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THE CONFERENCE "DYNAMICS IN SIBERIA", NOVOSIBIRSK, MARCH 1 – 6, 2021

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Abstract. In this article abstracts of talks of the Conference "Dynamics in Siberia" held in Sobolev Institute of Mathematics, March 1 – 6, 2021 are presented.

The conference was held in Sobolev Institute of Mathematics SB RAS (Novosibirsk) from March 1 to 6, 2021, supported by Mathematical Center in Akademgorodok under agreement No. 075 − 15 − 2019 − 1675 with the Ministry of Science and Higher Education of the Russian Federation. Members of the program committee were as follows: I.A. Dynnikov, A.E. Mironov, I.A. Taimanov, V.A. Timorin and A.Yu. Vesnin.

More than 70 experts on dynamical systems, mathematical physics, geometry and topology participated in the conference. The conference program consisted of 40–minutes plenary talks and 25–minutes short talks. The talks were made by well–known experts from Dubna, Moscow, Nizhny Novgorod, Novosibirsk, Rostov–on–Don, St. Petersburg, Saransk, Tomsk, Ufa, Vladivostok and others. About 45 young scientists, graduate and undergraduate students participated in the conference. Most of them gave short talks.

The conference "Dynamics in Siberia" has been held in Novosibirsk annually since 2016. Information about previous conferences and abstracts of presented talks can be found in [1]–[5].

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Program (Plenary talks)

March 1

10:00 – 10:40	V. Kozlov (*Moscow*). Weak ergodicity and non-equilibrium statistical mechanics.
10:45 – 11:25	A. Shafarevich. (*Moscow*). Reflection of Lagrangian manifolds and of Maslov complex germs in the Cauchy problem for the Schrödinger equation with a delta potential localized on a surface of codimension 1.
11:45 – 12:25	O. Pochinka. (*Nizhny Novgorod*). On the number of the classes of topological conjugacy of Pixton diffeomorphisms.

March 2

10:00 – 10:40	D. Treschev (*Moscow*). Volume preserving diffeomorphisms as Poincare maps for volume preserving flows.
10:45 – 11:25	A. Tsiganov. (*St. Petersburg*). Divisor arithmetic, integrable and superintegrable systems.
11:45 – 12:25	N. Kuznetsov. (*St. Petersburg*). Global stability boundary, hidden oscillations, and non-equilibrium dynamics in the models with cylindrical phase space: PLL and Sommerfeld effect.

March 3

10:00 – 10:40	S. Sokolov. (*Moscow*). Dynamics of a circular cylinder and two vortex filaments in an ideal fluid.
10:45 – 11:25	V. Timorin. (*Moscow*). A model for the cubic connectedness locus.
11:45 – 12:25	D. Orlov. (*Moscow*). Geometric realizations of algebraic objects and finite-dimensional algebras.

March 4

10:00 – 10:40	S. Bolotin. (*Moscow*). Separatrix map for slow-fast Hamiltonian systems.
10:45 – 11:25	S. Dobrokhotov. (*Moscow*). Real semiclassical approximation for the asymptotics with complex-valued phases and asymptotics of Hermitian type orthogonal polynomials.
11:45 – 12:25	A. Glutsyuk. (*Moscow*). Density of thin film billiard reflection pseudogroup in Hamiltonian symplectomorphism pseudogroup.

March 5

9:00 – 09:40	I. Vyugin. (*Moscow*). On Some Applications of Differential Equations to Problems in Additive Number Theory.
09:45 – 10:25	A. Buryak. (*Moscow*). A noncommutative generalization of Witten's conjecture.

Plenary talks

Short talks

Plenary talks

A noncommutative generalization of Witten's conjecture

A. Buryak (Moscow)

The classical Witten conjecture says that the generating series of integrals over the moduli spaces of curves of monomials in the psi-classes is a solution of the Korteweg - de Vries (KdV) hierarchy. Together with Paolo Rossi, we present the following generalization of Witten's conjecture. On one side, let us deform Witten's generating series by inserting in the integrals certain naturally defined cohomology classes, the so-called double ramification cycles. It turns out that the resulting generating series is conjecturally a solution of a noncommutative KdV hierarchy, where one spatial variable is replaced by two spatial variables and the usual multiplication of functions is replaced by the noncommutative Moyal multiplication in the space of functions of two variables.

Real semiclassical approximation for the asymptotics with complex-valued phases and asymptotics of Hermitian type orthogonal polynomials.

S. Dobrokhotov (Moscow)
Joint work with A. Aptekarev (Moscow),
D. Tulyakov (Moscow), A. Tsvetkova (Moscow)

The Hermitian type orthogonal polynomials $H_(n_1, n_2)(z, a)$ are determined by the pair of recurrence relations for $H_{n_1+1,n_2}(z, a)$, $H_{n_1,n_2+1}(z, a)$, $H_{n_1,n_2-1}(z, a)$, $H_{n_1,n_2-1}(z, a)$, $H_{n_1,n_2}(z, a)$. We obtain a uniform asymptotics of diagonal polynomials $H_{n,n}(z, a)(z, a)$ in the form of an Airy function for $n \gg 1$, which is a far-reaching generalization of the Plancherel-Rotach asymptotic formulas for Hermitian polynomials. We discuss one of the possible approach which we call "real semiclassics for asymptotics with complex-valued phases"(another approach based on the construction of decompositions of bases of homogeneous difference equations was recently developed by A.I.Aptekarev and D. N. Tulyakov). Introducing an artificial small parameter $h = O(1/n)$ and a continuous function $\phi(x, z, a)$ such that $H(z, a)(z, a) = \phi(kh, z, a)$, we reduce the described to a pseudo – differential equation for ϕ, where x is a variable and (z, a) are parameters. Seeking its solution in the WKB–form, one obtains the Hamilton-Jacobi equations with complex Hamiltonians connected with a third-order algebraic curve. This circumstance is the main difficulty of solving the problem and, as a rule, leads to the transition from the real variable x to the complex one. In this problem, we propose a different approach based on a reduction of the original problem to three equations, two of which have asymptotics with a purely imaginary phase, and the symbol of the third one is pure real and has the form $\cos p + V_0(x) + hV_1(x) + O(h^2)$. This ultimately allows us to represent the desired asymptotic uniformly through the Airy function of the complex but real-valued argument.

Density of thin film billiard reflection pseudogroup in Hamiltonian symplectomorphism pseudogroup

A. Glutsyuk (Lyon, France; Moscow)

Reflections from hypersurfaces in Euclidean space act by symplectomorphisms on the symplectic manifold of oriented lines. There is an important open question [3]: which symplectomorphisms are compositions of reflections? We consider an arbitrary C^∞-smooth hypersurface $\gamma \subset \mathbf{R}^{n+1}$ that is either strictly convex and closed, or a germ. We investigate *compositional ratios of reflections from* γ *and from its small deformations,* introduced and studied by Ron Perline in [2]. Perline had shown that their derivatives in the parameter are Hamiltonian vector fields and calculated their Hamiltonian functions. In the case, when γ is a global convex hypersurface, we show that *the pseudogroup generated by the above compositional ratios is dense in the pseudogroup of Hamiltonian diffeomorphisms between subdomains of the phase cylinder:* the space of oriented lines intersecting γ transversally [1]. We prove an analogous local result in the case, when γ is a germ. To prove the main results, we find the Lie algebra generated by the above Hamiltonian functions and prove its C^∞-density in the space of C^∞-smooth functions.

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Global stability boundary, hidden oscillations, and non-equilibrium dynamics in models with cylindrical phase space: PLL and Sommerfeld effect

N. Kuznetsov (St. Petersburg, Russia; Jyväskylä, Finland)

One of the central pratical problems of the dynamics analysis is the study of possible limiting behaviors and transient processes to establish the global stability or to reveal all local attractors and their basins. The rapid development of methods in the global stability theory, the theory of bifurcations, the theory of oscillations, and the chaos theory during the 20th century made it possible to significantly advance in solving this problem for smooth and discontinuous dynamical models [1,2,3,4,5]. One of the famous result in this direction was obtained by A.A. Andronov in 1944 [6]: for a nonlinear discontinuous model of machine controled by Watt's regulator he confirmed Vyshnegradsky's conjecture on local stability by the first approximation and moreover proved the global stability (the significance of these results was noted during the election of A.A. Andronov as a full member of the USSR Academy of Sciences in 1946 [7]). Nevertheless, by the turn of the century it happened that the available arsenal of the methods was insufficient both for solving a number of well-known fundamental problems (e.g., 16th Hilbert problem on the limit cycles of 2d polynomial systems), and for a reliable analysis of applied dynamical models to avoid accidents and disasters. These showed the necessity and urgency of revision and further development of analytical and numerical methods for analyzing stability and limit oscillations in dynamical systems.

This lecture is devoted to the study of global stability and hidden oscillations in dynamical models with the cylindrical phase space. In contrast to the Euclidean phase space, in the cylindrical phase space the global attractor may or may not contain equilibrium states [8]. For the models with equilibria an outer estimation of the global stability boundary in the space of parameters and the birth of self-exited oscillations in the phase space can be obtained by the linearisation around equilibria and the analysis of local bifurcations. Inner estimations of the global stability boundary can be obtained by special modifications of the classical sufficient criteria of global stability for cylindrical phase space [5,9]. In the gap between outer and inner estimations the exact boundary of global stability can be studied numerically. While trivial parts of the global stability boundary are defined by local bifurcations and behavior in vicinity of equilibria, the study of hidden parts requires analizing non-local bifurcations and the birth of hidden oscillations [10,11]. By analogy with Andronov-Vyshnegradsky conclusions, various famous conjectures on the coincidence of the outer estimation given the first approximation with the exact boundary (i.e. on the global stability by the first approximation) have been put forward for phase-locked loops models (PLL) [10,12]. However, phase-locked loop models may have there boundary of global stability with trivial and hidden parts, or may have a global attractor without equilibrium points. If there are no equilibria in the model then the task is to reveal all local attractors and their basins (remark, that even two-dimensional systems can have an infinite number of hidden local attractors). The practical significance of such task can be demonstrated [13,14] on famous Sommerfeld effect discovered in electromechanical systems in 1902 [15,16].

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On the number of the classes of topological conjugacy of Pixton diffeomorphisms

O. Pochinka (Nizhny Novgorod)

In the present report we state the existence of an invariant of the first order for Hops knots. This allows to model countable families of pairwise non-equivalent Hopf knots and, therefore, infinite set of topologically non-conjugate Pixton diffeomorphisms. The results were obtained in collaboration with P.M. Akhmet'ev.

Отражение лагранжевых многообразий и комплексного ростка Маслова в задаче Коши для уравнения Шредингера с дельта-потеницалом, локализованном на поверхности коразмерности 1

А. Шафаревич (Москва)

Хорошо известно, что квазиклассические решения эволюционных уравнений с гладкими коэффициентами описываются в терминах движения лагранжевых поверхностей (или комплексных векторных расслоений над изотропными поверхностями) вдоль траекторий классических гамильтоновых систем. В докладе обсуждается эволюционная задача с дельта-потенциалом; показано, что в этом случае указанные поверхности и векторные расслоения отражаются от носителя дельта-функции.

A model for the cubic connectedness locus

V. Timorin(Moscow)

This is a joint work with A. Blokh and L. Oversteegen. We construct a rough combinatorial model for the connectedness locus in the full parameter space of cubic polynomials, somewhat similar to the model of the Mandelbrot set obtained by collapsing the filled main cardioid and all baby Mandelbrot sets.

Volume preserving diffeomorphisms as Poincare maps for volume preserving flows

D. Treschev(Moscow)

Let q be a volume-preserving diffeomorphism of a smooth manifold M to itself. We study the possibility to present q as the Poincare map, corresponding to a volume-preserving vector field on the direct product of M and a circle. We plan to discuss applications of our results in hydrodynamics of an ideal fluid.

Divisor arithmetic, integrable and superintegrable systems

A. Tsiganov (St. Petersburg)

In the Jacobi separation of variables method, the motion of physical objects is replaced on the evolution of points (prime divisors) along algebraic curves on projective plane. The laws of motion, symmetries, hidden symmetries and other properties of the real motion in phase space are replaced on the Abel theorem, Riemann-Roch theorem, Brill-Noether theorem, Cantor alghorithm, etc. In recent years, these classical algebro-geometric methods have become the base for the development of modern alghoritms of cryptography, including post-quantum cryptography, and cryptocurrencies. We want to discuss various applications of these modern alghoritms in classical mechanics.

On Some Applications of Differential Equations to Problems in Additive Number Theory

I. Vyugin (Moscow)

In the talk, we plan to review some results on additive number theory, which use some ideas from the analytic theory of differential equations. In particular, we will talk about the upper bound for the cardinality of the set

$$G \cap (G+1) \cap ... \cap (G+k),$$

where G is a subgroup of the multiplicative group of the field modulo prime. Also we discuss the upper bound for the number of points $x, y \in G$ of an algebraic curve:

$$P(x,y) = 0$$

over a simple finite field. We also talk about some progress in the study of the Markov equation:

$$x^2 + y^2 + z^2 - 3xyz = 0,$$

considered over a field of characteristic p. (The talk is based on the results of joint papers with S.V. Konyagin, I.D. Shkredov and I.E. Shparlinski.)

Short talks

Asymptotic solution of the contact problem on indentation of poroelastic layer

S. Aizikovich (Rostov-on-Don)

We consider the mixed two-dimensional contact problem on indentation of a porous layer rigidly adhered to a non-deformable base. The micro-dilatation theory firstly developed by Cowin-Nunziato has been applied to describe the porous materials. We use the Fourier integral transform to reduce the problem to solving an integral equation for contact stresses. The solution of the integral equation is carried out by separating the main part of the integral operator, inversion of the singular integral operator and applying the method of successive approximations. Such approach allows us to obtain explicit analytical expressions for the distribution of contact stresses, indentation force, vertical and horizontal displacements, change in volume fraction, shear and normal stresses in the entire poroelastic layer. Computational experiments were carried out for a narrow indenter with a flat base. It is shown that in the limiting case the solution converges to the solution for an elastic medium. The dependence of the characteristics of contact interaction on such porosity parameters as voids diffusion parameter, coupling modulus and the void stiffness modulus is analyzed and illustrated by the numerical examples for different porous media.

Thanks. The investigation was carried out at the expense of the Megagrant no. 14.Z50.31.0046.

Sliding homoclinic bifurcations in a Lorenz-type system: stability with a positive saddle value

N.V. Barabash (Nizhny Novgorod),
V.N. Belykh (Nizhny Novgorod), I.V. Belykh (Atlanta, USA)

We consider piecewise-smooth dynamical Lorenz-type system composed of three systems of linear ODE [1] A_s, A_l, and A_r :
(1)

$$A_s:\ \begin{array}{l}\dot{x}=x,\\ \dot{y}=-\alpha y,\\ \dot{z}=-\nu z,\end{array}\quad A_l:\ \begin{array}{l}\dot{x}=-\lambda(x+1)+\omega(z-b),\\ \dot{y}=-\delta(y+1),\\ \dot{z}=-\omega(x+1)-\lambda(z-b),\end{array}\quad A_r:\ \begin{array}{l}\dot{x}=-\lambda(x-1)-\omega(z-b),\\ \dot{y}=-\delta(y-1),\\ \dot{z}=\omega(x-1)-\lambda(z-b),\end{array}$$

where α, δ, ν, ω, λ and b are positive parameters. This linear systems (subsystems) are defined on the following phase space partition G_s, G_l, и G_r, respectively:

$$G_s : |x| < 1, y \in \mathbb{R}^1, z < b,$$

$$G_l:\begin{cases} x\le -1 \quad \text{for} \quad z\le b,\\ x\le -1 \quad \text{for} \quad z>b, y\ge 0,\\ x<1 \quad \text{for} \quad z>b, y<0,\end{cases}\quad G_r:\begin{cases} x\ge 1 \quad \text{for} \quad z\le b,\\ x\ge 1 \quad \text{for} \quad z>b, y<0,\\ x>-1 \quad \text{for} \quad z>b, y\ge 0.\end{cases}$$

Linear subsystem A_s determines a dynamics of system (1) in the region G_s and has saddle O_s at the origin. Subsystem $A_{r,l}$ are defined in the regions $G_{r,l}$ and have symmetrical 3-D foci $e_{r,l} = \{\pm 1, \pm 1, b\}$, respectively. We assume that the parameters satisfy the condition

$$\frac{1}{2} < \nu < 1.$$

which implies that the saddle value $\sigma = 1 - \nu > 0$ is positive. Introduce new parameters

$$\gamma = be^{-\frac{3\pi\lambda}{2\omega}}, \quad \gamma_{cr} = 2\sqrt{1+\lambda^2/\omega^2}e^{-\frac{\delta}{\omega}\arctan\frac{\lambda}{\omega}},$$
$$\mu = (\gamma - 1)\gamma^{\frac{1}{\nu-1}}, \quad \varepsilon = (\gamma - \gamma_{cr})\gamma^{\nu-1}.$$

Then the following theorem is true.

Theorem 1. *(unstable homoclinic orbits generate a stable cycle)*

(1) *For $\mu < \varepsilon \leq 0$, system* (1) *has two stable foci e_l u e_r, and saddle O_s.*
(2) *At $\mu = 0$, $\varepsilon = 0$, two unstable homoclinic orbits of saddle O_s (homoclinic butterfly) arise in system* (1)*.*
(3) *For $\varepsilon > 0$, increasing $\mu \in (\varepsilon, \varepsilon + \varepsilon^{1/\nu})$ leads to emergence of stable period-2 limit cycle and two saddle limit cycles that were born simultaneously from the homoclinic butterfly.*

A proof of this theorem is considered in the talk. The presented results are published in the paper [2].

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Классификация периодических преобразований двумерного тора

Д. Баранов (Нижний Новгород), Е. Чилина (Нижний Новгород)

Результаты данного доклада были получены совместно с В.З. Гринесом и О.В. Починкой и посвящены классификации периодических преобразований тора.

Пусть S_p – замкнутая ориентируемая поверхность рода p и $f : S_p \to S_p$ – сохраняющий ориентацию периодический гомеоморфизм.

Гомеоморфизм f называется *периодическим*, если существует такое $n \in \mathbb{N}$, что $f^n = id$. Наименьшее из таких n называется периодом f.

Обозначим через $\overline{B}$ множество точек гомеоморфизма f, период которых меньше периода гомеоморфизма.

В силу результатов Я. Нильсена [1] множество $\overline{B}$ конечно и состоит из конечного числа орбит $\mathcal{O}_i$ ($i = 1, \ldots, k$) периода n_i, являющегося делителем n. Положим $\lambda_i = \frac{n}{n_i}$, тогда для каждого λ_i существует единственное взаимно простое с ним число $\delta_i \in \{1, \ldots, \lambda_i - 1\}$ такое, что для любой точки $\bar{x} \in \mathcal{O}_i$ существует окрестность $D_{\bar{x}}$, в которой гомеоморфизм $f^{n_i}|_{D_{\bar{x}}}$ топологически сопряжен с отображением комплексной плоскости: $z \to e^{\frac{2\pi\delta_i}{\lambda_i}i}z$.

Также с каждым периодическим преобразованием связаны следующие объекты:

- группа отображений $G = \{id, f, \ldots, f^{n-1}\}$, изоморфная $\mathbb{Z}_n = \{0, \ldots, n-1\}$, и действующая на S_p так, что *модульная поверхность* $\Sigma_g = S/G$ рода g является замкнутой поверхностью;

- *естественная проекция* $\pi : S_p \to \Sigma_g$, которая является n-листным накрытием всюду, кроме точек множества $B = \pi(\overline{B})$.

Каждому периодическому гомеоморфизму f однозначно соответствует набор *периодических данных* $(n, p, g, n_1, ..., n_k, \delta_1, ..., \delta_k)$.

Следуя [1] гомеоморфизмы $f, f' : S_p \to S_p$ называются *топологически сопряженными*, если существует сохраняющий ориентацию гомеоморфизм $h : S_p \to S_p$ такой, что $f' = h \circ f \circ h^{-1}$.

Также в [1] доказано, что два периодических преобразования f, f' поверхности S топологически сопряжены тогда и только тогда, когда их периодические данные совпадают.

Теорема 1 Пусть $f : \mathbb{T}^2 \to \mathbb{T}^2$ - сохраняющий ориентацию периодический гомеоморфизм периода n, тогда следующие условия эквивалентны:

(1) f – гомотопен тождественному отображению;
(2) $B = \emptyset$;
(3) $g = 1$;
(4) f топологически сопряжен диффеоморфизму $\Psi_n\left(e^{i2x\pi}, e^{i2y\pi}\right) = \left(e^{i2\pi\left(x+\frac{1}{n}\right)}, e^{i2y\pi}\right)$.

Теорема 2 Существует семь классов топологической сопряженности не гомотопных тождественному периодических гомеоморфизмов тора со следующими периодическими данными в каждом классе :

(1) f_1: $n = 6, k = 3, n_1 = 3, n_2 = 2, n_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 1$;
(2) f_2, $n = 3$, $k = 3$, $n_1 = n_2 = n_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 1$;
(3) f_3, $n = 2$, $k = 4$, $n_1 = n_2 = n_3 = n_4 = 1$, $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 1$;
(4) f_4, $n = 3$, $k = 3$, $n_1 = n_2 = n_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 2$;
(5) f_5, $n = 6, k = 3, n_1 = 3, n_2 = 2, n_3 = 1$, $\delta_1 = \delta_2 = 1$, $\delta_3 = 5$;
(6) f_6: $n = 4$, $k = 3$, $n_1 = 2$, $n_2 = n_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 1$;
(7) f_7, $n = 4$, $k = 3$, $n_1 = 2$, $n_2 = n_3 = 1$, $\delta_1 = 1$, $\delta_2 = \delta_3 = 3$.

Пусть A=$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ – унимодулярная целочисленная матрица. Тогда она индуцирует отображение $f_A : \mathbb{T}^2 \to \mathbb{T}^2$, заданное формулой

$$f_A : \begin{cases} \overline{x} = ax + by \pmod 1 \\ \overline{y} = cx + dy \pmod 1 \end{cases} .$$

Следующий результат был получен с помощью результатов в [2].

Теорема 3 В каждом классе топологической сопряженности не гомотопных тождественному периодических гомеоморфизмов тора существует алгебраический автоморфизм, индуцированный следующей матрицей в каждом классе:

$$A_1 = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix};\ A_2 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix};\ A_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix};\ A_4 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix};$$

$$A_5 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix};\ A_6 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix};\ A_7 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

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Реализация диаграмм Хассе с помощью А-диффеоморфизмов поверхностей с нетривиальными базисными множествами

М. Баринова (Нижний Новгород)

Результаты получены совместно с Починкой О.В. и Гогулиной Е.Ю.

Пусть f – диффеоморфизм замкнутого n-многообразия M^n. Говорят, что диффеоморфизм f удовлетворяет аксиоме A, если его неблуждающее множество является гиперболическим, и периодические точки плотны в нем. Для A-диффеоморфизмов справедлива теорема о спектральном разложении С. Смейла [1], устанавливающая единственное представление неблуждающего множества в виде конечного объединения попарно непересекающихся множеств, называемых базисными, каждое из которых является компактным, инвариантным и топологически транзитивным.

На множестве базисных множеств любого A-диффеоморфизма f можно ввести отношение С. Смейла [1]. Именно, пусть Λ_i, Λ_j – базисные множества A-диффеоморфизма f. Говорят, что Λ_i, Λ_j находятся в отношении $\prec$ $(\Lambda_i \prec \Lambda_j)$, если $W^s_{\Lambda_i} \cap W^u_{\Lambda_j} \neq \varnothing$. Последовательность, состоящая из базисных множеств $\Lambda_i = \Lambda_{i_0}, \Lambda_{i_1}, ..., \Lambda_{i_k} = \Lambda_j$ $(k \geq 1)$, такая что $\Lambda_{i_0} \prec \Lambda_{i_1} \prec \ldots \prec \Lambda_{i_k}$ называется цепью длины $k \in \mathbb{N}$, соединяющей периодические орбиты Λ_i и Λ_j.

Такая цепь называется максимальной, если в нее нельзя добавить ни одного нового базисного множества. Цепь называется циклом, если $\Lambda_i = \Lambda_j$. Диффеоморфизм $f : M^n \to M^n$ называется Ω-устойчивым, если C^1-близкие к f диффеоморфизмы топологически сопряжены на неблуждающих множествах. Согласно [2], диффеоморфизм $f : M^n \to M^n$ является Ω-устойчивым тогда и только тогда, когда он удовлетворяет аксиоме A и не имеет циклов.

Диаграммой Смейла Δ_f Ω-устойчивого диффеоморфизма $f : M^n \to M^n$ называется граф, вершины которого соответствуют базисным множествам, а ориентированные ребра последовательно соединяют вершины максимальных цепей. В действительности диаграмма Смейла является частным случаем диаграмм Хассе.

Диаграммой Хассе частично упорядоченного множества $(X, \prec)$ называется граф, вершинами которого являются элементы множества X, а пара (x, y) образует ребро, если $x \prec y$ и $\nexists z : \ x \prec z, \ z \prec y$.

В работе [1], в качестве проблемы (Проблема 6.6а) сформулирован следующий вопрос: какие диаграммы могут соотвествовать Ω-устойчивым диффеоморфизмам?

С помощью хирургической операции Смейла мы конструируем модельные диффеоморфизмы двумерного тора, реализующие диаграммы Хассе.

Теорема Любая связная диаграмма Хассе реализуется некоторым Ω-устойчивым диффеоморфизмом.

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Integrable hierarchies associated to infinite families of Frobenius manifolds

A. Basalaev (Moscow)

We propose a new construction of an integrable hierarchy associated to any infinite series of Frobenius manifolds satisfying a certain stabilization condition. We study these hierarchies for Frobenius manifolds associated to A, D and B singularities. As a side product to these results we illustrate the enumerative meaning of certain coefficients of A, D and B Frobenius potentials.

This is a joint work with S.M. Natanzon and P.I.Dunin-Barkowsky.

A topological classification of billiards bounded by confocal quadrics in three-dimensional Euclidean space

G. Belozerov (Moscow)

Let us consider a motion of material point inside a billiard table, i. e. inside a compact three-dimensional domain bounded by confocal quadrics. Also we assume that all dihedral angles on boundary of the billiard table are equal to $\frac{\pi}{2}$. No force acts on this material point and reflection is absolutely elastic. As it turns that out such billiards are integrable Hamiltonian systems. Let us consider rough Liouville equivalence relation of such billiards (isomorphism of bases of their Lagrangian fibrations with singularities). Author proved that there are exactly 25 types of rough Liouville nonequivalent billiards. Also it turns out that we can determinate homeomorphism class of regular isoenergy surfaces, if we know shape of billard table. Author proved that each regular isoenergy surface of the billiard is homeomorphic to S^5 or $S^4 \times S^1$ or $S^3 \times S^2$.

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Simulation of growth of a vapor bubble induced by laser heating of a liquid

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A laser energy source for heating various substances, including biological tissues and liquids, is widely used in various practical problems and applications, in particular, in endoscopic and puncture surgical interventions, which are the safest and most promising. One of the advantages of this method is the high intensity of the thermal effect, mainly localized, as well as a good degree of controllability. The essence of the method is as follows. Laser radiation is delivered through optical fiber, which is in contact with biological tissues or fluids (blood, lymph, liquid contents of cysts, etc.). The liquid is quickly heated near the end of the fiber. Its explosive boiling is initiated. Since the liquid as a whole is significantly subcooled, boiling is accompanied not only by the growth, but also by the collapse of vapor-gas bubbles, accompanied by the formation of hot submerged jets. These jets carry out a destructive effect on pathological formations. Despite the fact that a huge number of works have been devoted to the study of the boiling process, there are still many issues that require their solution. In particular, this concerns the boiling process of a locally superheated (under general subcooling) liquid.

In the present work, a mathematical model of the growth of a vapor bubble in a superheated liquid, which simultaneously takes into account both dynamic and thermal effects and includes the well-known classical equations, the Rayleigh equation and the energy equation, written in relation to the problem under consideration, taking into account the specifics associated with the process of liquid evaporation is proposed.

It is shown that the presented problem is reduced to solving a system of three ordinary differential equations of the first order. The obtained solution is in good agreement with direct numerical calculations in a wide range of overheating and at all stages of the process, including the transitional one, which is extremely necessary if one considers the growth of a bubble in a highly superheated liquid, especially at the initial stage.

It is shown that, at long times, the growth of a bubble is determined exclusively by the supply of heat to the interface. The temperature field around the bubble (in Lagrangian variables) becomes stationary, and the solution of the thermal problem becomes self-similar. The dependence of the bubble radius on time takes a root form, and the proportionality coefficient becomes a function of only the Jacob number. It is shown that at certain operating parameters of the process, this stage is unattainable at times that are foreseeable in real processes.

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On fractal cubes with finite intersection property

D. Drozdov (Novosibirsk)

Let $P^k = [0,1]^k$ be the unit k-dimensional cube (or, simply *k-cube*).

Let $n \geq 2$. Take a set $\mathcal{D} = \{\xi_1, ..., \xi_m\} \subset \{0, 1, \ldots, n-1\}^k$, $2 \leq \#D = m < n^k$, and call it a *digit set*. The elements ξ_i of the set $\mathcal{D}$ define the system $\mathcal{S} = \{S_1, ..., S_m\}$ of the similarities $S_i(x) = \dfrac{x + \xi_i}{n}$ in $\mathbb{R}^k$, mapping P^k to cubes with side $1/n$ which belong to the n^k partition of P^k. There is a unique non-empty compact set $F \subset P^k$ that satisfies the equation

$$F = \bigcup_{i=1}^{m} S_i(F) = \frac{F + \mathcal{D}}{n},$$

which we will call *a fractal k-cube* of the order n (see [2,3]).

A fractal 1-cube is called a fractal segment, a fractal 2-cube is a fractal square.

The intersections of a fractal k-cube F with *l-faces of the k-cube* P^k are called *l-faces* of F (for $0 \le l < k$). Obviously, the l-faces of a fractal k-cube are fractal l-cubes.

The partition cube $P_i^k = S_i(P^k)$ of the k-cube P^k can intersect each other only by the images of the respective pairs of opposite l-faces of the k-cube P^k. Since $S_i(F) \subset P_i^k$, these copies can intersect each other only by the images of respective pairs of opposite l-faces of F.

To verify that a fractal cube F has finite intersection property (see [1]), we need to study finite intersection conditions for pairs of fractal segments and fractal squares (the intersection of 0-faces is at most one-point).

Suppose $\mathcal{D}_1$ and $\mathcal{D}_2$ are the digit sets defining fractal segments K_1 and K_2 and suppose that $1 \in \mathcal{D}_1$ and $n-1 \in \mathcal{D}_2$. Then if $m \in \mathcal{D}_1 \cap (D_2+1)$, then the point $m/n \subset K_1 \cap K_2$. Such point is called a *transition point* for these fractal segments.

Theorem 1. *The fractal segments K_1 and K_2 of the the order n generated by the digit sets $\mathcal{D}_1$ and $\mathcal{D}_2$ have a finite intersection in the following cases:*
1. $\#(\mathcal{D}_1 \cap \mathcal{D}_2) = 1$, and K_1 and K_2 have no transition points, then $\#(K_1 \cap K_2) = 1$;
2. $\mathcal{D}_1 \cap \mathcal{D}_2 = \varnothing$, and K_1 and K_2 have s transition points, then $\#(K_1 \cap K_2) = s$.

Theorem 2. *Two fractal squares K_1 and K_2 of the order n, with digit sets $\mathcal{D}_1$ and $\mathcal{D}_2$ have a finite intersection in the following cases:*
1. $\#(\mathcal{D}_1 \cap \mathcal{D}_2) = 0$, there is at least one pair of vertex- or edge-adjacent copies $S_i(K_1) \subset K_1$ and $S_j(K_2) \subset K_2$ that have a non-empty finite intersection, and there is no such pair of edge-adjacent copies of K_1 and K_2 that intersect at an infinite number of points (see Theorem 1);
2. $\#(\mathcal{D}_1 \cap \mathcal{D}_2) = 1$, and any pair of vertex- or edge-adjacent copies of K_1 and K_2 has an empty intersection, then $\#(K_1 \cap K_2) = 1$.

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On the wave turbulence theory for the stochastically perturbed nonlinear Schrodinger equation

A. Dymov (Moscow)

The wave turbulence (WT) was developed in 1960's by V.E. Zakharov and his school as a heuristic tool to study small-amplitude oscillations in nonlinear Hamiltonian PDEs with periodic boundary conditions of large period. Since then WT has been intensively developed in physical works, which employ some different (but consistent) approaches. Non of them was ever rigorously justified, despite the strong interest in physical and mathematical communities to the questions, addressed by these works.

The principal assertion of WT is that one of the main characteristics of solution, called the energy spectrum, approximately satisfies a nonlinear kinetic equation, called the wave kinetic equation. I will talk about my joint work with S.B. Kuksin, in which we made an attempt to rigorously justify this assertion for the energy spectrum of the damped/driven nonlinear Schrodinger equation. This stochastic model for WT was earlier suggested by Zakharov and L'vov.

К вопросу о формировании высокоскоростной струи при коллапсе газового пузырька вблизи тонкого волокна погруженного в жидкость. Численное исследование

Р.В. Фурсенко (Новосибирск),
С.С. Минаев (Владивосток), В.М. Чудновский (Владивосток)

Недавние экспериментальные исследования [1] продемонстрировали возможность формирования высокоскоростных (от нескольких до сотен м/с) струй в результате коллапса паро-газового пузырька вблизи оптоволкна погруженного в жидкость и служащего проводником лазерного излучения. Энергоподвод с торца оптоволокна приводит к вскипанию прилегающих к нему слоев жидкости и образованию парового пузыря. После стадии роста пузырька происходит его сжатие, связанное с конденсацией за счет охлаждения пара вблизи границы раздела пар - холодная жидкость. Наконец, в результате схлопывания пузырька формируется струя жидкости, направленная от торца оптоволокна. Описанный процесс нашел многообещающие применения в медицине, например, при лечении биологических тканей. В то же время, механизмы формирования струи и влияние параметров задачи на ее характеристики недостаточно изучены. В этой работе, процесс формирования струи в результате коллапса газового пузырька вблизи тонкого (радиус 0.1-0.6 мм) волокна исследуется численно с помощью метода объема жидкости [2]. Результаты численного моделирования позволили описать некоторые детали механизма формирования струи и определить условия, необходимые для ее существования. Так же исследовано влияние радиуса волокна, радиуса парового пузыря и температуры окружающей жидкости на скорость струи. В частности обнаружено, что подходящее обезразмеривание начального радиуса пузырька и средней скорости струи приводит к тому, что все численные данные ложатся на одну линию [3].

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Derivations of the Lie algebra of type F_4 over the field of characteristic two

V. Galkin (Nizhny Novgorod)

The computation of derivations of classical Lie algebras is interesting in connection with the unsolved problem of classification of simple Lie algebras over fields of small characteristic. Over a field of characteristic two, one of the possibilities for obtaining new Lie algebras is to construct deformations of classical Lie algebras. The space of external differentiations can serve as an indicator of the non-isomorphism of the obtained Lie algebras.

In work was found space external differentiation of the lie algebra of type F_4 over a field of characteristic 2. To simplify computations space of external differentiation, or the first cohomology group $H^1(L, L)$ is regarded as a direct sum of weight subspaces of $H^1_\mu(L, L)$, and separately evaluated each weight subspace. Algorithms were written and implemented in the Maple environment.

A Lie algebra of type F_4 has dimension 52. The structural constants in the Chevalley basis of this algebra in characteristic 2 are calculated based only on the Cartan matrix, i.e., a matrix of size 4×4. The corresponding algorithm for obtaining these constants was implemented. Algorithms for obtaining weights and weight spaces of cochains are also implemented. With the help of these programs, the first group of weight μ cohomology was calculated. With the help of this program, the theorem is proved.

Theorem. The space of external differentiations $H^1(L, L)$ of a Lie algebra F_4 over a field of characteristic 2 is trivial.

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Необходимые и достаточные условия сопряженности декартовых произведений грубых преобразований окружности.

И. Голикова (Нижний Новгород)

Результаты были получены совместно с О.В. Починкой и посвящены топологической классификации декартовых произведений грубых преобразований окружности.

Как показал А. Г. Майер в [1], класс топологической сопряженности сохраняющего ориентацию грубого преобразования окружности однозначно определяется параметрами n, k, l, где k – период периодических точек, $2n$ – число периодических орбит, $\frac{l}{k}$ – число вращения преобразования. Таким образом, любой такой диффеоморфизм топологически сопряжен некоторому модельному преобразованию $\phi_{n,k,l} : \mathbb{S}^1 \to \mathbb{S}^1$. Соответственно, декартово произведение грубых преобразований окружности является градиентно-подобным диффеоморфизмом на двумерном торе, топологически сопряженным модельному преобразованию $\phi_{n_1,k_1,l_1} \times \phi_{n_2,k_2,l_2} : \mathbb{T}^2 \to \mathbb{T}^2$. При этом каждый модельный диффеоморфизм имеет $4n_1n_2k_1k_2$ периодических точек, а их период равен $q = HOK(k_1, k_2)$.

Основным результатом работы является доказательство следующей теоремы.

Теорема. *Диффеоморфизмы* $\phi_{n_1,k_1,l_1} \times \phi_{n_2,k_2,l_2}$, $\phi_{n'_1,k'_1,l'_1} \times \phi_{n'_2,k'_2,l'_2} : \mathbb{T}^2 \to \mathbb{T}^2$ *топологически сопряжены тогда и только тогда, когда* $n_1k_1 = n'_1k'_1$, $n_2k_2 = n'_2k'_2$ *и* $q = q'$.

Таким образом, для декартовых произведений грубых преобразований окружности их числа вращения не являются топологическими инвариантами. Доказательство классификационной теоремы существенно опирается на результаты работы [2], в которой установлено, что полным топологическим инвариантом градиентно-подобного диффеоморфизма поверхности является трехцветный граф. Изоморфизм графов при выполнении числовых равенств в теореме строится методами работы [3], где получена классификация декартовых произведений поворотов окружностей с точностью до сопряжения линейным преобразованием.

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On uniqueness of a cycle of one dynamical system

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Аннотация. We obtain conditions of uniqueness of a cycle in phase portrait of a Elowitz-Leibler type piecewise linear dynamical system which simulates functioning of one gene network.

Following [1], we consider mathematical model of one natural circular gene network

$$\frac{dm_1}{dt} = L_1(p_3) - k_1 m_1; \qquad \frac{dp_1}{dt} = \Gamma_1(m_1) - l_1 p_1; \qquad \frac{dm_2}{dt} = L_2(p_1) - k_2 m_2;$$
$$\frac{dp_2}{dt} = \Gamma_2(m_2) - l_2 p_2; \qquad \frac{dm_3}{dt} = L_3(p_2) - k_3 m_3; \qquad \frac{dp_3}{dt} = \Gamma_3(m_3) - l_3 p_3. \quad (1)$$

Here $m_j(t)$ are concentrations of three mRNAs, $p_j(t)$ are concentrations of corresponding proteins, and step-functions L_j, Γ_j are defined by $L_j(w) = A_j > k_j > 0$ for $0 \leq w < 1$, $L_j(w) = 0$ for $1 \leq w$; $\Gamma_j(w) = 0$ for $0 \leq w < 1$, $\Gamma_j(w) = B_j > l_j > 0$ for $1 \leq w$; $j = 1, 2, 3$. They describe negative, respectively, positive feedbacks in the gene network.

Let $a_j = A_j/k_j$, $b_j = B_j/l_j$, and $Q = [0, a_1] \times [0, b_1] \times [0, a_2] \times [0, b_2] \times [0, a_3] \times [0, b_3] \subset \mathbb{R}^6_+$.
Hyperplanes $m_j = 1$, $p_j = 1$ decompose Q to 64 blocks; we denote them by multi-indices $\{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5\varepsilon_6\}$, where $\varepsilon_{2j-1} = 0$ if $m_j < 1$ in this block, otherwise $\varepsilon_{2j-1} = 1$. Respectively, $\varepsilon_{2j} = 0$, if $p_j < 1$ in this block, and $\varepsilon_{2j} = 1$ otherwise.

It was shown in [2] that all trajectories of (1) which start in any block of the diagram (2) follow its arrows, and pass through all 12 blocks listed there. Let W_1 be their union, this is positively invariant domain of the system (1). This domain contains all blocks of Q such that trajectories of their points can pass out of each of them to **one** incident block only, we call them *one-valent blocks.*

$$\begin{tikzcd}
\{110011\} \arrow[r] & \{010011\} \arrow[r] & \{000011\} \arrow[r] & \{001011\} \arrow[d] \\
\{110010\} \arrow[u] & & & \{001111\} \arrow[d] \\
\{110000\} \arrow[u] & & & \{001101\} \arrow[d] \\
\{110100\} \arrow[u] & \{111100\} \arrow[l] & \{101100\} \arrow[l] & \{001100\} \arrow[l]
\end{tikzcd} \quad (2)$$

Similar State Transition Diagrams appear in combinatorial description of other piecewise linear and smooth gene networks models in different dimensions, see [3,4,5].

Theorem. *If $A_j > k_j$, and $B_j > l_j$, then the system* (1) *has a unique cycle $\mathcal{C} \subset W_1$; it passes through all one-valent blocks according arrows of the diagram* (2).

Let $F_0 = \{110011\} \cap \{010011\}$ be common face of two incident blocks in (2), and let $\Phi : F_0 \to F_0$ be the Poincaré map of the cycle $\mathcal{C}$. The proof of the Theorem is based on the fact that first derivatives of the coordinate functions of Φ are strictly positive in an appropriate coordinate system in F_0, and their second derivatives are strictly negative.

Similar considerations can be reproduced for various dynamical systems of the type (1) in other dimensions as well. For one analogous 4-dimension system

$$\frac{dx_1}{dt} = L(y_4) - k_1x_1;\ \frac{dy_2}{dt} = \Gamma_2(x_1) - k_2y_2;\ \frac{dy_3}{dt} = \Gamma_3(y_2) - k_3y_3;\ \frac{dy_4}{dt} = \Gamma_4(y_3) - k_4y_4,$$

considered in [5], we have described necessary and sufficient conditions of uniqueness and exponential stability of the cycle in its invariant neighborhood W_1.

For higher-dimensional systems of this type, non-invariant domains $Q \setminus W_1$ can contain invariant surfaces and other cycles, see [2,4] and references therein. Most of provious publications on piecewise linear systems of the type (1) were devoted to the "dimensionless case" $k_j = l_j = 1$.

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Об обратимой трехмерной системе, содержащей аттрактор и репеллер типа Лоренца

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В работе рассматривается трехмерная система вида

$$\begin{cases} \dot{x} = y, \\ \dot{y} = Fx + Cyz + Dx^3 + Exz^2, \\ \dot{z} = \mu + Az^2 + Bx^2, \end{cases} \tag{1}$$

где A, B, C, D, E, F и μ – ее параметры. Система (1) обладает симметрией, в точности такой же как у известной системы Лоренца, т.е. она инвариантна относительно замены координат $x \to -x, y \to -y, z \to z$. Принципиально важной особенностью системы (1) является ее обратимость – она инвариантна относительно обращения времени $t \to -t$ и замены координат вида $h : x \to x, y \to -y, z \to -z$, которая является инволюцией, т.е. $h^2 = Id$. Заметим, что множество $Fix(h)$ неподвижных точек инволюции h одномерно: $Fix(h) = \{y = 0,\ z = 0\}$.

Система (1) была предложена нами в связи с общей задачей исследования бифуркаций обратимых систем, приводящих к рождению симметричной пары "аттрактор Лоренца и репеллер Лоренца". В случае трехмерных отображений существование симметричных дискретных аттрактора Лоренца и репеллера Лоренца было впервые установлено работе [1] для неголономной модели кельтского камня. Глобальные бифуркации, приводящие к появлению в такой модели дискретного аттрактора (репеллера) Лоренца, были изучены в работе [2]. В связи с этими исследованиями, возникла новая задача о структуре локальных бифуркаций обратимых трехмерных отображений, приводящих к рождению симметричной пары "аттрактор и репеллер Лоренца". В случае обратимых трехмерных отображений с инволюцией R, у которой $\dim(Fix(R)) = 1$, как ожидается, такие локальные бифуркации могут быть у неподвижной симметричной относительно R точки с триплетом $(-1, -1, +1)$ мультипликаторов. В силу обратимости отображений, коразмерность такой бифуркации равна 2 (в общем случае, это локальная бифуркация коразмерности 3). Как мы показали, система (1) является локальной потоковой нормальной формой бифуркации симметричной неподвижной точки с триплетом $(-1, -1, +1)$ мультипликаторов для квадрата соответствующего обратимого отображения в некоторой малой окрестности этой точки. Построение такой нормальной формы проводится стандартным путем. Сначала рассматривается квадрат отображения и локально вкладывается в некоторый неавтономный трехмерный поток, от которого берется его основная автономная часть. Затем, в полученном трехмерном потоке делаются замены переменных и времени (рескейлинги) с целью упрощения и унификации его правых частей. В результате получается система (1) с семью независимыми параметрами. Это показывает, что структура бифуркаций в системе и сама ее динамика может быть весьма разнообразной. Но если ограничиться задачей исследования аттракторов и репеллеров Лоренца и их бифуркаций в системе, то она становится обозримой.

В принципе, система (1) может иметь 6 состояний равновесия

$$
\begin{aligned}
&O_1(0, 0, \sqrt{-\tfrac{\mu}{A}}),\ O_2(0, 0, -\sqrt{-\tfrac{\mu}{A}}),\\
&O_3(S, 0, Q),\ O_4(S, 0, -Q),\ O_5(-S, 0, Q),\ O_3(-S, 0, -Q),
\end{aligned}
$$

где

$$
S = \sqrt{\frac{\mu E - AF}{AD - BE}},\ Q = \sqrt{\frac{BF - \mu D}{AD - BE}}.
$$

В докладе рассматривается система (1) с $A = 1$, $B = 1$, $D = -25$, $E = 50$, $\mu = -1$, в которой параметры C и F являются управляющими. При этом координаты состояний равновесия имеют следующий вид: $O_1(0, 0, 1)$, $O_2(0, 0, -1)$, $O_3\left(\sqrt{\frac{F+50}{75}}, 0, \sqrt{\frac{25-F}{75}}\right)$, $O_4\left(\sqrt{\frac{F+50}{75}}, 0, -\sqrt{\frac{25-F}{75}}\right)$, $O_5\left(-\sqrt{\frac{F+50}{75}}, 0, \sqrt{\frac{25-F}{75}}\right)$, $O_6\left(-\sqrt{\frac{F+50}{75}}, 0, -\sqrt{\frac{25-F}{75}}\right)$.

В докладе представлены следующие результаты. На плоскости параметров C и F построена бифуркационная диаграмма состояний равновесия и гомоклинических петель, с помощью которой численно и аналитически изучены основные локальные и глобальные бифуркации, приводящие к возникновению симметричных аттратора и репеллера Лоренца. В системе (1) были прослежены два сценария возникновения аттрактора Лоренца. А также найден новый тип аттрактора Лоренца, который не содержит внутри его дырок состояний равновесия. В этом случае, при $F > 25$, система (1) имеет только два симметричных состояния равновесия, одно из которых принадлежит аттрактору, а другое репеллеру.

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Контрпримеры в дифференциальной теории Галуа

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Дифференциальная теория Галуа исследует симметрии решений систем линейных дифференциальных уравнений над абстрактным полем с дифференцированием. Ключевым понятием в данной теории являются так называемые расширения Пикара-Вессио — аналог полей разложения многочленов в обычной теории Галуа. Однако оказывается, что, в отличие от обычной теории Галуа, расширения Пикара-Вессио существуют не для любой системы линейных дифференциальных уравнений. Первый контрпример, т.е. пример такого феномена, был построен в 50-х годах при помощи явного вычисления. В докладе будет рассказано о том, как концептуально строить серии таких контрпримеров при помощи алгебро-геометрического подхода к данным вопросам.

О взаимоотношениях между базисными множествами структурно устойчивых диффеоморфизмов поверхностей

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Пусть M^n - замкнутое гладкое многообразие размерности $n \geq 1$.

В [1] доказано, что если неблуждающее множество $NW(f)$ структурно устойчивого диффеоморфизма $f : M^n \to M^n$ $(n \geq 3)$ содержит растягивающийся ориентируемый аттрактор Ω коразмерности один, то: 1) многообразие M^n гомотопически эквивалентно n-мерному тору $\mathbb{T}^n$; если $n \neq 4$, то M^n гомеоморфно $\mathbb{T}^n$; 2) множество $NW(f)\backslash\Omega$ состоит из конечного числа изолированных источников и седел.

Для случая $n = 2$ данное утверждение неверно. Несущая поверхность M^2 структурно устойчивого диффеоморфизма f, неблуждающее множество $NW(f)$ которого содержит ориентируемый аттрактор Ω, не обязательно является тором, а может быть любой ориентируемой поверхностью, отличной от сферы, и динамика на множестве $NW(f)\backslash\Omega$ может быть устроена гораздо более сложно.

Пусть M^2 - замкнутая поверхность (не обязательно ориентируемая) рода g, где $g \geq 0$.

Theorem 2. *Пусть $f : M^2 \to M^2$ - структурно устойчивый диффеоморфизм, все тривиальные базисные множества которого являются источниковыми* *периодическими точками* $\alpha_1, ..., \alpha_k$, *где* $k \geq 1$. *Тогда неблуждающее множество* $NW(f)$ *диффеоморфизма* f *состоит из точек* $\alpha_1, ..., \alpha_k$ *и в точности одного одномерного аттрактора* Λ.

В статье [2] было анонсировано обобщение хирургической операции С. Смейла на псевдоаносовские диффеоморфизмы произвольных поверхностей. Данная операция приводит к появлению структурно устойчивого диффеоморфизма той же поверхности с неблуждающим множеством, состоящим в точности из одного одномерного аттрактора и конечного числа источниковых периодических точек. Полученные в результате обобщённой хирургической операции диффеоморфизмы, а также диффеоморфизмы двумерной сферы, неблуждающее множество которых содержит аттрактор Плыкина, и DA-диффеоморфизмы двумерного тора являются примерами диффеоморфизмов, удовлетворяющих условиям теоремы 2.

Нетривиальное базисное множество Ω A-диффеоморфизма $f : M^2 \to M^2$ называется просторно расположенным, если не существует гомотопной нулю петли, образованной парой отрезков устойчивого и неустойчивого многообразий какой-либо точки $x \in \Omega$. Определение связки, используемое далее, см., например, в [3].

Corollary 1. *Пусть выполняются условия теоремы 2, аттрактор* Λ *не имеет связок степени один и* $g \geq 1$, *если поверхность* M^2 *ориентируемая,* $g \geq 3$, *если поверхность* M^2 *неориентируемая. Тогда аттрактор* Λ *просторно расположен на поверхности* M^2.

Corollary 2. *Пусть выполняются условия теоремы 2 и* $k = 1$ *(*α_1 *- неподвижная точка). Тогда поверхность* M^2 *имеет род* $g \geq 1$, *если она ориентируемая,* $g \geq 3$, *если она неориентируемая, аттрактор* Λ *просторно расположен на поверхности* M^2 *и не имеет связок степени один.*

В [3] (теорема 1) доказано, что если неблуждающее множество структурно устойчивого диффеоморфизма поверхности содержит одномерный аттрактор (репеллер), то оно также содержит источниковую (стоковую) периодическую точку. Следующая теорема дополняет данное утверждение.

Theorem 3. *Если неблуждающее множество структурно устойчивого диффеоморфизма* $f : M^2 \to M^2$ *содержит нетривиальное нульмерное базисное множество* Ω, *то оно содержит источниковую и стоковую периодические точки.*

Результаты, представленные в данном докладе, будут опубликованы в [4].

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Study on laser heating of biological fluids in the interests of medicine

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Ontological grounds have been formulated for carrying out studies of laser heating inside the tissues of the human body when developing methods of treating some common pathological formations. The physical laws of laser heating, boiling, and evaporation of biological fluids, through which the main heat transfer in tissues is carried out, are indicated. A model of interstitial contact laser heating is presented. Within its framework, submerged jets' formation is observed at the end of an optical fiber immersed in a liquid and in the end vicinity. These jets are shown to be capable of transferring heat at high speed through a relatively cold environment, dissecting tissues, foaming biofluids, and performing biological cleaning of surfaces. Using numerical and analytical methods to analyze the proposed model makes it possible to understand the laser heating processes' mechanism and put the basis for new technologies for the surgical treatment of a wide range of diseases.

Connections between dynamic, spectral and scattering inverse problems

S. Kabanikhin (Novosibirsk)

Connections between inverse problems for hyperbolic equations (dynamic inverse problems), inverse spectral and inverse scattering problems have been investigating by many authors starting from M. Krein. In 1D case the impulse-response function (data for dynamic inverse problem) is connected with spectral function and Jost function by explicit formulas. In multidimensional case even for the simplest wave equation the trace of the solution at the time-like surface has some special properties (R. Courant). We will discuss some of those properties.

Realization of topological invariants by integrable billiard books

I. Kharcheva (Moscow)

Let us consider a free motion of a particle in some fixed domain $\Omega \subset \mathbb{R}^2$ with elastic reflection at the boundary $P = \partial\Omega$. We obtain a Hamiltonian dynamical system with a Hamiltonian that equals to the scalar square of the velocity vector. Such dynamical systems and their generalizations usually are called billiards

If the domain's boundary P is a piecewise curve and consist of several arcs of confocal ellipses and hyperbolas then the billiard has a following special property: the straight lines containing the segments of the billiard trajectory are tangents to a certain quadric (ellipse or hyperbola). The parameter of this quadric is the value of the additional integral Λ (see [1]). Thus this billiard is integrable and called an elementary billiard.

A billiard book is a generalization of elementary billiards obtained by glueing them along the boundaries. A billiard book is still an integrable Hamiltonian system on a piecewise smooth phase space (see [2]).

Integrable Hamiltonian systems can be classified by topological invariants: 3-atoms, f-graphs, coarse and marked molecules (see [3]). Such invariants allow us to speak about Liouville equivalence of different dynamical systems.

Researching billiard books we try to realize well-known classical dynamical systems in terms of topological invariants (see, for example, [4]). So V. V. Vedyushkina and I. S. Kharcheva found an algorithm that constructs a billiard book which realizes 3-atoms and f-graphs. This algorithm can be extended to another one that realizes any coarse molecule. Details of this result will be presented.

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Cuspidal singularities in integrable systems of dynamics

V. Kibkalo (Moscow)

Integrable Hamiltonian systems usually have singularities, i.e. points, trajectories and Lagrangian fibers near which topology and dynamics of system differs from the regular one (rational or irrational motion on a half-dimensional Liouville torus).

Classification of singularities of IHS (for some equivalence relation) generates various solved and open problems. In non-degenerate case several results by L. Eliasson, A.Fomenko, A.Bolsinov, L.Lerman, N.Zung and A.Oshemkov described such singularities and their invariants in great detail (see book [1] and review [2]).

Case of degenerate singularities is much more wider and complicated for study. This area is also deeply connected with bifurcation theory. In a recent paper [3] by A.Bolsinov, L. Guglielmi and E.Kudryavtseva a class of cuspidal (parabolic) singularities was introduced, investigated and classified. Such singularities have a critical degenerate (not of Morse-Bott type) S^1-orbits which momentum map image is a cusp of the local bifurcation diagram. Topological and symplectic invariants of such singularities were determined and their structural stability was proved in [3].

In the talk we will discuss about appearance of cuspidal singulairites in integrable systems of dynamics. A lot of such systems (e.g. Zhukovsky, Klesch, Kovalevskaya cases) have cusp points on the curves of their bifurcation diagram and degenerate rk 1 orbits in pre-images of such points. As it turns out, cuspidal singularities indeed appear there.

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On invariant surfaces in the smooth gene network models

N. Kirillova (Novosibirsk)

Аннотация. In previous works we have already established the conditions for the existence of cycles for some smooth dynamical systems modeling the functioning of circle gene networks. Now for a six-dimensional system (1) we construct an invariant surface, containing the cycle. The reduction of phase portrait dimension simplifies the analysis of the trajectory behavior of such systems.

A dynamical system is considered here in the folloing form:

$$\frac{dm_j}{dt} = -k_j m_j + f_j(p_{j-1}); \;\; \frac{dp_j}{dt} = \mu_j m_j - \nu_j p_j; \;\; j = 1, 2, 3, \tag{1}$$

where $f_j(p_{j-1})$ are smooth monotonically decreasing functions of non-negative argument; p_j and m_j are the concentrations of some proteins and their corresponding mRNAs; μ_j, ν_j and k_j are positive constants, characterizing the rate of synthesis of these proteins and mRNAs. Here, if $j = 1$, then $j - 1 = n$, where $n = 3$.

A simplified version of such system was discussed in [3] as a model for the functioning of one gene network. Earlier, see [1], it was shown that parallelepiped $Q = \prod_{j=1}^{j=3}([0, A_j] \times [0, B_j])$ is an invariant domain of the system (1). Here $A_j := f_j(0)/k_j$ and $B_j := \mu_j A_j/\nu_j$, wherein the system (1) has exactly one equilibrium point S_0.

Divide the invariant parallelepiped Q by the planes parallel to the coordinate planes and passing through equilibrium point $S_0 = (m_1^0, p_1^0, m_2^0, p_2^0, m_3^0, p_3^0)$ on 64 smaller parallelepipeds, which we will call blocks and number binary multi-indices:

$$\mathcal{E} = \{\varepsilon_1\varepsilon_2\varepsilon_3\varepsilon_4\varepsilon_5\varepsilon_6\} =$$

$$\left\{\mathbf{X} \in Q \mid m_1 \gtrless_{\varepsilon_1} m_1^0;\; p_1 \gtrless_{\varepsilon_2} p_1^0; \ldots ; m_3 \gtrless_{\varepsilon_5} m_3^0;\; p_3 \gtrless_{\varepsilon_6} p_3^0 \right\},$$

where $\mathbf{X} = (m_1, p_1, m_2, p_2, m_3, p_3)$, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 \in \{0, 1\}$, and the order relations are defined as follows: the symbol $\gtrless_0$ corresponds $\le$, and the symbol $\gtrless_1$ corresponds $\ge$. For blocks of valency one a transition diagram is constructed, see [2]. Valence one means that from each block the trajectories can pass in only one adjacent block. Denote by Ω_1 the union of such blocks.

Let $\lambda_{1,2} = \alpha_1 \pm i\beta_1$ are eigenvalues of the linearization matrix M_6 of the system (1), where $\alpha_1 > 0$, $\beta_1 \neq 0$. Then these eigenvalues correspond to the real plane P_1^2, which is invariant subspace of the matrix M_6.

Definition. *An equilibrium point of a dynamical system is called hyperbolic if the eigenvalues of the corresponding linearization matrix have positive and negative real parts and they are not purely imaginary.*

Lemma. *If S_0 is hyperbolic, then $U \cap P_1^2 \subset \Omega_1$, where U is a neighborhood of S_0.*

Theorem. *If S_0 is hyperbolic equilibrium point, then an invariant surface passes through it, and this surface contains the cycle C of the system* (1).

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Эволюция скрытой бифуркацонной границы в возмущенном уравнении Пенлеве-2

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Осциллирующие при отрицательных значениях независимой переменной решения уравнения Пенлеве-2 претерпевают динамическую бифуркацию в малой окрестности нуля и при больших положительных значениях независимой переменной осциллируют либо около $\sqrt{x/2}$, либо около $-\sqrt{x/2}$. Есть еще однопараметрическое решение в окрестности нуля, но оно неустойчиво.

Ответ на вопрос о том, где будет осциллировать решение после бифуркации можно получить из теории изомонодромных бифуркаций, развитой для Пенлеве-2 Итсом, Капаевым и др.

Для неинтегрируемых возмущений можно воспользоваться теорией усреднения для параметров асимптотики, пригодной при больших отрицательных значениях независимой переменной, и получить ответ на вопрос об окрестности осцилляций после бифуркации для траекторий с заданной асимптотикой при $x \to -\infty$.

Основной результат изложен в препринте https://arxiv.org/abs/2012.07895

Einstein's equation on three-dimensional locally homogeneous (pseudo) Riemannian spaces with vectorial torsion

P. N. Klepikov (Barnaul), E. D. Rodionov (Barnaul), O. P. Khromova (Barnaul)

For the first time a metric connection with vectorial torsion, or a semi-symmetric metric connection, was discovered by E. Cartan. Later properties of this connections have been studied by many mathematicians. For example, K.Yano, I.Agricola and others mathematicians investigated the properties of the curvature tensor, geodesis, and behavior of connection under conformal deformations of the original metric. In this paper, we study the Einstein equation on three-dimensional locally homogeneous (pseudo) Riemannian manifolds with metric connection and invariant vector torsion. A theorem was proved that all such manifolds either are Einstein manifolds with respect to the Levi-Civita connection, or are conformally flat. Earlier, the authors investigated the Einstein equation in the case of three-dimensional locally symmetric (pseudo) Riemannian manifolds.

Asymptotic "bouncing ball" type eigenfunctions with focal points of the Schrödinger operator

A. I. Klevin (Moscow)

We consider a semiclassical spectral problem for the Schrodinger operator $-h^2\psi + V(x,y)\psi = E\psi$ with the parameter $h \to +0$. The corresponding Hamiltonian system with the Hamiltonian $H = p^2 + V(x,y)$ is generally nonintegrable. We consider the case when the potential $V(x,y)$ is non-negative, grows at infinity, and is a smooth even function of y. Then the Hamiltonian system has an invariant subspace $p_y = 0$, $y = 0$, which generates a family of libration motions along the x coordinate axis with two turning points. Such a family implies a series of asymptotic eigenfunctions localized in the vicinity of a trajectory with two focal points, and analogous to the eigenfunctions of the "bouncing ball" type of the spectral problem for the two-dimensional Laplace operator in a bounded domain with Dirichlet boundary conditions [1, 2]. A significant difference between the problem under consideration and [1, 2] is the presence of turning points in it, and the asymptotics is determined by Gaussian beams with focal points. The problem is solved using the theory of the Maslov's complex germ [3, 4] and its modification, which made it possible to write the solution globally using Gaussian exponents and Airy functions of complex argument [5]. The application of the results obtained in the three-dimensional quantum anisotropic Kepler problem (potential $(V = -(x^2 + y^2 + \gamma z^2)^{-1/2})$) is considered.

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Berezin-Toeplitz quantizations associated with Landau levels of the Bochner Laplacian on a symplectic manifold

Y. A. Kordyukov (Ufa)

We consider the Bochner Laplacian on high tensor powers of a positive line bundle on a compact symplectic manifold. First, we give a rough asymptotic description of its spectrum in terms of the spectra of certain model operators. It allows us to prove clustering of the spectrum near some points, which can be naturally called Landau levels, under some assumption on the Riemannian metric. We develop the Toeplitz operator calculus with the quantum space, which is the eigenspace of the Bochner Laplacian with eigenvalues from a fixed cluster. We show that it provides a Berezin-Toeplitz type quantization of the symplectic manifold.

Dynamics of curved gauge vortex in parity-breaking media

A.A. Kozhevnikov (Novosibirsk)

The shape and dynamics of the curved vortex in parity-broken medium is considered in the framework of Abelian Higgs gauge field model. It is shown that the static solution is a helix with the specific relation between the curvature and torsion of the vortex line, depending on the strength of the parity-breaking term in the energy functional. Nonlinear dynamical equation of the vortex motion is linearized in the case of small oscillations around the static solution, and the dispersion law of the propagating waves is obtained.

The singular braid group and its representations

T.A. Kozlovskaya (Tomsk)

The group of singular braids SB_n arrives in the theory of invariants of finite types (invariants of Vassiliev-Goussarov). The singular pure braid group SP_n is the kernel of the natural homomorphism of SB_n to the symmetric group S_n. We find a finite set of generators and defining relations for SP_n, describe some properties of this group. Also we construct local linear representations and representation by automorphisms of free group for SB_n.

Thanks. The work is supported by the Ministry of Science and Higher Education of Russia.

Агентное моделирование и построение сценариев прогноза распространения эпидемии COVID-19 в Новосибирской области

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Пандемия COVID-19 поставила вопросы создания современных инструментов для тестирования стратегий снижения ущерба и разработки эффективных мер сдерживания.

В докладе проведен анализ архитектуры агент-ориентированных моделей распространения эпидемий и выявлены основные компоненты для моделирования эпидемических процессов [1]. Рассмотрены преимущества агентного подхода, позволяющие имитировать динамику распространения инфекционных заболеваний в неоднородной популяции и моделировать схемы и механизмы передачи заболевания с учетом демографических, социально-экономических и территориально-пространственных факторов. Использование агентного подхода дает возможность моделировать сценарии эпидемических вспышек, тестировать стратегии борьбы с эпидемией и оценивать влияние на динамику эпидемий многокомпонентных мер и ограничений. При создании агент-ориентированной модели распространения эпидемии в условиях Новосибирской области учитывается социальный статус агентов, состояние здоровья, демографические показатели региона и уровень системы здравоохранения.

Первый этап моделирования включает в себя анализ данных о ежедневном количестве диагностируемых, тестируемых, госпитализированных, критических и смертельных случаев, основанный на методах машинного обучения. Это позволяет определить особенности строения данных, их корреляцию, сезонность, стационарные участки для оптимального использования при решении задачи уточнения параметров модели.

На втором этапе неизвестные параметры модели, такие как число репродукции вируса, количество бессимптомных больных, вероятности протекания легких и тяжелых форм, уточняются по дополнительной информации о количестве ежедневно диагностируемых, тестируемых и смертельных случаев, а также другой статистической информации о заболеваемости [2]. Для минимизации квадратичного целевого функционала используется алгоритм OPTUNA, в основе которого лежит алгоритм древовидных оценок Парзена [3].

На третьем этапе моделируются сценарии распространения COVID -19 и анализируется влияние карантинных мер.

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On functional moduli of surface non-singular flows with two limit cycles

V. Kruglov (Nizhny Novgorod)

Two flows $f^t, f'^t\colon M \to M$ are called *topologically equivalent* if there exists a homeomorphism $h\colon M \to M$ sending trajectories of f^t into trajectories of f'^t preserving orientations of the trajectories. In difference with it, two flows are called *topologically conjugate* if $h \circ f^t = f'^t \circ h$, it means that h sends trajectories into trajectories preserving not only directions but in addition the time of moving. To find an invariant showing the class of topological equivalence or topological conjugacy of each flow from some class of flows means to construct *topological classification* for the class. Note that for some classes their classifications in sense of equivalence and conjugacy coincide; for other classes these classifications completely differ. The second case is about the class that we consider in this paper.

The *Morse-Smale flows* were introduced on the plane for the first time in the classical paper of A.A. Andronov and L.S. Pontryagin in [1]. The non-wandering set of such flows consists of a finite number of hyperbolic fixed points and finite number of hyperbolic limit cycles, besides, saddle separatrices cross-sect only transversally (which means that saddle points of a flow on the plain can not be connected by a separatrix). This important class of flows was topologically classified for many times on different manifolds during the twentieth century. The most important combinatorial invariants are the *Leontovich-Maier's scheme* [2,3] for flows on the plane, the *Peixoto's directed graph* [5] for Morse-Smale flows on any closed surface and the *Oshemkov-Sharko's three-colour graph* [4] for Morse-Smale flows on any closed surface.

All these invariants classify flows only in sense of topological equivalence. The next step is conjugacy. In the work [6] it was proved that for gradient-like flows (i.e. Morse-Smale flows without limit cycles) classes of topological equivalence and topological conjugacy on surfaces coincide. But any limit cycle generates infinite many conjugacy classes for each equivalence class (even two cycles with different periods cannot be conjugate). For two saddles connected by a separatrix the invariant (the so-called *modulus of stability* or *modulus of topological conjugacy*) was found by J. Palis in [7].

Any limit cycle obviously gives at least one modulus equal to its period. In this talk there is considered the class of non-singular flows on the annulus with only two limit cycles on the annulus's boundary components. For these flows there is proved that they have infinite number of moduli, also there is constructed topological classification in sense of topological conjugacy for the considered class of flows.

Thanks. The results have been obtained in collaboration with O. Pochinka and G. Talanova. The authors are partially supported by Laboratory of Dynamical Systems and Applications NRU HSE, grant No 075-15-2019-1931 of the Ministry of Science and Higher Education of Russian Federation.

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Realization of homeomorphisms of surfaces of algebraically finite type by Morse-Smale diffeomorphisms with orientable heteroclinic

A. Morozov (Nizhny Novgorod)

In this paper, we describe the realization of each homotopy class of type T_2 by a Morse-Smale diffeomorphism with an orientable heteroclinic set. Such diffeomorphisms have relatively simple dynamics, since, by virtue of the results of A.N. Bezdezhnykh and V.Z. Grines, have only a finite number of heteroclinic orbits. Moreover, we prove that the type of the homotopy class of any Morse-Smale diffeomorphism with a finite number of heteroclinic orbits is uniquely determined by the index of its heteroclinic intersection.

Let $S_{g,k}$, $g \geq 0$, $k \geq 0$ – be a connected compact orientable surface of genus g with the boundary consisting of k connected components. We set $S_{g,0} = S_g$. Everywhere below, surface mappings are assumed to preserve orientation.

A homeomorphism $h : S_{g,k} \to S_{g,k}$ is called a *periodic homeomorphism* if there exists $m \in \mathbb{N}$, such that $h^m = id$, where id — is the identity transformation. The smallest of these numbers m is called the *period* of the periodic homeomorphism.

A homeomorphism $h : S_g \to S_g$, $g \geq 1$ is called a *reducible* by system C of disjoint simple closed curves C_i, $i = 1, \dots, l$, non-homotopic to zero and pairwise not homotopic to each other if the system of curves C is invariant under h.

A reducible nonperiodic homeomorphism $h : S_g \to S_g$, $g \geq 1$ is called a *homeomorphism of algebraically finite type*, f there exists an h-invariant neighborhood $\mathbb{C}$ of curves of the set C, which consists of the union two-dimensional anulus and such that for each connected component S_{g_j,k_j}, $j = 1, \dots, n$ of the set $S_g \setminus int\,\mathbb{C}$ there is a number $m_j \in \mathbb{N}$ such that $h^{m_j}|_{S_{g_j,k_j}} : S_{g_j,k_j} \to S_{g_j,k_j}$ – is a periodic homeomorphism.

Recall that a diffeomorphism $f : S_g \to S_g$ is called a *Morse-Smale diffeomorphism* if

1) the non-wandering set Ω_f consists of a finite number of hyperbolic orbits;
2) the invariant manifolds W^s_p, W^u_q intersect transversally for any non-wandering points p, q.

Denote by $MS(S_g)$ the set of Morse-Smale diffeomorphisms. In the set of periodic orbits of any diffeomorphism $f \in MS(S_g)$ one can introduce a total order relation, which is a continuation of the partial order introduced by S. Smale [6]. Precisely, let $\mathcal{O}_i, \mathcal{O}_j$ — be the periodic orbits of the Morse-Smale diffeomorphism f. They say that the orbits $\mathcal{O}_i, \mathcal{O}_j$ *are in the relation* $\prec$ $(\mathcal{O}_i \prec \mathcal{O}_j)$, if

$$W^s_{\mathcal{O}_i} \cap W^u_{\mathcal{O}_j} \neq \emptyset.$$

A sequence of different periodic orbits $\mathcal{O}_i = \mathcal{O}_{i_0}, \mathcal{O}_{i_1}, ..., \mathcal{O}_{i_k} = \mathcal{O}_j$ $(k \geqslant 1)$, such that $\mathcal{O}_{i_0} \prec \mathcal{O}_{i_1} \prec \dots \prec \mathcal{O}_{i_k}$ is called a *chain of length k, connecting periodic orbits* $\mathcal{O}_i$ *and* $\mathcal{O}_j$. The chain connecting the periodic orbits of saddle points will be called *saddle chain*. Since the non-wandering set is finite, for any diffeomorphism $f \in MS(M^n)$ there is a well-defined number equal to the length of the maximal saddle chain, which is denoted by

$$beh(f).$$

Let σ_i, σ_j — be saddle points of the diffeomorphism f such that $W^s_{\sigma_i} \cap W^u_{\sigma_j} \neq \emptyset$. Recall that the intersection $W^s_{\sigma_i} \cap W^u_{\sigma_j}$ is a countable set and each point of this set is called *heteroclinic point*, and each orbit of a heteroclinic point is called a *heteroclinic orbit*. For any heteroclinic point $x \in W^s_{\sigma_i} \cap W^u_{\sigma_j}$ For any heteroclinic point $(\vec{v}^u_x, \vec{v}^s_x)$, where:

- $\vec{v}^u_x$ — the tangent vector to the unstable manifold of the point σ_j at the point x;
- $\vec{v}^s_x$ — the tangent vector to the stable manifold of the point σ_i at the point x.

Following [**?**](or see for example [2, p. 7]), we call a heteroclinic intersection of the diffeomorphism f *orientable*, if the ordered pairs of vectors $(\vec{v}^u_x, \vec{v}^s_x)$ set the same orientation of the bearing surface S_g. Otherwise, the heteroclinic intersection is called *non-orientable*.

Two homeomorphisms $h, h' : S_g \to S_g$ are called *homotopic*, if there exists a continuous mapping $H : S_g \times [0, 1] \to S_g$ such that $H(x, 0) = h(x)$? $H(x, 1) = h'(x)$. By $[h]$ we denote the *homotopy class* of the homeomorphism h.

Theorem 4. *In every homotopy class $[h]$ of the homeomorphism $h : S_g \to S_g$, $g \geq 1$ of algebraically finite type, there exists a Morse-Smale diffeomorphism $f : S_g \to S_g$ with orientable heteroclinic intersection.*

In [4], it was announced and then proved in [3] that any diffeomorphism $f \in MS(S_g)$ with orientable heteroclinic intersections has $beh(f) = 1$. This fact was also proved in the work [5] using the factorization method.

Let $f : S_g \to S_g$ be an orientation-preserving Morse-Smale diffeomorphism such that $beh(f) \leq 1$ (that is, the diffeomorphism f has a finite number of heteroclinic orbits). Let us denote by $MS_1(S_g)$ the set of such diffeomorphisms. By virtue of [7], the dynamics of any diffeomorphism $f \in MS_1(S_g)$ can be represented as follows.

The set Ω_f of periodic orbits of the maps f can be divided into subsets $\Omega^i_f, i \in \{\omega, s, u, \alpha\}$ as follows:

* Ω^ω_f — the set of all sink orbits;
* Ω^s_f — s the set of saddle orbits whose unstable manifolds do not contain heteroclinic points;
* Ω^u_f — the set of the remaining saddle orbits of the system;
* Ω^α_f — the set of source orbits.

Let

$$\mathcal{A}_f = \Omega^\omega_f \cup W^u_{\Omega^s_f},\ \mathcal{R}_f = \Omega^\alpha_f \cup W^s_{\Omega^u_f},\ V_f = S_g \setminus (\mathcal{A}_f \cup \mathcal{R}_f).$$

By construction, all heteroclinic points of the diffeomorphism f belong to the set V_f, which consists of a finite number of connected components V_i, $i = 1, \dots, m$. Each component V_i is homeomorphic to an open two-dimensional ring and is invariant with respect to some power $q_i \in \mathbb{N}$ of the diffeomorphism f. Each heteroclinic orbit $\mathcal{O}_x \subset V_i$ of the diffeomorphism f^{q_i} is assigned the index $\xi_{\mathcal{O}_x}$, equal to $+1(-1)$, if the orientation of the carrier the surface (not) coincides with the orientation defined by the pair of vectors $(\vec{v}^u_x, \vec{v}^s_x)$. Since the diffeomorphism f preserves orientation, the index $\xi_{\mathcal{O}_x}$ does not depend on the choice of a point in the orbit $\mathcal{O}_x$. We set

$$\xi_i = \sum_{\mathcal{O}_x \subset V_i} \xi_{\mathcal{O}_x},\ \xi_f = \sum_{i=1}^m |\xi_i|$$

and we will call the number ξ_f *the index of the heteroclinic intersection* of the diffeomorphism $f \in MS_1(S_g)$.

It follows directly from the definition that the heteroclinic intersection index is a non-negative number. The next result shows that it uniquely determines the type of the homotopy class $[f]$ of the diffeomorphism $f \in MS_1(S_g)$.

Theorem 5. *Let $f \in MS_1(S_g)$. Then $[f]$ is of type T_1, if $\xi_f = 0$ and $[f]$ is of type T_2, if $\xi_f > 0$.*

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On bifurcations changing the homotopy type of the closure of an invariant saddle manifold of a surface diffeomorphism

E. Nozdrinova (Nizhny Novgorod)

The results have been obtained in collaboration with Olga Pochinka.

It is well known from the homotopy theory of surfaces that an ambient isotopy does not change the homotopy type of a closed curve. In the language of dynamical systems, this means that any arc in the space of diffeomorphisms connecting isotopic diffeomorphisms with invariant closed curves from different homotopy classes necessarily passes through a bifurcation value. In this paper, we describe a scenario that changes the homotopy type of the closure of the invariant manifold of a saddle point of a polar diffeomorphism on a two-dimensional torus to any given homotopy nontrivial type. Moreover, the arc constructed is stable in the space of diffeomorphisms and does not change the topological conjugacy class of the original diffeomorphism. The ideas proposed in this paper for constructing such an arc for a two-dimensional torus can be naturally generalized to surfaces of a larger genus.

The problem of the existence of an arc with no more than a countable (finite) number of bifurcations connecting structurally stable systems (Morse-Smale systems) on manifolds is included in the list of fifty Palis-Pugh problems [5] under number 33. The report will present a solution this problem for polar gradient-like diffeomorphisms of a torus.

In 1976, S. Newhouse, J. Palis, F. Takens [2] introduced the concept of a stable arc connecting two structurally stable systems on a manifold. Such an arc does not change its quality properties with little movement. In the same year, S. Newhouse and M. Peixoto [3] proved the existence of a simple arc (containing only elementary bifurcations) between any two Morse-Smale flows. From the result of the work of J. Fleitas [1] it follows that a simple arc constructed by Newhouse and Peixoto can always be replaced by a stable one. For Morse-Smale diffeomorphisms given on manifolds of any dimension, examples of systems that cannot be connected by a stable arc are known. In this connection, the question naturally arises of finding an invariant that uniquely determines the equivalence class of the Morse-Smale diffeomorphism with respect to the connection relation by a stable arc (*is a component of stable connection*).

In this report, a stable arc will be constructed connecting any two cascades from the class in question. Note that it was shown in [4] that polar cascades on a two-dimensional sphere are always connected by an arc without bifurcations. For a two-dimensional torus, the situation is different due to the fact that the closures of the invariant manifolds of saddle points of the polar cascade are circles belonging to any previously defined homotopy class. It follows directly from this that in the general case there is no arc without bifurcations between the systems under consideration. Nevertheless, the authors of this paper established the following result.

Theorem. Any diffeomorphisms $f_0, f_1 \in G$ belong to the same class of stable isotopy connection. Moreover, there exists a stable arc connecting them, all the bifurcation points of which are saddle-nodes.

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Квази-симметричная форма двумерных интегрируемых систем

М. Павлов (Москва)

В докладе будет показано, что многие многокомпонентные интегрируемые системы (например, все двумерные редукции иерархии КП) могут быть записаны в квази-симметричной форме.

Построение квази-симметричной формы будет продемонстрировано на примере систем "сцепленных КдФ".

Именно в этом случае, будет показано, что такая квази-симметричная форма является гамильтоновой, причём, зависящей от числа произвольных параметров, равного числу полевых переменных в системе. Различными предельными переходами можно получить уже известный результат М. Антоновича и А. Форди о максимальном числе локальных гамильтоновых структур для интегрируемых систем уравнений.

On possible rates of convergence for the ergodic averages

I. Podvigin (Novosibirsk)

New facts are presented about positive sequences tending to zero, which are estimates of the pointwise rate of convergence for the ergodic averages. In particular, by representing such sequences in the form of the ratio ϕ/n, it is shown that ϕ is separated from zero on a subset of natural numbers with positive lower asymptotic density.

On the non-integrability and dynamics of discrete models of threads
I. Polekhin (Moscow)

In the talk we will consider the dynamics of planar n-gons, which can be considered as discrete models of threads. The main result is that, under some weak assumptions, these systems are not integrable in the sense of Liouville. This holds for both completely free threads and for threads with fixed points that are placed in external force fields. We will present sufficient conditions for the positivity of topological entropy in such systems. We will briefly consider other dynamical properties of discrete threads and we will also consider discrete models of inextensible yet compressible threads.

Двойственность Долгачева-Никулина и зеркальная симметрия трехмерных многообразий Фано
В. Пржиялковский (Москва)

Двойственность Долгачева-Никулина для поверхностей типа К3, грубо говоря, меняет местами решетки алгебраических и трансцендентных циклов во второй группе когомологий. Ожидается, что слой модели Ландау-Гинзбурга трехмерного многообразия Фано двойственен по Долгачеву-Никулину антиканоническому сечению этого многообразия. Мы обсудим, как доказать это ожидание, и как оно связано с теоретико-ходжевыми гипотезами зеркальной симметрии.

Integrable magnetic billiard in circular domain
S. Pustovitov (Moscow)

Mathematical billiard is a dynamical system that describes the motion of a material point inside a closed bounded domain (billiard table). The material point moves along a smooth trajectory until it hits a boundary of the table and then reflects from it according to the usual reflection law. We know that the classical planar mathematical billiard, where a velocity vector of the point is constant, is a Hamiltonian system. The integrability of this billiard depends on shape of the table. For example, the billiard inside an ellipse or some domain bounded by confocal ellipses and hyperbolas is integrable, i.e. there exist a first integral that is functionally independent with the Hamiltonian. Such billiards were considered by V. V. Vedyushkina in [2].

Consider a billiard obtained from the classical planar billiard by adding a magnetic field forcing on a material point. Suppose the magnetic field is uniform in signature b and orthogonal to the billiard table. It terns out that if the table is bounded by an ellipse then there doesn't exist two functionally independent first integrals, i.e. system is not integrable anymore [3]. Also the general problem of the integrability of the magnetic billiard was investigated by M. Bialy and A. E. Mironov [4]. The following theorem guarantees the integrability of a circular billiard.

Theorem 6. *Consider a planar billiard inside a circle with orthogonal uniform magnetic field. A trajectory of the material point is piecewise smooth curve which consists of circular arcs (they are called the Larmor circles) of a common radius A and centers of these circles are equidistant from the center of the table at a distance R. Therefore there are two first integrals R and A, which are functionally independent.*

Theorem 7. *Pre-image of every regular value of the integrals R and A is homeomorphic to a torus in a phase space.*

That means one can treat the system using the Fomenko-Zieschang theory of marked molecules [1]. In particular, by fixing some values of the integral R one can obtain a marked molecule of a kind A–A with $r = \infty$ and $\epsilon = -1$, which was not observed for billiards before.

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Dynamic Inverse Problems and Conservation Laws

M.A. Shishlenin (Novosibirsk), S.I. Kabanikhin (Novosibirsk),
N.S. Novikov (Novosibirsk), D.V. Klyuchinskiy (Novosibirsk)

Conservation laws play a distinguished role in mathematical physics. They have practical applications in several areas related to differential equations, including integrability theory, asymptotic integrability, and the construction of geometric numerical integration schemes.

The mathematical model of the acoustic tomography is based on the conservation laws [2], which not only describes such effects as diffraction, refraction, reaction, and acoustic absorption of inhomogeneous media on a physical level and allows us to simulate the radiation patterns of sources and receivers [5], to decrease the smoothness of the sought coefficients [1].

We investigate the mathematical model of the 2D acoustic waves propagation in homogeneous and heterogeneous areas. The hyperbolic first-order system of partial differential equations is considered and solved by the Godunov method of the first order of approximation. This is a direct problem with non-reflecting boundary conditions.

As the main aim of the work, we solve the coefficient inverse problem of recovering density and speed of sound propagation of the medium [4]. The inverse problem is reduced to an optimization problem which is solved by the gradient descent method [3].

The hyperbolic first-order system allows us to propose a more realistic model from the physical point of view. These equations are obtained directly from the conservation laws of continuum mechanics. It allows us to control the preservation of the basic invariants during the solution of direct and inverse problems. This is important for solving unstable problems, as the conservation laws of the main invariants are the only criterion of the well-posedness of the solution.

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Realization of homeomorphisms of surfaces of algebraically finite type by Morse-Smale diffeomorphisms with orientable heteroclinic

V. Shmukler (Nizhny Novgorod)

In this paper, we consider the class G of orientation-preserving Morse-Smale diffeomorphisms defined on a closed 3-manifold whose non-wandering set consists of exactly four points of pairwise different Morse indices. It is known that the two-dimensional saddle separatrices of any such diffeomorphism always intersect and their intersection necessarily contains non-compact heteroclinic curves, but may also contain compact ones. The main result of this work is the construction of a path in the space of diffeomorphisms connecting the diffeomorphism $f_0 \in G$ with the diffeomorphism $f_1 \in G$, which does not have compact heteroclinic curves. This result is an important step in solving the open problem of describing the topology of 3-manifolds admitting gradient-like diffeomorphisms with wildly embedded saddle separatrices.

Despite the simple structure of the non-wandering set, the class under consideration contains diffeomorphisms with wildly embedded saddle separatrices [2]. It was proved in [1] that for any diffeomorphism $f \in G$ the set $H_f = W^s_{\sigma_1} \cap W^u_{\sigma_2}$ is not empty and contains at least one non-compact heteroclinic curve. According to [3], in the case of a manual embedding of the closures of one-dimensional separatrices of the diffeomorphism $f \in G$, the bearing manifold M^3 admits a Heegaard decomposition of genus 1 and, therefore, is a lens space. In the case of wild embedding, the description of the topology of the supporting manifold is an open problem formulated in [1].

In the present paper, an important step has been taken in solving this problem, namely, the following fact is proved.

Theorem 8. *Let the manifold M^3 admit a diffeomorphism $f_0 \in G$. Then the same manifold admits a diffeomorphism $f_1 \in G$, a wandering set that does not contain compact heteroclinic curves.*

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Automorphism groups of Severi-Brauer varieties

C. Shramov (Moscow)

A Severi-Brauer variety is a form of a projective space, and its automorphism group is an inner form of the group PGLn. I will prove a sharp multiplicative bound for the orders of finite subgroups of the latter group provided that the base field contains all roots of unity (e.g. it is a function field on some algebraic variety over the complex numbers).

On non-singular flows on n-manifolds with two limit cycles

D. Shubin (Nizhny Novgorod)

The results were obtained in collaboration with O. Pochinka. This talk is devoted to the so-called *NMS-flows* (non-singular Morse-Smale flows) which are Morse-Smale flows without fixed points. Such flows have close connection with topology of ambient manifold. Exhaustive classification of this systems with exactly two limit cycles on closed n-manifolds was obtained and will be presented.

General theory (see e.g. [1]) implies that ambient manifold M^n is the union of stable manifolds and simultaneously the union of unstable manifolds. Thus, on of this trajectories is attracting and another is repelling.

Due to Poincaré-Hopf theorem, Euler characteristic of the ambient manifold is 0. It leaves only torus and Klein bottle for two-dimensional manifold. Classification of such flows is a part of the problem solved in [2,3,4]. Namely, there are two equivalent classes of considered flows on the torus and three on the Klein bottle.

For three-dimensional manifolds the fact that Euler characteristic is equal to zero does not contract the class of manifolds since all three-dimensional manifolds have Euler characteristic equal to zero. Necessary and sufficient conditions follow from [5] where author considers wider class of dynamical systems. However, the results are not contain realisation and it is impossible to judge whether one or other flow is admissible.

In case of two non-twisted orbits the topology of ambient manifolds is known from [6]: they are so called *lens spaces*, which are obtained by attaching of two solid tori along their boundary. We establish that every lens space, except sphere, admits exactly two equivalent classes of considered flows. On the sphere S^3 there is unique class due to [6]. If orbits are twisted then there is only one ambient manifold admitting such flow. It is $\mathbb{S}^2\widetilde{\times}\mathbb{S}^1$.

This result was generalized for any lens space and also for any dimension $n \geqslant 3$. So, ambient manifold $M^n,\ n > 3$ similarly to the previous case is two generalised solid tori $\mathbb{D}^{n-1} \times \mathbb{S}^1$ glued along boundaries. The results [7] and [8] imply that only two manifolds which admit NMS-flow with two limit cycles are $\mathbb{S}^{n-1} \times \mathbb{S}^1$ and $\mathbb{S}^{n-1}\widetilde{\times}\mathbb{S}^1$.

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Quasi-periodic Henon-like attractors: universal scenario of appearance and radio-physical applications

N.V. Stankevich (Nizhny Novgorod)

Chaos is a typical attribute of nonlinear dynamical systems in various fields of science and technology [1-2]. One of the conventional indicator of chaotic dynamics is the largest Lyapunov exponent. Chaos is implemented in a situation when the spectrum of Lyapunov exponents have one positive, one zero and at least one negative exponents for a flow. In this work, we consider a somewhat different situation, when the spectrum of Lyapunov exponents of chaotic attractor contains an additional zero Lyapunov exponent, it means it includes one positive, two zero and several negative exponents [3]. Such kind of attractors called Quasi-periodic Henon-like attractors because it's can be represented as multiplication of torus and Henon attractor. As part of the work, examples of flow systems will be presented in which this type of chaotic dynamics is observed: modified Anishchenko-Astakhov generator, two-mode vand Pol oscillator, coupled generators of quasi-periodic oscillations and ensemble of five van der Pol oscillators. A universal scenario such chaotic attractors occurrence associated with torus-doubling bifurcations and homoclinc bifurcation of unstable torus will be discussed.

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Quasi-periodic Henon-like attractors: universal scenario of appearance and radio-physical applications

A. Tetenov (Novosibirsk), O. Chelkanova (Gorno-Altaisk)

A Jordan arc $\gamma \subset \mathbb{R}^n$ is called *self-similar* if there is a system $\mathcal{S} = \{S_1, ..., S_m\}$ of contracting similarities of $\mathbb{R}^n$ such that $\gamma = S_1(\gamma) \cup \ldots \cup S_m(\gamma)$. The self-similarity condition is quite restrictive for Jordan arcs. As it was proved in [3], if the semigroup $G(\mathcal{S})$, generated by $S_1, ..., S_m$ contains a sequence of pairs of elements $g_n, h_n \in G(\mathcal{S})$ such that the intersections $g_n^{-1}h_n(\gamma) \cap \gamma$ converge to γ, then γ is a straight line segment. This last property implies that each self-similar Jordan arc γ can be obtained by iterating some finite polygonal line whose vertices lie on γ on each step of the iteration process. Moreover, unless γ is a line segment, its subarcs have the same Hausdorff dimension $s > 1$ and none of these subarcs admits a one-to-one projection to a line segment [2].

In the case of self-affine Jordan arcs the situation remained unclear. From one hand, smooth affine fractal interpolation functions on $[0, 1]$ form a dense subspace of $C([0, 1]$; and their graphs are naturally projected to $[0, 1]$. From the other hand, it was proved in [1] that each C^2-smooth self-affine arc is a segment of a parabola.

A Jordan arc $\gamma \subset \mathbb{R}^n$ is called *locally self-affine*, if for any proper subarc $\gamma' \subset \gamma$ there is a non-degenerate affine map S, such that $S(\gamma) \subset \gamma'$.

A injective affine map S of $\mathbb{R}^2$ is called a *affine shift* of a Jordan arc γ, if S has no fixed points on γ and both $\gamma \setminus S(\gamma)$ and $S(\gamma) \setminus \gamma$ are proper subarcs in γ and $S(\gamma)$ respectively.

We prove the following rigidity theorem for locally self-affine arcs:

Theorem 9. *Let $\gamma \subset \mathbb{R}^2$ be a locally self-affine Jordan arc such that there is a sequence of affine shifts f_k of the arc γ, which converges to* Id.

Then γ is a segment of a parabola or straight line.

This theorem allows to prove that each self-affine Jordan arc γ can be obtained by affine iterations of some finite polygonal line whose vertices lie on γ on each step of the iteration process.

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Lagrangian Mironov cycles in complex Grassmannians

N.A. Turin (Moscow, Dubna)

Every algebraic variety can be studied as a real symplectic manifold equipped by a Kahler form coming from an embedding to an appropriate projective space. Therefore it is natural problem to examine which lagrangian submanifolds can (or can not) appear in this framework for a given algebraic variety and a choosen polarization. This problem is important for both Geometric Quantization and Mirror Symmetry; and this problem is widely open even for the basic algebraic variety such as complex projective spaces. We study this problem for complex Grassmanians taking as the symplectic form the Kahler form lifted by the Plucker embedding. It is not hard to see that $Gr_{\mathbb{C}}(k, n+1)$ admits Hamiltonian action of real torus T^n, such that the corresponding moment maps $\mu_1, ..., \mu_n$ can be described in pure geometrical terms; on the other hand this variety admits natural anti holomorphic involution with real part $Gr_{\mathbb{R}}(k, n+1)$. It follows one can apply the construction, proposed by A. Mironov for the projective spaces, results with a wide class of lagrangian submanifolds, which we call "Mironov cycles". In my talk I present two "ends"of this class for $Gr_{\mathbb{C}}(k, n+1)$, constructed with a single moment map (the case of homogeneity 1) and with n moment maps (the case of homogeneity n).

The Schur-Sato theory for quasi-elliptic rings and some of its

A.B. Zheglov (Moscow)

The notion of quasielliptic rings appeared as a result of an attempt to classify a wide class of commutative rings of operators found in the theory of integrable systems, such as rings of commuting differential, difference, differential-difference, etc. operators. They are contained in a certain non-commutative "universe"ring - a purely algebraic analogue of the ring of pseudodifferential operators on a manifold, and admit (under certain mild restrictions) a convenient algebraic-geometric description. An important algebraic part of this description is the Schur-Sato theory - a generalisation of the well known theory for ordinary differential operators. I'll talk about this theory in dimension n and about some of its unexpected corollaries: new short proofs of the generalized Birkhoff decomposition and of the Abhyankar formula.

Morse-Bott function for topological flows with a finite hyperbolic chain-reccurent set

S. Kh. Zinina (Saransk)

We introduce a class G of continuous flows f^t on M^n that generalize the concept of Morse-Smale flows. Such flows have a hyperbolic (in the topological sense) chain-recurrent set R_{f^t} consisting of a finite number of orbits (chain components). Each non-wandering orbit is either a fixed point or a periodic orbit. The main result is the following theorem: each flow $f^t \in G$ has a Morse-Bott energy function whose critical points are either nondegenerate or have a degeneracy degree of 1.

Numerical solution and stability analysis of the inverse problem for the diffusion-logistic model

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The diffusion-logistic model describes the process of information dissemination in social networks [1]. The type of information is determined by the coefficients of the mathematical model and the initial conditions of the problem. In the inverse problem [2], it is necessary to determine the function of the initial data using additional information. This problem was reduced to the problem of minimizing a quadratic functional and solved using a combination of the particle swarm and Nelder-Mead methods, the gradient descent method, and the multilevel gradient method. The singular value decomposition analysis was carried out for two discrete operators of linearized inverse problems. It helps one to control the degree of ill-posedness of inverse problem and to construct the regularization algorithm.

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