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ENUMERATION OF STRICTLY DEZA GRAPHS WITH AT MOST 21 VERTICES

S.V. GORYAINOV, D.I. PANASENKO, L.V. SHALAGINOV

Abstract. A Deza graph Γ with parameters (v, k, b, a) is a k-regular graph with v vertices such that any two distinct vertices have b or a common neighbours, where $b \geqslant a$. A Deza graph of diameter 2 which is not a strongly regular graph is called a strictly Deza graph. We find all 139 strictly Deza graphs up to 21 vertices and list corresponding constructions and properties.

Keywords: Deza graph, strictly Deza graph, strongly regular graph, dual Seidel switching.

1. Introduction

Deza graphs were introduced in 1999 [3] as a generalisation of strongly regular graphs. A *Deza graph* Γ with parameters (v, k, b, a) is a k-regular graph with v vertices for which the number of common neighbours of two distinct vertices takes just two values, b or a, where $b \geqslant a$. A *strongly regular graph* G with parameters (v, k, λ, μ) is a k-regular graph with v vertices such that any two adjacent vertices have λ common neighbours and any two non-adjacent vertices have μ common neighbours. A Deza graph of diameter 2 which is not a strongly regular graph is called a *strictly Deza graph*.

In 1999 [3] the complete list of strictly Deza graphs with at most 13 vertices was presented and different constructions for those graphs were discussed. In 2011 [6] this list was extended up to 16 vertices. In 2014 S. Goryainov and L. Shalaginov [5] found all Cayley-Deza graphs with $a > 0$ up to 59 vertices and listed all corresponding groups. These results are available on the web pages

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http://alg.imm.uran.ru/dezagraphs/dezatab.html
and
http://alg.imm.uran.ru/dezagraphs/deza_cayleytab.html.

A k-regular graph is called a *divisible design graph* if its vertex set can be partitioned into m classes of size n, such that two distinct vertices from the same class have exactly λ_1 common neighbors, and two vertices from different classes have exactly λ_2 common neighbors. The definition implies that all divisible design graphs are Deza graphs. Divisible design graphs were first studied in master's thesis by M.A. Meulenberg [10] and the list of feasible parameters of divisible design graphs up to 50 vertices was presented. In 2011-2013 in the following papers [2, 7] divisible design graphs were studied in more details and the existence of graphs was resolved in all but one cases for graphs up to 27 vertices.

A *coherent configuration* $\mathcal{X}$ on a finite set V can be thought as a special partition of $V \times V$ for which the diagonal of $V \times V$ is a union of classes [1]. If in a coherent configuration the diagonal of $V \times V$ is a single class then this coherent configuration is an association scheme.

Each graph has a specific coherent configuration associated with it, known as *WL-closure*, which can be obtained using *Weisfeiler-Leman algorithm* [9]. Given a graph G with the vertex set $V(G)$ and the edge set $E(G)$, this algorithm constructs the smallest coherent configuration on $V(G)$ for which $E(G)$ is a union of classes. The number of classes in WL-closure is called *WL-rank*.

In this paper we find all strictly Deza graphs up to 21 vertices. It turns out that the number $Num(v)$ of non-isomorphic strictly Deza graphs with $v \leqslant 21$ is given by the following table:

v	8	9	10	11	12	13	14	15	16	17	18	19	20	21
$Num(v)$	3	2	1	0	6	1	1	1	10	3	13	11	56	31

In previous papers [3, 6] corresponding constructions were given for almost all graphs. However, among graphs we found, the constructions of almost half of the graphs were unknown. Therefore, we give a list of the found graphs and indicate some known constructions. This shows for which graphs the constructions are unknown, thus giving one of the approaches to finding new constructions.

This paper is organised as follows. In Section 2 we describe the algorithm used for enumerating Deza graphs. In Section 3 we give an overview of some known constructions of Deza graphs. In Section 4 we present tables with enumeration results, corresponding properties, constructions and WL-ranks.

2. Enumeration algorithm

2.1. **Search for feasible parameters.** Let Γ be a Deza graph with parameters (v, k, b, a). For a fixed vertex u in Γ, define

$$\alpha = |\{w \in V(\Gamma) : |N(u) \cap N(w)| = a\}|$$

and

$$\beta = |\{w \in V(\Gamma) : |N(u) \cap N(w)| = b\}|,$$

where $V(\Gamma)$ is the vertex set of Γ and $N(u), N(w)$ are the neighborhoods of u and w, respectively.

In [3, Proposition 1.1] it was proved that α and β do not depend on u and can be computed as follows:

$$\alpha = \frac{b(v-1)-k(k-1)}{b-a},\ \beta = \frac{a(v-1)-k(k-1)}{a-b} \text{ if } a \neq b$$

and

$$\alpha = \beta = \frac{k(k-1)}{a} \text{ otherwise.}$$

At the first step, for a fixed number of vertices, we calculate all feasible parameters of Deza graphs satisfying restrictions given by the following lemma.

Lemma 1. [3, Corollary 1.2] *If there is a Deza graph with parameters* (v, k, b, a), *then the following statements hold:*
(i) $b - a$ *divides* $b(v-1) - k(k-1)$;
(ii) if $\alpha \neq 0$, *then* $v \geqslant 2k - a$;
(iii) if $\alpha, \beta \neq 0$, *then* $a(v-1) < k(k-1) < b(v-1)$.

The number $Num'(v)$ of feasible parameters of Deza graphs with $v \leqslant 21$ is given in the table below:

v	8	9	10	11	12	13	14	15	16	17	18	19	20	21
$Num'(v)$	14	10	24	19	34	26	44	34	73	40	74	60	86	77

2.2. **Constructing adjacency matrices.** Given feasible parameters (v, k, b, a) of a Deza graph Γ, we initially construct the first two rows of the adjacency matrix using the following method.

Let us consider three possible cases.

Case 1: $\alpha < k$ holds. Then there exist two adjacent vertices in Γ that have exactly b common neighbors. Without loss of generality, we may assume that the first two rows of the adjacency matrix of Γ look like this:

$$\begin{array}{llllll} 01 & \overbrace{1\ldots1}^{b} & \overbrace{1\ldots1}^{k-b-1} & \overbrace{0\ldots0}^{k-b-1} & \overbrace{0\ldots0}^{v-2k+b} \\ 10 & 1\ldots1 & 0\ldots0 & 1\ldots1 & 0\ldots0 \end{array}$$

Case 2: $\alpha = k$ holds. Then there exist two adjacent vertices in Γ that have exactly b common neighbors (otherwise, Γ is strongly regular). Without loss of generality, we may assume that the first two rows of the adjacency matrix of Γ are the same as in Case 1.

Case 3: $\alpha > k$ holds. Then there exist two non-adjacent vertices in Γ that have exactly a common neighbors. Without loss of generality, we may assume that the first two rows of the adjacency matrix of Γ look like this:

$$\begin{array}{llllll} 00 & \overbrace{1\ldots1}^{a} & \overbrace{1\ldots1}^{k-a} & \overbrace{0\ldots0}^{k-a} & \overbrace{0\ldots0}^{v-2k+a-2} \\ 00 & 1\ldots1 & 0\ldots0 & 1\ldots1 & 0\ldots0 \end{array}$$

To enumerate the next three rows of the matrix, we use the following approach.

Assume we have specified the first two rows of the adjacency matrix of a Deza graph Γ:

$$\begin{array}{cl} 0\spadesuit 1 & 1\dots1\ 1\dots1\ 0\dots0\ 0\dots0 \\ \spadesuit 01 & 1\dots1\ 0\dots0\ 1\dots1\ 0\dots0, \end{array}$$

where $\spadesuit$ denotes the same symbol ('1' in Cases 1 and 2; '0' in Case 3).

Further, we construct the third row. Notice that the columns can be divided into four blocks as follows:

$$\begin{array}{cc|c|c|c} 0\spadesuit 1 & 1\dots1 & 1\dots1 & 0\dots0 & 0\dots0 \\ \spadesuit 01 & 1\dots1 & 0\dots0 & 1\dots1 & 0\dots0 \\ 110 & *\dots* & *\dots* & *\dots* & *\dots* \end{array}.$$

and changing the order of 1s inside each block gives an equivalent matrix. We say that two partially filled matrices are *equivalent* if the graphs determined by them are isomorphic. Note that, for a partially filled matrix, changing the order of vertices inside each block gives an equivalent matrix.

For example, these two matrices are equivalent:

$$\begin{array}{cc|c|c|c} 011 & 111 & 111 & 000 & 00 \\ 101 & 111 & 000 & 111 & 00 \\ 110 & 110 & 100 & 110 & 10 \end{array} \qquad \begin{array}{cc|c|c|c} 011 & 111 & 111 & 000 & 00 \\ 101 & 111 & 000 & 111 & 00 \\ 110 & 101 & 001 & 011 & 10 \end{array}$$

For the next rows, this division into blocks can be extended, where the number of blocks multiplies by 2 with each row (8 blocks for the 4th row, 16 blocks for the 5th row, etc.).

Thus, to construct the next row of the adjacency matrix, we consider all possible numbers of 1s in each block. Then the obtained matrices are checked for equivalence using Magma, and the procedure for adding a new row repeats for all nonequivalent matrices. Since the equivalent matrices will give isomorphic graphs at the end, leaving all nonequivalent options does not reduce the exhaustiveness of the algorithm.

For the remaining $v-5$ rows of the matrix we use exhaustive search of possible rows: all possible combinations of required number of 1s in $v-i$ positions, where i is the number of current row. For each added row we also check if the resulting matrix is the adjacency matrix of the Deza graph.

We use Magma to check whether graphs are isomorphic after the completion of the enumeration. In case of $a=0$ we calculate the diameter of the resulting graphs. If the diameter does not equal to 2, then this graph is not a strictly Deza graph, so we exclude it.

3. Constructions of Deza graphs

(1) Cayley graphs. The way to check whether a Deza graph is a Cayley graph was described in [3, Proposition 2.1]. In the resulting table 1 below, we denote Cayley-Deza graphs as 'cay'.

(2) Association schemes. The way to construct Deza graphs from association schemes was described in [3, Theorem 4.2]. In the resulting table 1 below, we denote Deza graphs obtained from association schemes as 'as'.

(3) Dual Seidel switching. An involutive automorphism of a graph is called *Seidel automorphism* if it interchanges only non-adjacent vertices. In [3, Theorem 3.1] the method for obtaining Deza graphs from strongly regular graphs with Seidel automorphism, called dual Seidel switching, was described. In [8, Theorem 5, 6] the generalisation of this method for strongly regular graphs and Deza graphs was presented.

In the resulting table 1 below, we denote Deza graphs obtained by dual Seidel switching as ‘dss’, Deza graphs obtained by generalised dual Seidel switching as ‘gdss’ and Deza graphs obtained by generalised dual Seidel switching from other Deza graphs as ‘gdss(n)’, where n denotes the serial number of the used Deza graph. We also note two more constructions related to Seidel automorphisms from [8]: [8, Theorem 7] as ‘sa1’ and [8, Theorem 8] as ‘sa2’.

(4) Lexicographic product of graphs. [3, Proposition 2.3] described the condition under which the lexicographic product of a strongly regular graph and a Deza graph is a Deza graph. In this paper we restrict ourselves to three applications of this construction, which describe Deza graphs with parameters $(v, k, k-1, a)$.

In the resulting table 1 below, we denote Deza graphs obtained by [3, Example 2.4] as ‘lp1’, by [4, Construction 1] as ‘lp2’ and by [4, Construction 2] as ‘lp3’.

4. Enumeration results

In the table below, # gives a serial number, v, k, b, a are the parameters of a Deza graph, ‘egv’ denotes the number of distinct eigenvalues, ‘int’ denotes whether a graph has an integral spectrum, ‘ddg’ means divisible design graph, ‘WL-rank’ denotes WL-rank of a graph. The column ‘constructions’ describes constructions from Section 3, which can be used to obtain this graph.

Note that sometimes generalised dual Seidel switching produces Deza graphs from Deza graphs with an unknown construction. For example, we can obtain the graph with serial number 29 from the graph with serial number 30 and vice versa. These cases are not presented in the resulting table.

Table 1: Strictly Deza graphs with at most 21 vertices

#	v	k	b	a	egv	int	ddg	WL-rank	constructions
1	8	4	2	0	4	+	+	4	cay, as
2	8	4	2	1	5	−	−	5	cay, as
3	8	5	4	2	4	+	+	4	cay, as, lp1
4	9	4	2	1	5	+	−	10	dss, gdss
5	9	4	2	1	5	−	−	5	cay, as
6	10	5	4	2	4	−	+	4	cay, as, lp2
7	12	5	2	1	4	+	+	4	cay, as
8	12	6	3	2	5	−	−	10	gdss(9)
9	12	6	3	2	4	+	+	5	cay, as
10	12	7	4	3	5	+	+	6	cay, as
11	12	7	6	2	4	+	+	4	cay, as, lp1
12	12	9	8	6	4	+	+	4	cay, as, lp1
13	13	8	5	4	4	−	−	4	cay, as
14	14	9	6	4	4	−	−	4	cay, as
15	15	6	3	1	5	+	−	16	dss, gdss

#	v	k	b	a	egv	int	ddg	WL-rank	constructions
16	16	5	2	1	7	–	–	16	cay
17	16	7	4	2	5	+	–	6	cay, as, sa1, sa2
18	16	7	4	2	5	+	–	6	cay, as, sa1, sa2
19	16	8	4	2	6	–	–	8	cay, as
20	16	9	6	4	5	+	–	6	cay, as, dss, gdss
21	16	9	6	4	5	+	–	12	dss, gdss
22	16	9	8	2	4	+	+	4	cay, as, lp1
23	16	11	8	6	5	+	–	5	cay, as, sa1, sa2
24	16	12	10	8	5	+	–	5	cay, as
25	16	13	12	10	4	+	+	4	cay, as, lp1
26	17	8	4	3	10	–	–	93	–
27	17	8	4	3	13	–	–	83	–
28	17	8	4	3	13	–	–	83	–
29	18	8	4	2	18	–	–	162	–
30	18	8	4	2	12	–	–	34	–
31	18	8	4	2	13	–	–	65	–
32	18	8	4	2	10	–	–	18	cay
33	18	8	4	2	11	–	–	54	–
34	18	8	4	2	8	–	–	19	gdss(32, 35)
35	18	8	4	2	5	–	–	5	cay, as
36	18	8	4	3	13	–	–	98	–
37	18	9	6	4	7	–	–	36	gdss(38)
38	18	9	6	4	5	–	+	5	cay, as
39	18	9	8	4	5	+	+	13	lp3
40	18	9	8	4	4	+	+	4	cay, as, lp2
41	18	13	12	8	4	+	+	4	cay, as, lp1
42	19	6	2	1	13	–	–	65	–
43	19	6	2	1	13	–	–	65	–
44	19	6	2	1	13	–	–	65	–
45	19	6	2	1	13	–	–	65	–
46	19	6	2	1	4	–	–	4	cay, as
47	19	6	2	1	13	–	–	65	–
48	19	8	4	2	9	–	–	55	–
49	19	8	4	2	14	–	–	93	–
50	19	8	4	2	18	–	–	361	–
51	19	12	8	7	8	–	–	24	–
52	19	12	8	7	13	–	–	61	–
53	20	6	2	1	11	–	–	42	–
54	20	6	2	1	10	–	–	100	–
55	20	6	2	1	18	–	–	200	–

#	v	k	b	a	egv	int	ddg	WL-rank	constructions
56	20	6	2	1	5	–	–	6	as
57	20	6	2	1	10	–	–	80	–
58	20	6	2	1	20	–	–	400	–
59	20	6	2	1	20	–	–	400	–
60	20	6	2	1	19	–	–	400	–
61	20	6	2	1	19	–	–	200	–
62	20	6	2	1	18	–	–	200	–
63	20	6	2	1	20	–	–	202	–
64	20	6	2	1	20	–	–	400	–
65	20	6	2	1	19	–	–	400	–
66	20	6	2	1	16	–	–	202	–
67	20	6	2	1	20	–	–	400	–
68	20	6	2	1	20	–	–	202	–
69	20	6	2	1	20	–	–	400	–
70	20	6	2	1	18	–	–	200	–
71	20	6	2	1	18	–	–	400	–
72	20	6	2	1	20	–	–	400	–
73	20	6	2	1	20	–	–	202	–
74	20	6	2	1	8	–	–	42	–
75	20	6	2	1	16	–	–	202	–
76	20	6	2	1	20	–	–	200	–
77	20	6	2	1	20	–	–	200	–
78	20	6	2	1	8	–	–	42	gdss(56)
79	20	6	2	1	7	–	–	31	gdss(80)
80	20	6	2	1	7	–	–	27	gdss(56)
81	20	6	2	1	20	–	–	202	–
82	20	6	2	1	20	–	–	400	–
83	20	6	2	1	18	–	–	200	–
84	20	6	2	1	18	–	–	200	–
85	20	6	2	1	20	–	–	200	–
86	20	6	2	1	10	–	–	80	–
87	20	6	2	1	20	–	–	202	–
88	20	6	2	1	10	–	–	40	–
89	20	6	2	1	11	–	–	40	–
90	20	7	3	2	4	+	+	4	cay, as
91	20	7	6	2	4	+	+	4	cay, as, lp2
92	20	7	6	2	5	+	+	19	lp3
93	20	8	4	2	15	–	–	122	–
94	20	8	4	2	14	–	–	59	–
95	20	8	4	2	14	–	–	100	–

#	v	k	b	a	egv	int	ddg	WL-rank	constructions
96	20	8	4	2	5	–	–	20	cay
97	20	8	4	2	13	–	–	208	–
98	20	10	6	4	5	–	–	40	–
99	20	10	6	4	13	–	–	200	–
100	20	10	6	4	13	–	–	200	–
101	20	10	6	4	9	–	–	36	–
102	20	10	6	4	5	–	–	7	cay, as
103	20	10	6	4	7	–	–	40	–
104	20	11	10	2	4	+	+	4	cay, as, lp1
105	20	13	9	8	5	+	+	6	cay, as
106	20	13	12	8	4	+	+	4	cay, as, lp2
107	20	14	10	9	5	–	–	6	as
108	20	17	16	14	4	+	+	4	cay, as, lp1
109	21	8	3	2	8	–	–	50	–
110	21	8	3	2	8	–	–	35	–
111	21	8	3	2	5	–	–	12	cay, as
112	21	8	4	2	7	–	–	129	gdss(122)
113	21	8	4	2	20	–	–	225	–
114	21	8	4	2	7	–	–	28	gdss(130)
115	21	8	4	2	21	–	–	225	–
116	21	8	4	2	15	–	–	441	gdss(122)
117	21	8	4	2	11	–	–	117	gdss(122)
118	21	8	4	2	21	–	–	225	–
119	21	8	4	2	19	–	–	225	gdss(123)
120	21	8	4	2	21	–	–	225	–
121	21	8	4	2	7	–	–	38	gdss(130)
122	21	8	4	2	7	–	–	71	gdss(130)
123	21	8	4	2	8	–	–	63	gdss(122)
124	21	8	4	2	19	–	–	225	–
125	21	8	4	2	12	–	–	78	–
126	21	8	4	2	12	–	–	27	–
127	21	8	4	2	12	–	–	30	–
128	21	8	4	2	6	–	–	26	–
129	21	8	4	2	12	–	–	27	–
130	21	8	4	2	4	–	–	4	cay, as
131	21	10	5	4	8	–	–	32	–
132	21	10	5	4	11	–	–	63	–
133	21	10	5	4	5	+	–	46	gdss
134	21	10	5	4	5	+	–	117	gdss(133)
135	21	10	6	3	5	+	–	16	dss, gdss

#	v	k	b	a	egv	int	ddg	WL-rank	constructions
136	21	12	7	5	7	–	–	46	gdss(137)
137	21	12	7	5	4	–	–	4	cay, as
138	21	12	7	6	5	–	–	5	cay, as
139	21	12	7	6	5	–	–	12	cay, as

Among 139 Deza graphs we found there are 30 graphs with integral spectra. These graphs and their spectra are listed in the table bellow.

Table 2: Strictly Deza graphs with integral spectra

#	v	k	b	a	non-principal eigenvalues			
1	8	4	2	0	-2^3	0^3	2^1	
3	8	5	4	2	-3^1	-1^4	1^2	
4	9	4	2	1	-2^3	-1^2	1^2	2^1
7	12	5	2	1	-2^6	1^3	2^2	
9	12	6	3	2	-2^6	0^2	2^3	
10	12	7	4	3	-2^6	-1^1	1^2	2^2
11	12	7	6	2	-5^1	-1^6	1^4	
12	12	9	8	6	-3^2	-1^6	1^3	
15	15	6	3	1	-3^4	-1^3	1^6	3^1
17	16	7	4	2	-3^4	-1^5	1^4	3^2
18	16	7	4	2	-3^4	-1^5	1^4	3^2
20	16	9	6	4	-3^4	-1^6	1^3	3^2
21	16	9	6	4	-3^5	-1^3	1^6	3^1
22	16	9	8	2	-7^1	-1^8	1^6	
23	16	11	8	6	-3^4	-1^6	1^4	3^1
24	16	12	10	8	-4^1	-2^6	0^6	2^2
25	16	13	12	10	-3^3	-1^8	1^4	
39	18	9	8	4	-3^5	-1^6	1^3	3^3
40	18	9	8	4	-3^4	-1^9	3^4	
41	18	13	12	8	-5^2	-1^9	1^6	
90	20	7	3	2	-2^{12}	2^4	3^3	
91	20	7	6	2	-3^4	-1^{10}	3^5	
92	20	7	6	2	-3^5	-1^7	1^3	3^4
104	20	11	10	2	-9^1	-1^{10}	1^8	
105	20	13	9	8	-3^1	-2^{12}	2^4	3^2
106	20	13	12	8	-3^5	-1^{10}	3^4	
108	20	17	16	14	-3^4	-1^{10}	1^5	
133	21	10	5	4	-3^2	-2^{11}	2^3	3^4

#	v	k	b	a	non-principal eigenvalues
134	21	10	5	4	-3^4 -2^8 2^6 3^2
135	21	10	6	3	-4^5 -1^4 1^{10} 4^1

5. Conclusion

The complete list of strictly Deza graphs up to 21 vertices is available by `http://alg.imm.uran.ru/dezagraphs/dezatab.html`.
This web page provides access to adjacency matrices, WL-closures and spectra of the graphs we found.

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Sergey Viktorovich Goryainov
Chelyabinsk State University,
129, Bratiev Kashirinykh str.,
Chelyabinsk, 454001, Russia
Email address: sergey.goryainov3@gmail.com

Dmitriy Igorevich Panasenko
Chelyabinsk State University,
129, Bratiev Kashirinykh str.,
Chelyabinsk, 454001, Russia
N.N. Krasovskii Institute of Mathematics and Mechanics,
16, S. Kovalevskaya str.,
Yekaterinburg, 620108, Russia
Email address: makare95@mail.com

Leonid Viktorovich Shalaginov
Chelyabinsk State University,
129, Bratiev Kashirinykh str.,
Chelyabinsk, 454001, Russia
Email address: 44sh@mail.com