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4TH INTERNATIONAL CONFERENCE GROUPS AND QUANDLES IN LOW-DIMENSIONAL TOPOLOGY, TOMSK, JULY 5 – 8, 2021

N.V. ABROSIMOV, V.G. GORBOUNOV, S.K. NECHAEV, M. SINGH, A.YU. VESNIN

Abstract. In this article, abstracts of talks of the 4th International Conference Groups and quandles in low-dimensional topology held in Tomsk, July 5 — 8, 2021, are presented.

The 4th International Conference Groups and quandles in low-dimensional topology held in Tomsk from July 5 to 8, 2021.

A typical feature of the modern low-dimensional topology is the active use of such rich algebraic structures as groups, semigroups, quandles (distributive groupoids), and algebras. This allows us to construct strong invariants of knots and links, three-dimensional manifolds, braids and virtual braids. At the conference, new results on these topological objects obtained using algebraic methods were presented and discussed. The Conference was co-organized by Regional Scientific and Educational Mathematical Center of Tomsk State University, Faculty of Mathematics of Higher School of Economics, and Interdisciplinary Scientific Center J.-V. Poncelet. Members of the program committee were as follows: V.G. Gorbounov, S.K. Nechaev, M. Singh and A.Yu. Vesnin.The Conference homepage is http://gqlt.rmc.math.tsu.ru.

More than 30 experts on geometry and topology, knot theory, qroups and quandles were participated in the conference. The conference program consisted

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of plenary and contributed talks. The talks were made by well-known experts from Moscow, Novosibirsk, Tomsk, and also from Mohali (India), Osaka (Japan), Pusan (South Korea), Ropar (India), and Shijiazhuang (China). About 15 young scientists, graduate and undergraduate students participated in the conference. Most of them gave contributed talks. Unfortunately, because of COVI-19 talks by A. Egorov, S. Nechaev, and V. Yakhin were canceled.

Program

July 5

10:00 – 10:30 Vassily Gorbounov (*Moscow*), The electrical Lie algebra

10:40 – 11:10 Ivan Dynnikov (*Moscow*), Markov's theorem for links in the projective three-space

11:30 – 12:00 Jie Wu (*Shijiazhuang, China*), Algebraic topology methods in knot theory

12:10 – 12:40 Mahender Singh (*Mohali, India*), A Wells type exact sequence for solutions of the Yang–Baxter equation

12:50 – 13:20 Manpreet Singh (*Mohali, India*), Some properties of link quandles

15:00 – 15:20 Vadim Leshkov (*Novosibirsk*), Category of weighted 2-complexes and generalized Coxeter groups

15:30 – 15:50 Bogdan Chuzhinov (*Novosibirsk*), Representations of flat virtual braids

16:00 – 16:20 Maxim Ivanov (*Novosibirsk*), Connected sums of virtual knots

July 6

10:00 – 10:30 Dmitri Talalaev (*Moscow*), Invariants of 2-knots, quandles and Zamolodchikov equation

10:40 – 11:10 Valeriy Bardakov (*Novosibirsk*), Tetrahedron equation and related algebraic systems

11:30 – 12:00 Sang Youl Lee (*Pusan, Korea*), Ternary quasigroups and invariants of classical links and surface-links

12:10 – 12:40 Madeti Prabhakar (*Ropar, India*), The Gordian complex of theta-curves

12:50 – 13:20 Neha Nanda (*Mohali, India*), Alexander and Markov theorems for virtual doodles

15:00 – 15:20 Anton Kazakov (*Moskow*), Tutte polynomials for vertex-weighted graphs and group cohomology

15:30 – 15:50 Mikhai Chernavskikh (*Moscow*), Algorithm for constructing a rectangular diagram of the Seifert surface

16:00 – 16:20 Valeriy Yakhin (*Novosibirsk*), Idempotents in quande rings and representations of flat virtual braids

July 7

10:00 – 10:30 Andrei Vesnin (*Tomsk*), Right-angled knots and links

10:40 – 11:10 Mikhail Neschadim (*Novosibirsk*), Skew-braces constructed on free groups

11:30 – 12:00 Seichi Kamada (*Osaka, Japan*), Commutator identities related to curves on a surface

12:10 – 12:40 Krishnendu Gongopadhyay (*Mohali, India*), Reversibility of Hermitian isometries

12:50 – 13:20 Sergei Nechaev (*Moscow*), Fractal Brownian motion meets topology: statistical and topological properties of condensed macromolecules

15:00 – 15:20 Nikolay Abrosimov (*Novosibirsk*), Geometrical properties and volume of hyperbolic cone manifold m125(m/0, n/0)

15:30 – 15:50 Bao Vuong (*Novosibirsk*), Antiprisms in three-dimensional spaces of constant curvature

16:00 – 16:20 Andrei Egorov (*Novosibirsk*), Right-angled hyperbolic polyhedra

July 8

10:00 – 10:30 Timur Nasybullov (*Novosibirsk*), Verbal quandles with one parameter

10:40 – 11:10 Denis Fedoseev (*Moscow*), Quandle-like structures and presentations of G_n^3-related groups

11:30 – 12:00 Boris Bychkov (*Moscow*), Topological recursion for maps and fully simple maps

Contents

Abstracts

Geometrical properties and volume of hyperbolic cone manifold $m125(m/0, n/0)$

Nikolay Abrosimov (Novosibirsk)

We investigate geometrical properties of the 3-dimensional hyperbolic manifold $m125$ homeomorphic to the complement of the pretzel link of type $(?2, 3, 8)$, which is also known as Whitehead sister link WS. In spite of $m125$ and WS are homeomorphic, they have different peripheral systems of longitudes and meridians. In particular, the orbifold Dehn surgery $m125(m/0, n/0)$ on the first manifold is equivalent to Dehn surgery $WS(3m/-m, n/0)$ on the second one.

We obtain trigonometric identities relating the complex lengths of singular geodesics and cone angles of cone manifold $m125(m/0, n/0)$. They form Sine rule and Tangent rule similar to those in the classical trigonometry. Then these identities and the Schläfli formula will be used to produce exact integral formula for volume of the cone manifold under investigation.

This is our joint work with Alexander Mednykh.

Tetrahedron equation and related algebraic systems

Valeriy Bardakov (Novosibirsk)

In my talk, I will recall some known facts about the Yang–Baxter equation and algebraic systems associated with it: groups, quandles, biquandles. Then I will tell about the tetrahedron equations, I will define the groups associated with it. Further, ternoid will be defined, that is an algebraic system with 3-ary algebraic operations. The connection of ternoids with solutions of the tetrahedron equation will be described. In particular, solutions of the tetrahedron equation will be constructed on an arbitrary Abelian group.

Topological recursion for maps and fully simple maps

Boris Bychkov (Novosibirsk)

I will talk about the enumeration of maps: we count weighted numbers of ways to combinatorially glue the surface of genus g from M ordered polygons along their sides, such that the numbers of sides of the first n polygons are fixed. Using the fact that the generating function for maps satisfies some universal recursion, called topological recursion, I will show that the generating function for fully simple maps, that is maps with an important additional requirement, also satisfies topological recursion. The generating functions for maps and fully simple maps are modifications of the Kadomtsev–Petviashvili tau function of hypergeometric type, which is known to be the generating function for weighted double Hurwitz numbers and satisfies topological recursion.

This talk is based on the recent joint works with P. Dunin-Barkowski, M. Kazarian and S. Shadrin (arXiv:2106.08368, arXiv:2012.14723, arXiv:2012.14723).

Algorithm for constructing a rectangular diagram of the Seifert surface

Mikhai Chernavskikh (Moscow)

We present an algorithm for constructing a rectangular diagram of a Seifert surface for any link, represented by a rectangular diagram. We estimated the complexity of a resulting diagram of a surface.

Representations of flat virtual braids

Bogdan Chuzhinov (Novosibirsk)

Invariants arising from representations of braid groups have an important role in the classical knot theory and its generalizations. L. Kaufman introduced virtual knots and links, which have been intensively studied in recent years. There is an algorithm for constructing an invariant of virtual links from some representation of the virtual braids group. In the talk we will propose a new representation of the flat virtual braids group by automorphisms of the free group and also describe the kernel of this representation.

Markov's theorem for links in the projective three-space

Ivan Dynnikov (Moscow)

A few years ago, in a private communication, Oleg Viro proposed, without proof, an analogue of Markov's theorem and Alexander's lemma for links in the three-dimensional projective space. Any oriented link in the projective space can be presented as the closure of a braid, and two braids representing equivalent links are related by a sequence of moves that resemble the classical Markov moves. I will present a simple proof of these facts using rectangular diagrams. This is a joint work with Stepan Orevkov.

The work is supported by the Russian Science Foundation under grant no. 19-11-00151.

Right-angled hyperbolic polyhedra

Andrei Egorov (Novosibirsk)

In three-dimensional Lobachevsky space consider right-angled polyhedra. We will discuss the theorems of Pogorelov and Andreev, which give necessary and sufficient conditions for the existence of such polyhedra in the Lobachevsky space. We also will consider the bounds on the volume of these polyhedra, fullerenes and some other things.

Quandle-like structures and presentations of G_n^3-related groups

Denis Fedoseev (Moscow)

Groups G_n^k were introduced by Vassily Manturov as a method to capture and describe algebraically the bahaviour of dynamical systems of moving points on a configurational space, satisfying certain natural conditions. On the other hand, those groups are related to non-Reidemeister knot theories and non-Artin braid theories.

In the present talk I will discuss a partial case of the groups G_n^3 and their relatives. Presentations of these groups are useful to tackle actual problems of distinguishing

between dynamical systems. The presentations may be constructed in a variety of ways, including the use of quandle-like structures.

The talk is based on a joint work with V.O. Manturov and S. Kim.

Reversibility of Hermitian isometries

Krishnendu Gongopadhyay (Mohali, India)

An element g in a group G is called reversible (or real) if it is conjugate to $g-1$ in G, i.e. there is an h in G such that $g-1 = hgh-1$. The element g in G is called strongly reversible (or strongly real) if g is a product of two involutions (i.e. order one or two elements) in G. In this talk, we shall discuss reversibility in the isometry group of a Hermitian space.

The talk is based on joint work with Tejbir Lohan.

The electrical Lie algebra

Vassily Gorbounov (Moscow)

We will discuss the electrical Lie algebra introduce by Lam and Pyliavskyy for studying the the electrical networks. This algebra has many interesting features, in particular, it is closely related to the Temperley–Lieb algebra, the six vertex quantum spin chain, and hence to the representation theory of the Yangian.

Connected sums of virtual knots

Maxim Ivanov (Novosibirsk)

It is known that connected sum of two virtual knots is not uniquely determined and depends on knot diagrams and a choice of splicing points. In this talk we discuss how flat virtual knots may be used to distinguish different connected sums.

Commutator identities related to curves on a surface

Seichi Kamada (Osaka, Japan)

R. Fenn and P. Taylor introduced the notion of a doodle and showed that a doodle diagram, which is a collection of curves on the 2-sphere, induces an identity among commutators in a free group. This idea was observed more closely in a recent paper by A. Bartholomew, R. Fenn, N. Kamada and S. Kamada, arXiv:2006.08871v2. On the other hand, the notion of a doodle on the sphere was extended to any surface with positive genus by the same authors, Bartholomew, Fenn, Kamada and Kamada [JKTR 27 (2018)]. We recall these and then discuss commutator identities related to a doodle on a surface. It is a part of our ongoing project.

Tutte polynomials for vertex-weighted graphs and group cohomology

Anton Kazakov (Moscow)

In the focus of my talk is a generalization of the Tutte polynomial for vertex-weighted graphs, for which the coefficients of the "contraction-deletion" relation depend nontrivially on the vertex weights. We demonstrate that the corresponding coefficient relation coincides with the symmetric 2-cocycle relation in the group cohomology. We show that our construction is a natural generalization of the symmetrized chromatic Stanley polynomial and obtain a representation of

our polynomial by summing over subgraphs. Finally, we demonstrate that our polynomial is an reach source for constructing 4-invariants of graphs, which are very important in the knot theory.

The report is based on a joint article B.S. Bychkov, A.A. Kazakov, and D.V. Talalaev, The Tutte polynomial of graphs with weighted vertices and group cohomology.

Ternary quasigroups and invariants of classical links and surface-links

Sang Youl Lee (Pusan, South Korea)

Constructions of invariants for classical links in 3-space and surface-links in 4-space in terms of coloring their diagrams by algebraic systems such as groups, quandles, biquandles and more have a long history. A ternary quasigroup is a set X equipped with a ternary operation $T : X^3 \to X$ such that for a quadruple (x_1, x_2, x_3, x_0) of elements of X satisfying $T(x_1, x_2, x_3) = x_0$, specification of any three elements of the quadruple determines the remaining one uniquely. Recently, colorings of complement regions of a link diagram by algebraic systems now known as (knot-theoretic) ternary quasigroups were introduced and used to define invariants for oriented classical links and surface-links by M. Niebrzydowski. In this talk, I would like to review Niebrzydowski's construction, and introduce a way of obtaining invariants for unoriented classical links and non-orientable surface-links by using special vertical ternary quasigroups.

Category of weighted 2-complexes and generalized Coxeter groups

Vadim Leshkov (Novosibirsk)

We will define the category of weighted 2-complexes, which has initial and final objects, as well as a product and a coproduct. A functor will be constructed from the category of weighted 2-complexes to the category of groups. The image of this functor is generalized Coxeter groups — a class of groups containing Coxeter groups, as well as a number of other well-known groups. Moreover, a product of 2-complexes corresponds to a direct product of groups, and a coproduct of 2-complexes corresponds to a free product of groups.

Alexander and Markov theorems for virtual doodles

Neha Nanda (Mohali, India)

Study of certain isotopy classes of a finite collection of immersed circles without triple or higher intersections on closed oriented surfaces can be thought of as a planar analogue of virtual knot theory where the genus zero case corresponds to classical knot theory. Alexander and Markov theorems for the genus zero case are known, the role of groups is played by twin groups, a class of right-angled Coxeter groups with only far commutativity relations. In the talk, Alexander and Markov theorems for higher genus case, where the role of groups is played by a new class of groups called virtual twin groups, will be discussed. This work is in collaboration with Mahender Singh.

Verbal quandles with one parameter

Timur Nasybullov (Novosibirsk)

A quandle is an algebraic structure whose axioms are derived from the Reidemeister moves on oriented link diagrams. They were first introduced by Joyce and Matveev as an invariant for knots in the three-sphere. The knot quandle is a very strong invariant, therefore, it is logical to assume that this invariant is practically impossible to use. Indeed, in order to understand if two knots are equivalent or not it is necessary to understand if two knot quandles are isomorphic or not, while the isomorphism problem for quandles is known to be as difficult as it gets in the sense of Borel reducibility (A. Brooke-Taylor, S. Miller, 2020).

Sometimes homomorphisms from knot quandles to simpler quandles provide useful information that helps determine whether two knot quandles are isomorphic. This potential utility leads to the necessity of studying some special classes of quandles, which are not necessarily knot quandles. A lot of quandles can be constructed from groups. In particular, conjugation quandles, core quandles, Takasaki quandles, (G, A)-quandles are all constructed from groups. Another example of quandles defined on a group is a verbal quandle, i. e. a quandle $(G, *)$, where G is a group, and $a * b$ is a word in the free group in terms of a, b. Verbal quandles are classified in [V. Bardakov, T. Nasybullov, M. Singh, General constructions of biquandles and their symmetries, arXiv:1908.08301]. A verbal quandle with one parameter is a quandle $(G, *)$, where G is a group, and $a * b$ is a word in the free group in terms of a, b, c, where c is a fixed element of G. During the talk we will provide a classification of verbal quandles with one parameter and then discuss some applications of such quandles.

Fractal Brownian motion meets topology: Statistical and Topological Properties of Condensed Macromolecules

Sergei Nechaev (Moscow)

We investigate statistical and topological properties of fractal random walks with short-range volume interactions. Attention is paid to the statistical properties of collapsed conformations with fractal dimension $Df \geq 2$ in three-dimensional space, which are analyzed both numerically and by mean-field methods. It is known that compressed polymer rings without knots form a compact hierarchical "fractal"globule (FG) with fractal dimension $Df = 3$ at large scales. The question is asked: is the opposite true, i.e. is it true that a fractal Brownian motion with fractal dimension $Df = 3$ is significantly less knotted than a compact Browinan motion with $Df = 2$? If this is so, then an unknotted FG can be obtained by forgetting about topology and generating an ensemble of paths with fractal dimension $Df = 3$, which will greatly simplify the problem of generating compact conformations. We use Monte Carlo simulations to prepare an equilibrium ensemble of inflated chains with different fractal dimensions. We show that as Df increases, typical globular (compact) conformations become less knotted. The distributions of knot complexity for different fractal dimensions indicate a close relationship between the statistical and topological properties of fractal pathways.

Skew-braces constructed on free groups

Mikhail Neschadim (Novosibirsk)

Left skew-brace $(G, \cdot, \circ)$ is the algebraic system with two operations where $(G, \cdot)$ and $(G, \circ)$ are groups and the compatibility condition $a \circ (bc) = (a \circ b)a - 1(a \circ c)$ holds for all $a, b, c \in G$, where $a - 1$ denotes the inverse of a with respect to the group $(G, \cdot)$. Using technique of [1] we construct left skew-braces on free groups $(G, \cdot)$. For some cases we obtain left skew-braces $(G, \cdot, \circ)$ such that $(G, \cdot)$ and $(G, \circ)$ are free groups but skew-braces $(G, \cdot, \circ)$ are not trivial.

References

[1] V.G. Bardakov, M.V. Neshchadim, M.K. Yadav, On λ-homomorphic skew braces, arXiv:2004.05555

The Gordian complex of theta-curves

Madeti Prabhakar (Ropar, India)

In this talk, I will discuss the Gordian metric on the set of all theta curves and give a lower bound of this function. I also define the Gordian complex of theta-curves, which is a simplicial complex whose vertices consist of all theta curves in the 3-dimensional Euclidean space. I will conclude by showing that for any given theta curve, there exists an infinite family of theta curves containing the given theta curve such that the Gordian distance between any two distinct members is equal to one.

This is joint work with my Ph.D. student Mr. Sahil Joshi.

A Wells type exact sequence for solutions of the Yang–Baxter equation

Mahender Singh (Mohali, India)

Cycle sets are known to give non-degenerate unitary solutions of the Yang–Baxter equation and linear cycle sets are enriched versions of these algebraic systems. The paper explores the recently developed cohomology and extension theory for linear cycle sets. We derive a four term exact sequence relating 1-cocycles, second cohomology and certain groups of automorphisms arising from central extensions of linear cycle sets. We also compare the exact sequence for linear cycle sets with that for their underlying abelian groups known due to Wells. This is a recent joint work with Valeriy Bardakov.

Some properties of link quandles

Manpreet Singh (Mohali, India)

In this talk, we discuss residual finiteness and orderability of link quandles. It is known that all link groups are residually finite and left-orderable. We prove that all link quandles are residually finite, and deduce that the word problem is solvable in link quandles. We show that knot quandles of many fibered prime knots are right-orderable. Considering the fact that all link groups are left-orderable, it is reasonable to speculate that link quandles are left(right)-orderable. In contrast, we prove that the knot quandle of the trefoil knot is neither left nor right-orderable. We also prove that link quandles of many non-trivial torus links are not right-orderable. As an application, we recover a result of Perron and Rolfsen, which states that the

knot group of a non-trivial torus knot is not bi-orderable. In this talk, we discuss residual finiteness and orderability of link quandles. It is known that all link groups are residually finite and left-orderable. We prove that all link quandles are residually finite, and deduce that the word problem is solvable in link quandles. We show that knot quandles of many fibered prime knots are right-orderable. Considering the fact that all link groups are left-orderable, it is reasonable to speculate that link quandles are left(right)-orderable. In contrast, we prove that the knot quandle of the trefoil knot is neither left nor right-orderable. We also prove that link quandles of many non-trivial torus links are not right-orderable. As an application, we recover a result of Perron and Rolfsen, which states that the knot group of a non-trivial torus knot is not bi-orderable.

Invariants of 2-knots, quandles and Zamolodchikov equation

Dmitri Talalaev (Moscow)

I will talk about the problem of constructing invariants of 2-knots and well-known results in this area: the Carter-Saito invariant given by the 3-cocycle of the corresponding quandle cohomology, as well as quasi-invariants of 2-knots constructed from special solutions of the Zamolodchikov tetrahedron equation.

This problem is of special interest in the context of quantum loop gravity, in particular, in the theory of spin foams. In the talk, I will pay special attention to the question: which partition functions of 2-knot diagrams can be generalized to observables on spin foams, as well as to how the above constructions are related to quantum topological field theories.

Right-angled knots and links

Andrei Vesnin (Tomsk)

A hyperbolic link is said to be right-angled if its complement can be decomposed into ideal hyperbolic polyhedra with all dihedral angles equal to $\pi/2$. Among known examples of right-angled links are 2-component Whitehead link and 3-component Borromean rings link. Complements of these links can be decomposed into one and two right-angled octahedra, respectively. In [1] there was stated a conjecture that there are no right-angled hyperbolic knots. We will demonstrate that computation of volumes of ideal right-angled hyperbolic polyhedra [2] support the conjecture for knot with small number of crossings.

References

[1] A. Champanerkar, I. Kofman, J. Purcell, Right-angled polyhedra and alternating links, arXiv:1910.13131.

[2] A. Vesnin, A. Egorov, Ideal right-angled polyhedra in Lobachevsky space, Chebyshevskii Sbornik 21 (2020), 65–83.

Antiprisms in three-dimensional spaces of constant curvature

Bao Vuong (Novosibirsk)

This is our joint work with Nikolay Abrosimov. Consider antiprisms in Euclidean, hyperbolic and spherical space. An antiprism is a convex polyhedron with a symmetry group S_{2n} (according to the Schönflis classification), which is generated by a mirror-rotational symmetry of the order of $2n$, that is, a composition of rotation through an angle π/n and reflection relative to a plane perpendicular to the axis of rotation.

We have established necessary and sufficient conditions for the existence of antiprisms in $\mathbb{E}^3, \mathbb{H}^3$ and $\mathbb{S}^3$. The relations between dihedral angles and edge lengths are found in the form of cosine theorems. Explicit integral formulas are obtained for the volumes of antiprisms in each of the three geometries.

Algebraic topology methods in knot theory

Jie Wu (Shijiazhuang, China)

Algebraic topology methods have been largely used in knot theory. In this talk, we will display some of algebraic topology methods used in knot theory. After giving a brief review on knot spaces studied by V.A. Vassiliev, F. Cohen, Kohno and etc., we will talk how to introduce knot invariants given by simplicial groups. We will also talk how to use homotopy theory to determine certain intersecting subgroups of Brunnian link groups.

Idempotents in quande rings and representations of flat virtual braids

Valeriy Yakhin (Novosibirsk)

Quandles and braid groups are algebraic systems used to construct knot and link invariants. They provide a powerful tool for determining equivalence, but their use is often hampered by the complex internal structure of these objects. Therefore, it is of great interest to study quandles, quandle rings, and braid groups from an algebraic point of view.

The question of describing nontrivial idempotents in quandle rings was formulated in [1]. In the talk, the sets of idempotents for quandle rings over some types of quandles are considered and described.

The flat virtual braid group is a generalization of the flat braid group. It differs from the latter, among other things, by the presence of two forbidden relationships. In the talk, we considered a representation of the group of flat virtual braids that does not preserve these forbidden relations. The corresponding problem was formulated in [2].

References

[1] Valeriy G. Bardakov, Inder Bir S. Passi, Mahender Singh, Quandle rings, J. Algebra Appl. 18 (2019), no. 8, 1950157, 23 pp.

[2] R. Fenn, D. Ilyutko, L. Kauffman, V. Manturov, Unsolved problems in virtual knot theory and combinatorial knot theory, arXiv: 1409.2823

Nikolay Vladimirovich Abrosimov
Sobolev Institute of Mathematics,
4, Koptyuga ave.,
Novosibirsk, 630090, Russia

Novosibirsk State University,
1, Pirogova str.,
Novosibirsk, 630090, Russia
Email address: abrosimov@math.nsc.ru

Vassily Gennadievich Gorbounov
National Research University "Higher School of Economics",
20, Myasnitskaya str.,
Moscow, 101978, Russia
Email address: vgorb10@gmail.com

Sergei Konstantinovich Nechaev
Interdisciplinary Scientific Center J.-V. Poncelet, Russia
11, Bolshoy Vlasyevskiy Pereulok,
Moscow, 119002, Russia
Email address: sergei.nechaev@gmail.com

Mahender Singh
Indian Institute of Science Education and Research Mohali,
Knowledge city, Sector 81, SAS Nagar, Manauli PO 140306
SAS Nagar, Manauli PO 140306, India
Email address: mahender@iisermohali.ac.in

Andrey Yurievich Vesnin
Tomsk State University,
36, Lenina ave.,
Tomsk, 634050, Russia
Email address: vesnin@math.nsc.ru