(x, \lambda) \in C[0, 1) \cap C^2(0, 1), \tag{11}$$

$$\psi_1(x, \lambda) \in C^1\left[0, \frac{1}{2}\right] \cap C^3\left(0, \frac{1}{2}\right), \ \ \psi_2(x, \lambda) \in C\left[0, \frac{1}{2}\right] \cap C^2\left(0, \frac{1}{2}\right). \tag{12}$$

Note that the problem $AG_{1\lambda}$ is studied in [40] for the case $\lambda = \mu = 0$ of the equation (7). Any regular solution of the equation (7) is represented in the following form [2, 28]:

$$u(x, y, \lambda) = \upsilon(x, y, \lambda) + w(x, \lambda), \tag{13}$$

where $\upsilon(x, y, \lambda)$ is the solution of the equation

$$\frac{\partial}{\partial x}\left(\upsilon_{xx} - \upsilon_{yy} - \lambda^2 \upsilon\right) = 0 \tag{14}$$

and $w\,(x,\,\lambda)$ is a solution of the following ordinary differential equations

$$w'''(x,\,\lambda)-\lambda^2\,w'(x,\,\lambda)-\mu\,w\,(x,\,\lambda)=\mu\,\upsilon\,(x,\,0,\,\lambda). \tag{15}$$

Remark 1. Taking into account that the function $a\,e^{\lambda\,x}+b\,e^{-\lambda\,x}+c$ satisfies the equation (14) and we solve the problem $AG_{1\lambda}$ assuming that

$$w\,(0,\,\lambda)=w'(0,\,\lambda)=w''(0,\,\lambda)=0. \tag{16}$$

Now we solve the Cauchy problem for the equation (15) with the conditions (16) with respect to $w\,(x,\,\lambda)$. The characteristic equation corresponding to the homogeneous equation (15) has the form

$$m^3-\lambda^2 m-\mu=0. \tag{17}$$

1). If $\Delta\equiv\frac{\mu^2}{4}-\frac{\lambda^6}{27}>0$, then it is known (see [44]) that the equation (17) has one real and two complex conjugate roots, which have the forms

$$m_1=p_1+q_1,\;\; m_{2,\,3}=-\frac{1}{2}\big(p_1+q_1\big)\pm\frac{\sqrt{3}}{2}i\big(p_1-q_1\big),$$

where

$$p_1=\sqrt[3]{\frac{\mu}{2}+\sqrt{\Delta}},\;\; q_1=\sqrt[3]{\frac{\mu}{2}-\sqrt{\Delta}}.$$

Thus, the solution of the Cauchy problem for the equation (15) with the conditions (16) for $\Delta>0$ has the form

$$w\,(x,\,\lambda)=\int\limits_0^x T_1(x,\,t;\,\lambda,\,\mu)\,\upsilon\,(t,\,0,\,\lambda)\,d\,t, \tag{18}$$

where

$$T_1(x,\,t,\,\lambda,\,\mu)=\frac{\mu}{3\big(p_1^2+p_1q_1+q_1^2\big)}\Bigg[e^{\frac{3}{2}(p_1+q_1)(x-t)}+ \tag{19}$$

$$+\frac{\sqrt{3}(p_1+q_1)}{p_1-q_1}\sin\frac{\sqrt{3}}{2}(p_1-q_1)(t-x)-\cos\frac{\sqrt{3}}{2}(p_1-q_1)(t-x)\Bigg]e^{-\frac{1}{2}(p_1+q_1)(x-t)};$$

2). If $\Delta=0$, then the equation (17) has three real roots, two of them being equal:

$$m_1=\frac{3\mu}{\lambda^2},\;\; m_2=m_3=-\frac{3\mu}{2\lambda^2}.$$

Solution of the Cauchy problem for the equation (15) with conditions (16) and $\lambda^2=-3\big(\mu/2\big)^{\frac{2}{3}}$ has the form

$$w\,(x,\,\lambda)=\int\limits_0^x T_2(x,\,t,\,\mu)\,\upsilon(t,\,0,\,\lambda)\,d\,t, \tag{20}$$

where

$$T_2(x,\,t,\,\mu)=\frac{2}{9}\Big(\frac{\mu}{2}\Big)^{\frac{1}{3}}e^{\sqrt[3]{\frac{\mu}{2}}(x-t)}\Big(e^{\sqrt[3]{\frac{\mu}{2}}(x-t)}-3\Big(\frac{\mu}{2}\Big)^{\frac{1}{3}}(x-t)-1\Big). \tag{21}$$

3). If $\Delta<0$, then the equation (17) has three different real roots, which has the form:

$$m_1=2|\sqrt[3]{r}|\cos\frac{\phi}{3},\;\; m_2=2|\sqrt[3]{r}|\cos\frac{\phi+2\pi}{3},\;\; m_3=2|\sqrt[3]{r}|\cos\frac{\phi+4\pi}{3},$$

where

$$r = \Big|\Big(\frac{\lambda^2}{3}\Big)^{\frac{3}{2}}\Big|, \ \cos\phi = \frac{\mu}{2}\big|\big(\frac{\lambda^2}{3}\big)^{\frac{3}{2}}\big|^{-1}.$$

Accordingly, the solution of the Cauchy problem for the equation (15) with conditions (16) and $\Delta < 0$ has the form

$$w\,(x,\,\lambda) = \int_0^x T_3(x,\,t,\,\lambda,\,\mu)\,\upsilon\,(t,\,0,\,\lambda)dt, \tag{22}$$

where

$$T_3(x,\,t,\,\lambda,\,\mu) = \tag{23}$$

$$= \frac{\mu\big\{\big(m_2 - m_3\big)e^{m_1(x-t)} + \big(m_3 - m_1\big)e^{m_2(x-t)} - \big(m_2 - m_1\big)e^{m_3(x-t)}\big\}}{\big(m_2 - m_1\big)\big(m_3 - m_1\big)\big(m_2 - m_3\big)}.$$

3.1. **Existence and uniqueness of the solution to the problem $AG_{1\lambda}$.**

Theorem 1. *Let the conditions* (11) *and* (12) *be fulfilled. Then the solution of the problem $AG_{1\lambda}$ for the equation* (7) *exists and is unique in the domain D.*

Proof. Existence of a solution of the problem $AG_{1\lambda}$.

By virtue of representation (13) the problem $AG_{1\lambda}$ replace with the problem $AG^*_{1\lambda}$ of finding a regular solution $\upsilon\,(x,\,y,\,\lambda)$ for the equation (14), satisfying in the domain D the following conditions

$$\upsilon_y(x,\,y,\,\lambda)_{|y=0} = \varphi\,(x,\,\lambda), \ 0 \le x < 1, \ \lambda \in \mathbb{R}, \tag{24}$$

$$\upsilon\,(x,\,y,\,\lambda)_{|\sigma_1} = \psi_1(x,\,\lambda) - w\,(x,\,\lambda), \ 0 \le x \le \frac{1}{2}, \ \ \lambda \in \mathbb{R}, \tag{25}$$

$$\frac{\partial\,\upsilon\,(x,\,y,\,\lambda)}{\partial\,n}\Big|_{\sigma_1} = \psi_2(x,\,\lambda) - \frac{1}{\sqrt{2}}w'(x,\,\lambda), \ 0 \le x \le \frac{1}{2}, \ \lambda \in \mathbb{R}, \tag{26}$$

where

$$w\,(x,\,\lambda) = \int_0^x T_j(x,\,t,\,\lambda,\,\mu)\,\upsilon\,(t,\,0,\,\lambda)\,d\,t, \ \ j = \overline{1,\,3}, \tag{27}$$

and $T_j(x,\,t,\,\lambda,\,\mu), \ j = \overline{1,\,3}$ are defined from (18), (20) and (22).

Solution of the problem $AG^*_{1\lambda}$ with boundary value problem (24)–(26) for the equation (14) in domain D one can represent as (see [40]):

$$\upsilon\,(x,\,y,\,\lambda) = \int_0^{x+y} \varphi(t,\,\lambda)\,I_0\big[\lambda\sqrt{(x+y-t)(x-y-t)}\big]dt - \tag{28}$$

$$-\widetilde{\psi}_1(0,\,\lambda)I_0\big[\lambda\sqrt{(x^2-y^2)}\big] + \widetilde{\psi}_1(\frac{x+y}{2},\,\lambda) + \widetilde{\psi}_1(\frac{x-y}{2},\,\lambda) +$$

$$+\int_0^{x+y} \big(\lambda\widetilde{\psi}_1(t,\,\lambda) - \frac{\sqrt{2}\widetilde{\psi}'_2(t,\,\lambda)}{\lambda}\big)\sin\lambda\big(t - \frac{x+y}{2}\big)dt +$$

$$+\int_0^{\frac{x-y}{2}} \big[\lambda\widetilde{\psi}_1(t,\,\lambda) - \frac{\sqrt{2}\widetilde{\psi}'_2(t,\lambda)}{\lambda}\big]\sin\lambda\big(t - \frac{x-y}{2}\big)dt -$$

$$-2\int\limits_0^{\frac{x-y}{2}}\widetilde{\psi}_1(t,\,\lambda)B_t\big(0,\,2t;\,x+y,\,x-y\big)dt+\int\limits_0^{y}\big[\lambda\widetilde{\psi}_1(-t,\,\lambda)-\frac{\sqrt{2}\widetilde{\psi}_2'(-t,\,\lambda)}{\lambda}\big]\sin\lambda\big(y-t\big)dt-$$

$$-2\int\limits_0^{\frac{x-y}{2}}B_t\big(0,\,2t;\,x+y,\,x-y\big)dt\int\limits_0^{t}\big[\lambda\widetilde{\psi}_1(z,\,\lambda)-\frac{\sqrt{2}\widetilde{\psi}_2'(z,\,\lambda)}{\lambda}\big]\sin\lambda\big(-t+z\big)dz,$$

where

$$B\big(t,\,z;\,x+y,\,x-y\big)=$$

$$=\begin{cases} I_0\big[\lambda\sqrt{(x+y-t)(x-y-z)}\big], & z>x+y,\\ I_0\big[\lambda\sqrt{(x+y-t)(x-y-z)}\big]+I_0\big[\lambda\sqrt{(x+y-z)(x-y-t)}\big], & z<x+y,\end{cases}$$

$$\widetilde{\psi}_1(x,\,\lambda)=\psi_1(x,\,\lambda)-w(x,\,\lambda),\ \widetilde{\psi}_2(x,\,\lambda)=\psi_2(x,\,\lambda)-\frac{1}{\sqrt{2}}w'(x,\,\lambda),$$

$I_0[z]$ is modified Bessel function [45].

We set $y=0$ in (28) and taking (27) into account, we obtain the following functional relation brought from the domain D into J:

$$\tau\,(x,\,\lambda)+\int\limits_0^{x}K\,(x,\,t;\,\lambda,\,\mu)\,\tau\,(t,\,\lambda)dt=G\,(x,\,\lambda),\ 0\le x\le 1,\tag{29}$$

where $\tau\,(x,\,\lambda)=\upsilon\,(x,\,0,\,\lambda)$,

$$K\,(x,\,t;\,\lambda,\,\mu)=\begin{cases} K_1(x,\,t;\,\lambda,\,\mu), & 0\le t\le\dfrac{x}{2},\\ 0, & \dfrac{x}{2}\le t\le x,\end{cases}\tag{30}$$

$$K_1(x,\,t;\,\lambda,\,\mu)=\tag{31}$$

$$=2T_j(\frac{x}{2},\,t;\,\lambda,\,\mu)+\lambda^2x\int\limits_{2t}^{x}T_j\left(\frac{s}{2},\,t;\,\lambda,\,\mu\right)\bar{I}_1\left[\lambda\sqrt{x(x-s)}\right]ds+$$

$$+\int\limits_{2t}^{x}K_2(\frac{x}{2},\,t;\,\lambda)\big[\lambda T_j(\frac{s}{2},\,t;\,\lambda,\,\mu)-\frac{1}{2\lambda}T_j'(\frac{s}{2},\,t;\,\lambda,\,\mu)\big]ds,$$

$$G\,(x,\,\lambda)=2\psi_1(\frac{x}{2},\,\lambda)-\psi_1(0,\,\lambda)\,I_0[\lambda\,x]+\int\limits_0^{x}\varphi(t,\,\lambda)\,I_0[\lambda(x-t)]dt+\tag{32}$$

$$+\lambda^2\,x\int\limits_0^{x}\bar{I}_1[\lambda\sqrt{x(x-t)}]\psi_1(\frac{t}{2},\,\lambda)dt+$$

$$+\int\limits_0^{x}K_2(\frac{x}{2},\,\frac{t}{2};\,\lambda)\,\big[\lambda\psi_1(\frac{t}{2},\,\lambda)-\frac{\sqrt{2}}{2\lambda}\psi_2'(\frac{t}{2},\,\lambda)\big]dt,$$

$$K_2(x,\,t;\,\lambda)=\sin\lambda(t-x)+\lambda^2\int\limits_t^{x}x\bar{I}_1[\lambda\sqrt{x(x-2s)}]\sin\lambda(t-s)ds,\tag{33}$$

$\overline{I}_1(x) = \frac{I_1(x)}{x}$, $I_0(x)$, $I_1(x)$ is modified Bessel function.

By virtue of (11), (12) and properties of Bessel function (see [45]), from (30)–(33) we deduce that

$$K(x, t; \lambda, \mu) \in C([0, 1] \times [0, 1]), \quad G(x, \lambda) \in C[0, 1] \cap C^2(0, 1), \tag{34}$$

$$|K(x, t; \lambda, \mu)| \le \text{const}, \quad |G(x, \lambda)| \le \text{const}. \tag{35}$$

Thus, due to (35) the equation (29) is a Volterra integral equation of the second kind. According to the theory of Volterra integral equations of the second kind (see [46]), we conclude that the integral equation (29) is uniquely solvable in the class $C[0, 1] \cap C^2(0, 1)$ and its solution is represented by the formula

$$\tau(x, \lambda) = G(x, \lambda) - \int_0^x K^*(x, t; \lambda, \mu)\, G(t, \lambda) dt, \ 0 \le x \le 1, \tag{36}$$

where $K^*(x, t; \lambda, \mu)$ is resolvent of kernel $K(x, t; \lambda, \mu)$.

Substituting (36) into (27) we define the function $w(x, \lambda)$ as:

$$w(x, \lambda) = \int_0^x T_j(x, t; \lambda, \mu) \Big[G(t, \lambda) - \\ - \int_0^t K^*(t, z; \lambda, \mu)\, G(z, \lambda) dz \Big] dt, \quad j = \overline{1, 3}. \tag{37}$$

By virtue of (11), (12), (19), (21), (23), and taking (34) and (35) into account, from (37) implies that

$$w(x, \lambda) \in C[0, 1] \cap C^2(0, 1). \tag{38}$$

Then, substituting (37) into (28), we obtain a solution to the problem $AG^*_{1\lambda}$ for the equation (14) in domain D.

Thus, the solution of the problem $AG_{1\lambda}$ for the equation (7) in domain D exists and and we determine it from presentation (13), where $\upsilon(x, y, \lambda)$ and $w(x, \lambda)$ are determined from (28) and (39), respectively.

Uniqueness of the solution $AG_{1\lambda}$. Let be

$$\varphi(x, \lambda) \equiv \psi_1(x, \lambda) \equiv \psi_2(x, \lambda) \equiv 0. \tag{39}$$

Then from (32) we obtain $G(x, \lambda) \equiv 0$, and from (36) we have: $\tau(x, \lambda) \equiv 0$. Hence, from (37) we find

$$w(x, \lambda) \equiv 0. \tag{40}$$

By virtue of (39) and (40), from the solution of the analogue of the Cauchy–Goursat problem (see (28)) for the equation (14) in the domain D we deduce that

$$\upsilon(x, y, \lambda) \equiv 0, \ (x, y) \in \overline{D}. \tag{41}$$

This proves the uniqueness of the solution of the problem $AG^*_{1\lambda}$ for the equation (14). Further, taking into account (40) and (41), from (13) we have

$$u(x, y, \lambda) \equiv 0, \ (x, y) \in \overline{D}. \tag{42}$$

This implies the uniqueness of the solution of the problem $AG_{1\lambda}$ for the equation (7). Consequently, the problem $AG_{1\lambda}$ is one valued solvable. Theorem 1 is proved.

3.2. **Estimate for solution of the problem $AG_{1\lambda}$ for large values of parameter $|\lambda|$.** To ensure the existence of the integral (6) and the fulfilment of the conditions (5), it is necessary to evaluate the solution of the problem $AG_{1\lambda}$ for large values of parameter $|\lambda|$. It is known that any regular solution of the equation (14) can be represented in the form (see [40]):

$$\upsilon(x, y, \lambda) = \rho(x, y, \lambda) + \omega(y, \lambda), \tag{43}$$

where $\rho(x, y, \lambda)$ is regular solution of the equation

$$\rho_{xx} - \rho_{yy} - \lambda^2 \rho = 0, \tag{44}$$

$\omega(y, \lambda)$ is arbitrary twice continuously differentiable function.

Then by virtue of the presentation (43) the problem $AG_{1\lambda}$ is reduced to the problem $\widetilde{AG}_{1\lambda}$ of finding of regular in domain D solution $\rho(x, y, \lambda)$ of the equation (44) satisfying the conditions

$$\rho_y(x, y, \lambda)_{|y=0} = f_0(x, \lambda), \; 0 \le x < 1, \; \lambda \in \mathbb{R}, \tag{45}$$

$$\rho(x, y, \lambda)_{|\sigma_1} = f_1(x, \lambda), \; 0 \le x \le \frac{1}{2}, \; \lambda \in \mathbb{R}, \tag{46}$$

$$\frac{\partial \rho(x, y, \lambda)}{\partial n}\Big|_{\sigma_1} = f_2(x, \lambda), \; 0 \le x \le \frac{1}{2}, \; \lambda \in \mathbb{R}, \tag{47}$$

where

$$f_0(x, \lambda) = \varphi(x, \lambda) - \omega'(0, \lambda),$$
$$f_1(x, \lambda) = \psi_1(x, \lambda) - w(x, \lambda) - \omega(-x, \lambda),$$
$$f_2(x, \lambda) = \psi_2(x, \lambda) - \frac{1}{\sqrt{2}}[w'(x, \lambda) + \omega(-x, \lambda)],$$

$w(x, \lambda)$ is determined from (37), $\omega(y, \lambda)$ is determined from the formula

$$\omega(y, \lambda) = \sqrt{2}\psi_2'(-y, \lambda) - \lambda^2 \psi_1(-y, \lambda), \; -\frac{1}{2} \le y \le 0. \tag{48}$$

Theorem 2. *If fulfilled the following conditions*

$$f_0(x, \lambda) = O\left(\frac{1}{|\lambda|^k \cosh|\lambda|}\right), \; w(x, \lambda) = O\left(\frac{1}{|\lambda|^k}\right), \; \omega(y, \lambda) = O\left(\frac{1}{|\lambda|^k}\right), \tag{49}$$

$$f_1(x, \lambda) = O\left(\frac{1}{|\lambda|^{k+1} \cosh|\lambda|}\right), \; f_1'(x, \lambda) = O\left(\frac{1}{|\lambda|^k \cosh|\lambda|}\right), \tag{50}$$

$$f_2(x, \lambda) = O\left(\frac{1}{|\lambda|^k}\right), \; k > 3, \tag{51}$$

then the function $u(x, y, \lambda)$ for large value of $|\lambda|$ has a solution

$$u(x, y, \lambda) = O\left(\frac{1}{|\lambda|^k}\right), \; k > 3. \tag{52}$$

Proof. By the aid of extremum principle (see [40, p. 10]) we find an a priori estimate (52), of solution of the problem $AG_{1\lambda}$.

Note that for the coefficients of the equation (44) in the domain D the well-known Agmon–Nirenberg–Protter ($A-N-P$) conditions (see [47, P. 53] or [47, Paragraph 2, Chap. 2] are not fulfilled, i.e. for equation (44) we have $c(x, y) = -\lambda^2 < 0$, instead $c(x, y) \ge 0$. Therefore, the extremum principle for hyperbolic equations fails immediately. Despite this, using some trick from [36] one can obtain an estimate for the function $\rho\,(x,\,y,\,\lambda)$, from which follows the uniqueness of the solution of the problem $\widetilde{AG}_{1\lambda}$.

We introduce a new unknown function $\tilde{\rho}\,(x,\,y,\,\lambda)$ by the formula

$$\rho\,(x,\,y,\,\lambda) = \exp\left\{\int\limits_{-\infty}^{\infty} g\,(x,\,\lambda)\,d\,x\right\}\cdot\tilde{\rho}(x,\,y,\,\lambda), \tag{53}$$

where $g\,(x,\,\lambda)$ is continuously differentiable and non-negative solution on $[0,\,1]$ of the differential equation

$$\frac{d\,g\,(x,\,\lambda)}{d\,x} + g^2(x,\,\lambda) - \lambda^2 = 0,\ \ 0\le x\le 1,\ \ \lambda\in\mathbb{R} \tag{54}$$

and function $\tilde{\rho}\,(x,\,y,\,\lambda)$ is solution of the equation

$$L[\tilde{\rho}] \equiv \tilde{\rho}_{yy} - \tilde{\rho}_{xx} - 2g\,(x,\,\lambda)\tilde{\rho}_x = 0,\ \ (x,\,y)\in D, \tag{55}$$

which coefficient in the domain D satisfies the conditions $A-N-P$.

The solution to the equation (54) with the required properties can be the function

$$g\,(x,\,\lambda) = |\lambda|\,\tanh(|\lambda|x). \tag{56}$$

We formulate the maximum principle in the form of the following lemma.

Lemma 1. *Let the function $\tilde{\rho}\,(x,\,y,\,\lambda)$ has properties:*

1). $\tilde{\rho}\,(x,\,y,\,\lambda)\in C\,(\overline{D})\cap C^2(D)$, $\tilde{\rho}_x(x,\,y,\,\lambda)\in C\,(D\cup\sigma_1)$, $\tilde{\rho}_y(x,\,y,\,\lambda)\in C\,(D\cup\sigma_0\cup\sigma_1)$*;*

2). *$\tilde{\rho}(x,\,y,\,\lambda)$ satisfies the equation (55) in domain D and inequality $L[\tilde{\rho}]\ge 0$;*

3). *Coefficients of the operator $L[\tilde{\rho}]$ satisfies the conditions $A-N-P$;*

4). *The difference $\tilde{\rho}_y(-y,\,y,\,\lambda)-\tilde{\rho}_x(-y,\,y,\,\lambda)\ge 0$, $-\frac{1}{2}\le y\le 0$ is a non-decreasing function of y.*

Then the positive maximum of function $\tilde{\rho}(x,\,y,\,\lambda)$ in $\overline{D}$ is reached on the segment $\overline{\sigma}_0$.

The lemma 1 is proved in exactly the same way as in Theorem [48, Theorem 2.4, P. 53]. Now, using this lemma 1, we will obtain a priori estimate (52). For this purpose, we define the function $\mu\,(x,\,y,\,\lambda)$ as:

$$\mu\,(x,\,y,\,\lambda) = -\tilde{\rho}(x,\,y,\,\lambda) + M, \tag{57}$$

where M is non-negative constant and $\tilde{\rho}(x,\,y,\,\lambda)$ is regular solution of the equation (55).

Note that

$$L(\mu) \equiv \tilde{\rho}_{yy} - \tilde{\rho}_{xx} - 2g\,(x,\,\lambda)\tilde{\rho}_x = 0.$$

Further, we choose M such, that the function $\mu\,(x,\,y,\,\lambda)$ was non-decreasing with respect to y on characteristic σ_1. For this, it is enough to put

$$M = \max_{\sigma_1}\left|\tilde{\rho}_y(-y,\,y,\,\lambda) - \tilde{\rho}_x(-y,\,y,\,\lambda)\right|. \tag{58}$$

Thus, the function $\mu\,(x,\,y,\,\lambda)$ satisfies all conditions of the lemma 1. So, we have

$$|\,\mu\,(x,\,y,\,\lambda)\,| \le \max_{\overline{\sigma}_0}|\,\mu\,(x,\,y,\,\lambda)\,|$$

or by virtue of (57) we obtain

$$|\tilde{\rho}(x,y,\lambda)| \le \max_{\overline{\sigma}_0}|\tilde{\rho}(x,y,\lambda)| + 2\,M. \tag{59}$$

We put $\overline{g}(x,\lambda) = \exp\{-\int g(x,\lambda)\,dx\}$. Taking (56) into account we derive $\overline{g}(x,\lambda) = \frac{1}{\cosh(|\lambda|x)}$. Since $\tilde{\rho}(x,y,\lambda) = \overline{g}(x,\lambda)\cdot\rho(x,y,\lambda)$, then, passing in (58) to function $\rho(x,y,\lambda)$, we find

$$M = \max_{\sigma_1} |\overline{g}(x,\lambda)(\rho_y - \rho_x) - \overline{g}'(x,\lambda)\rho|\,.$$

Hence taking (46) into account we obtain the following estimate

$$M \le \big\{ \max_{\overline{\sigma}_1} \overline{g}(x,\lambda)|\, f_1'(x,\lambda)\,| + |\,\lambda\,| \max_{\overline{\sigma}_1} \overline{g}(x,\lambda)|\, f_1(x,\lambda)\,|\big\}. \tag{60}$$

Taking into account $\max_{\overline{\sigma}_1} \overline{g}(x,\lambda) = 1$, from the estimate (59) by virtue of (53), (56) and (60), we obtain the solution of the problem $\widetilde{AG}_{1\lambda}$:

$$|\,\rho(x,y,\lambda)\,| \le \cosh(|\,\lambda\,|\,x)\big\{ \max_{\overline{\sigma}_0} |\, f_0(x,\lambda)\,| + \max_{\overline{\sigma}_1} |\, f_1'(x,\lambda)\,| + \tag{61}$$
$$+|\,\lambda\,| \max_{\overline{\sigma}_1} |\, f_1(x,\lambda)\,|\big\}.$$

By virtue of (37) and (48), from (13), (43) and (61) for the large values of parameter $|\lambda|$ we obtain the following estimate for solution of the problem $AG_{1\lambda}$:

$$|\,u(x,y,\lambda)\,| \le |\,z(x,y,\lambda)\,| + |\,\omega(x,\lambda)\,| + |\,w(x,\lambda)\,| \le |\,\omega(x,\lambda)\,| + \tag{62}$$
$$+|\,w(x,\lambda)\,| + \cosh|\,\lambda\,|\big\{ \max_{\overline{\sigma}_0} |\, f_0(x,\lambda)\,| + \max_{\overline{\sigma}_1} |\, f_1'(x,\lambda)\,| + |\,\lambda\,| \max_{\overline{\sigma}_1} |\, f_1(x,\lambda)\,|\big\}.$$

Thus, by virtue of (49)–(51) from (62) we obtain an estimate

$$|\,u(x,y,\lambda)\,| \le \frac{c}{|\lambda|^k}, \quad k > 3, \quad c = \text{const} > 0.$$

The theorem 2 is proved.

Remark 2. From the estimate (62) implies also the uniqueness of solution of the problem $AG_{1\lambda}$ and continuous dependence of the solution on the given functions for any fixed λ.

3.3. Existence and uniqueness of the solution to the problem AG_1.

Theorem 3. *Let be fulfilled the condition* (5) *and* $\Phi(x,z) \equiv 0, \forall x \in [0,1]$, $\Psi_1(x,z) \equiv \Psi_2(x,z) \equiv 0$, $\forall x \in \left[0, \frac{1}{2}\right]$, $z \in \mathbb{R}$, *then in the domain* Ω *the solution of the problem* AG_1 *for the equation* (1) *is unique.*

Proof. By virtue of the conditions of the theorem and (5), from the formula (10) we obtain (39). Taking into account (39) from (37), (48) and given data of problem $\widetilde{AG}_{1\lambda}$ we obtain

$$\omega(y,\lambda) \equiv w(x,\lambda) \equiv f_0(x,\lambda) \equiv f_1(x,\lambda) \equiv f_2(x,\lambda) \equiv 0. \tag{63}$$

Taking (63) into account from the estimate (62) implies that $u(x,y,\lambda) \equiv 0$, $(x,y) \in \overline{D}$. Hence, taking into account (4) from (6) we obtain $U(x,y,z) \equiv 0$, $(x,y) \in \overline{\Omega}$. Thus, the uniqueness of the solution to the problem AG_1 is proved for the equation (1). Theorem 3 is proved.

Theorem 4. *We suppose the the solution* $u(x,y,\lambda)$ *of the plane problem* $AG_{1\lambda}$ *for the equation* (7) *exists and presents as*

$$u(x,y;\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} U(x,y,z)\cdot e^{i\lambda z} dz, \tag{64}$$

and for large values of $|\lambda|$ *has an estimate* (52)*. Then in the domain* Ω *the solution of the problem* AG_1 *for the equation* (1) *exists and this solution determines from the formula* (6)*. In addition, the function* $u(x, y, \lambda)$ *in* (6) *defines from* (13)*. The functions* $\upsilon(x, y, \lambda)$ *and* $w(x, \lambda)$ *define from* (28) *and* (37)*, respectively.*

Proof. Let function $U(x, y, z)$ be a solution of the problem AG_1 for the equation (1) in domain Ω. Then the function in (64) is unique solution of the problem $AG_{1\lambda}$ for the equation (7). This solution (64) satisfies conditions (8) and (9). In this case, $\varphi(x, \lambda)$, $\psi_1(x, \lambda)$, $\psi_2(x, \lambda)$ define from (10).

Thus, the problem AG_1 for the equation (1) is reduced to the problem $AG_{1\lambda}$, for which the unique solvability was studied above.

Inversely, if $u(x, y, \lambda)$ is a solution of plane problem $AG_{1\lambda}$ for the equation (7) and for large values of $|\lambda|$ has an estimate (52), then the solution of the problem AG_1 for the equation (1) we obtain by the aid of Fourier inverse transform to (6).

Taking into account the estimate (52) for the function $u(x, y, \lambda)$, we can for the functions $\Phi(x, z)$, $\Psi_1(x, z)$ and $\Psi_2(x, z)$ impose conditions ensuring the existence of the integral (6). Let functions $\Phi(x, z)$, $\Psi_1(x, z)$ and $\Psi_2(x, z)$ be such that for functions $\omega(y, \lambda)$, $w(x, \lambda)$, $f_0(x, \lambda)$, $f_1(x, \lambda)$ and $f_2(x, \lambda)$ hold estimates (49)–(51).

Note the estimate (52) ensures the existence of the integral (6), which is solution of the problem AG_1. This estimate (52) implies from the estimate (62).

In addition, using the Fourier transforms, one can prove the validity of the estimate (4), according to which the functions $U(x, y, z)$, $U_x(x, y, z)$, $U_y(x, y, z)$ and $U_z(x, y, z)$ trend to zero as $|z| \to \infty$ and all corresponding improper integrals exist (see [37]).

Thus, the solution of the problem AG_1 for the equation (1) with the conditions (2)-(4) exists and is found by the formula (6). The theorem 4 is proved.

Note that Theorem 4 implies the equivalence of the problems AG_1 and $AG_{1\lambda}$.

Remark 3. Similarly, with the above method, one can investigate the unique solvability of the following problem:

Problem AG_2. Find a regular in domain Ω solution $U(x, y, z)$ of the equation (1), satisfying all the conditions of the problem AG_1, except (2), which is replaced by the condition

$$\lim_{y \to -0} U(x, y, z) = F(x, z), \ 0 \le x \le 1, \ z \in \mathbb{R},$$

where $F(x, z)$ is given enough smooth function, and there $F(0, z) = \Psi_1(0, z)$, $\lim\limits_{|z| \to \infty} F(x, z) = 0$.

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