S©MR ISSN 1813-3304

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru

Том 18, №2, стр. 827–833 (2021) УДК 517.955.8
DOI 10.33048/semi.2021.18.061 MSC 35C20

RECONSTRUCTION OF A HIGH-FREQUENCY SOURCE TERM OF THE WAVE EQUATION FROM THE ASYMPTOTICS OF THE SOLUTION. CASE OF THE CAUCHY PROBLEM

E.V. KORABLINA, V.B.LEVENSHTAM

Abstract. We consider The Cauchy problem for the wave equation with an unknown right hand side, that rapidly oscillates in time. This right hand side is reconstructed from the three-term asymptotics of a solution, which are given at one point of the domain. In this case, an approach developed earlier by one of the authors of this article is used to solve the inverse problems with rapidly oscillating data.

Keywords: wave equation, Cauchy problem, asymptotics of a solution, reconstruction of an unknown high-frequency source term.

Introduction

In this paper we consider the Cauchy problem for a one-dimensional wave equation, with a right hand side, that can be expressed as a product of two functions, with one being a function of a time variable and a space variable and the other one being a function of a time variable and a so-called fast time variable (Прим. смысл "быстрой временной"переменной в статье не уточняется, в препринте статьи [5] используется термин "fast variable"). In the considered problem the second function is unknown. We study the way of reconstructing it using the three-term asymptotics of a solution, which are given at a certain point of the domain (in fact, much less data is needed, see last paragraph of the introduction). In this paper

Korablina, E.V., Levenshtam, V.B., Reconstruction of a high-frequency source term of the wave equation from the asymptotics of the solution. Case of the Cauchy problem.

Thie research is supported by a grant from the Russian Science Foundtaion (project No. 20-11-20141).

Received April, 28, 2021, published July, 25, 2021.

we use the algorithm [1-5] for solving coefficient inverse problems with rapidly oscillating data.

Inverse problems of mathematical phyiscs (including the source term reconstruction problem) are widely researched and one can find many monographs (see, for instance, [6-11]) and papers on these topics. However, inverse problems for equations with rapidly oscillating (high-frequency) data were not studied as a part of classical theory. At the same time they are often encountered in mathematical modeling of physical, chemical and other processes, which take place in environments with unknown characteristics, and, in addition, are subject to high-frequency actions of electromagnetic, acoustic, vibrational and other fields. Wave equation (1.1) refers to equations of the indicated type; it simulates the vibrations of a string under the influence of high-frequency external forces. This demonstrates the need for the development of the theory of high-frequency coefficient inverse problems.

In [1-5] as well as in this paper, we use a new method for stating and solving high-frequency coefficient inverse problems. This method is a combination of techniques from theories of asymptotic methods and inverse problems. Because of that, in every one of the aforementioned works the process of solving the inverse problems is split into two steps, each corresponding to one of these fields, which also raises the need to track coherence of these parts (for instance, smoothness of the obtained functions). The papers mentioned above study the inverse problems for parabolic and hyperbolic equations with the unkown rapidly oscillating source term (cases of initial boundary value problems are covered in [1-5], while this work studies the case of the Cauchy problem). Additionally, this method is special in a sense that the overdetermination condition does not use the exact solution, as is usually the case in more classical techniques, but its partial asymptotic, the length of which is computed during the first step of solving the inverse problem. Moreover, the information, required for the overdetermination condition is given not for all, but for some "basis" coefficients (or their fast terms), from which the data for other coefficients is uniquely determined, and the "basis" coefficients are used to uniquely determine the data for other coefficients. For example, in this paper the overdetermination condition in the statement of the inverse problem uses the coefficients of the three-term asymptotics, that are computed in the certain point of the domain, which are represented by functions $q(t)$, $\phi(t)$ and $\psi(t) + \chi(t, \omega t)$, however we explicitly set only two of them: $q(t)$ and $\chi(t, \tau)$. In that case, the inverse problem is uniquely solvable.

1. Construction of the asymptotics

Let $T > 0$, consider the strip $\Pi = \{(x,t) : x \in R, t \in [0,T]\}$ and the infinite right parallelepiped $\Omega = \{(x,t,\tau) : (x,t) \in \Pi, \tau \in [0,\infty)\}$. On the set Π consider the following Cauchy problem with the large parameter ω

$$\begin{cases} u_{tt} - u_{xx} = f(x,t,\omega t) \\ u|_{t=0} = 0 \\ u_t|_{t=0} = 0 \end{cases} \tag{1.1}$$

Here $f(x,t,\tau)$ is a real-valued function, which is defined (and continuous) on the set Ω and is also 2π-periodic in τ. In this paper, the term "solution" is understood in the classical sense. By $F(x,t)$ denote the mean value of the function $f(x,t,\tau)$ for the argument τ, i.e

$$F(x,t) =< f(x,t,\tau) >\equiv \frac{1}{2\pi}\int_0^{2\pi} f(x,t,\tau)d\tau.$$

Assume that $F(x,t)$ is differentiable with respect to x on Π and both the function and its derivative are continuous on Π. Consider the function $\phi(x,t,\tau) = f(x,t,\tau) - F(x,t)$. Assume that it is four times differentiable and is, along with all of these derivatives, continuous on Ω.

Let $u_\omega(x,t)$ be the solution of (1.1). Its asymptotics can be formally presented as the following series

$$u_\omega(x,t) \sim u_0(x,t) + \frac{1}{\omega}u_1(x,t) + \frac{1}{\omega^2}(u_2(x,t) + v_2(x,t,\omega t)) + \\ ... + \frac{1}{\omega^k}(u_k(x,t) + v_k(x,t,\omega t)) + ..., \quad (1.2)$$

where functions $u_k(x,t)$ and $v_k(x,t,\tau)$ are defined and continuous on Π and Ω and are two times continuously differentiable in x and (t,τ), with $v_k(x,t,\tau)$ also being 2π-periodic in τ such that

$$< v_k(x,t,\tau) >= 0.$$

All the functions considered in this paper are real-valued.

To state the theorem we introduce some notation. Firstly, we define two types of linear uniquely solvable problems. The first type contains the following problem for the ordinary second-order differential equation

$$\begin{cases} \frac{\partial^2 s(x,t,\tau)}{\partial \tau^2} = \psi(x,t,\tau) \\ s(x,t,\tau+2\pi) = s(x,t,\tau) \\ < s(x,t,\tau) >= 0, \end{cases} \quad (1.3)$$

where $\psi(x,t,\tau)$ is defined and continuous on Ω and is a 2π-periodic function in $\tau \in [0;\infty)$ with zero mean value. It is known that the solution for (1.3) can be represented as follows.

$$s(x,t,\tau) = \int_0^\tau \Big(\int_0^z \psi(x,t,s)ds - \Big\langle \int_0^\tau \psi(x,t,s)ds\Big\rangle\Big)dz - \\ -\Big\langle \int_0^\tau \Big(\int_0^z \psi(x,t,s)ds - \Big\langle \int_0^\tau \psi(x,t,s)ds\Big\rangle\Big)dz\Big\rangle.$$

In this paper the mean value $< ... >$ is always computed with respect to the variable τ. The second type contains the following Cauchy problem for the second-order wave equation on the strip $(x,t) \in \Pi$

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) = g(x,t) \\ u(x,t)|_{t=0} = a(x) \\ u_t(x,t)|_{t=0} = b(x), \end{cases} \quad (1.4)$$

where functions $g(x,t)$ and $b(x)$ are continuously differentiable with respect to x, defined on Π and R respectively (with g_x being continuous on Π), while $a(x)$ is a two times continuously differentiable function, defined on R. The solution for (1.4) can be represented by the d'Alembert formula:

$$u(x,t) = \frac{1}{2}(a(x-t) + a(x+t)) + \frac{1}{2}\int_{x-t}^{x+t} b(\xi)d\xi + \frac{1}{2}\int_0^t ds \int_{x-(t-s)}^{x+(t-s)} g(\xi,s)d\xi.$$

For every positive M define the rectangle $\Pi_M = \{(x,t) : |x| \leq M, t \in [0,T]\}$ and consider the $(k+1)$-term $(k = 1,2)$ asymptotics of the solution for (1.1) (see 1.2):

$$\overset{k}{u}_\omega (x,t) = u_0(x,t) + \frac{1}{\omega}u_1(x,t) + ... + \frac{1}{\omega^k}(u_k(x,t) + v_k(x,t,\omega t)).$$

Theorem 1. *The following asymptotic formula*

$$||u_\omega(x,t) - \overset{2}{u}_\omega (x,t)||_{C(\Pi_M)} = O(\omega^{-3}), \omega \to \infty,$$

where $u_n(x,t), n = 0,1,2,$ and $v_2(x,t,\tau)$ are solutions for problems of type (1.4) and (1.3) respectively, holds for every $M > 0$.

Доказательство. We start by formally substituting the series (1.2) into (1.1):

$$\begin{cases} \frac{\partial^2 u_0(x,t)}{\partial t^2} + \frac{1}{\omega}\frac{\partial^2 u_1(x,t)}{\partial t^2} + \frac{1}{\omega^2}\Big(\frac{\partial^2 u_2(x,t)}{\partial t^2} + \frac{\partial^2 v_2(x,t,\omega t)}{\partial t^2} + 2\omega\frac{\partial^2 v_2(x,t,\omega t)}{\partial t\partial\tau} + \omega^2\frac{\partial^2 v_2(x,t,\omega t)}{\partial\tau^2}\Big) - \\ -\frac{\partial^2 u_0(x,t)}{\partial x^2} - \frac{1}{\omega}\frac{\partial^2 u_1(x,t)}{\partial x^2} - \frac{1}{\omega^2}\Big(\frac{\partial^2 u_2(x,t)}{\partial x^2} + \frac{\partial^2 v_2(x,t,\omega t)}{\partial x^2}\Big) + ... = f(x,t,\omega t) \\ \Big[u_0(x,t) + \frac{1}{\omega}u_1(x,t) + \frac{1}{\omega^2}\Big(u_2(x,t) + v_2(x,t,\omega t)\Big) + ...\Big]\Big|_{t=0} = 0 \\ \Big[\frac{\partial u_0(x,t)}{\partial t} + \frac{1}{\omega}\frac{\partial u_1(x,t)}{\partial t} + \frac{1}{\omega^2}\Big(\frac{\partial u_2(x,t)}{\partial t} + \frac{\partial v_2(x,t,\omega t)}{\partial t} + \omega\frac{\partial v_2(x,t,\omega t)}{\partial\tau}\Big) + ...\Big]\Big|_{t=0} = 0. \end{cases}$$

In each one of the last three equalities, equate the coefficients of $\omega^0, \omega^{-1}, \omega^{-2}, \omega^{-3}$ and then apply the averaging operation with respect to $\tau, \tau = \omega t$. As a result we obtain the following set of problems

$$\begin{cases} \frac{\partial^2 u_0(x,t)}{\partial t^2} - \frac{\partial^2 u_0(x,t)}{\partial x^2} = F(x,t) \\ u_0(x,t)|_{t=0} = 0 \\ \frac{\partial u_0(x,t)}{\partial t}\Big|_{t=0} = 0; \end{cases} \quad (1.5)$$

$$\begin{cases} \frac{\partial^2 v_2(x,t,\tau)}{\partial\tau^2} = \phi(x,t,\tau) \\ v_2(x,t,\tau+2\pi) = v_2(x,t,\tau) \\ < v_2(x,t,\tau) >= 0; \end{cases} \quad (1.6)$$

$$\begin{cases} \frac{\partial^2 u_1(x,t)}{\partial t^2} - \frac{\partial^2 u_1(x,t)}{\partial x^2} = 0 \\ u_1(x,t)|_{t=0} = 0 \\ \frac{\partial u_1(x,t)}{\partial t}|_{t=0} + \frac{\partial v_2(x,t,\tau)}{\partial\tau}|_{t,\tau=0} = 0; \end{cases} \quad (1.7)$$

$$\begin{cases} \frac{\partial^2 v_3(x,t,\tau)}{\partial\tau^2} = -2\frac{\partial^2 v_2(x,t,\tau)}{\partial t\partial\tau} \\ v_3(x,t,\tau+2\pi) = v_3(x,t,\tau) \\ < v_3(x,t,\tau) >= 0; \end{cases} \quad (1.8)$$

$$\begin{cases} \frac{\partial^2 u_2(x,t)}{\partial t^2} - \frac{\partial^2 u_2(x,t)}{\partial x^2} = 0 \\ u_2(x,t)|_{t=0} + v_2(x,t,\tau)|_{t,\tau=0} = 0 \\ \frac{\partial u_2(x,t)}{\partial t}|_{t=0} + \frac{\partial v_2(x,t,\tau)}{\partial t}|_{t,\tau=0} + \frac{\partial v_3(x,t,\tau)}{\partial\tau}|_{t,\tau=0} = 0; \end{cases} \quad (1.9)$$

$$\begin{cases} \frac{\partial^2 v_4(x,t,\tau)}{\partial\tau^2} = \frac{\partial^2 v_2(x,t,\tau)}{\partial x^2} - \frac{\partial^2 v_2(x,t,\tau)}{\partial t^2} - 2\frac{\partial^2 v_3(x,t,\tau)}{\partial t\partial\tau} \\ v_4(x,t,\tau+2\pi) = v_4(x,t,\tau) \\ < v_4(x,t,\tau) >= 0; \end{cases} \quad (1.10)$$

$$\begin{cases} \frac{\partial^2 u_3(x,t)}{\partial t^2} - \frac{\partial^2 u_3(x,t)}{\partial x^2} = 0 \\ u_3(x,t)|_{t=0} + v_3(x,t,\tau)|_{t,\tau=0} = 0 \\ \frac{\partial u_3(x,t)}{\partial t}|_{t=0} + \frac{\partial v_3(x,t,\tau)}{\partial t}|_{t,\tau=0} + \frac{\partial v_4(x,t,\tau)}{\partial\tau}|_{t,\tau=0} = 0. \end{cases} \quad (1.11)$$

Set $\overset{3}{\hat{u}_\omega}(x,t) = \overset{3}{u_\omega}(x,t) + \frac{1}{\omega^4} v_4(x,t,\omega t)$. Then:

$$\begin{cases} (\overset{3}{\hat{u}_\omega}(x,t))_{tt} - (\overset{3}{\hat{u}_\omega}(x,t))_{xx} = f(x,t,\omega t) + z(x,t,\omega t) \\ \overset{3}{\hat{u}_\omega}(x,t)|_{t=0} = w(x) \\ (\overset{3}{\hat{u}_\omega}(x,t))_t|_{t=0} = r(x), \end{cases} \quad (1.12),$$

where functions $z(x,t,\tau)$, $w(x)$ and $v(x)$ can be written as follows:

$$z(x,t,\tau)$$
$$= \frac{1}{\omega^3}\Big(\frac{\partial^2 v_3(x,t,\tau)}{\partial t^2} - \frac{\partial^2 v_4(x,t,\tau)}{\partial x^2} + 2\frac{\partial^2 v_4(x,t,\tau)}{\partial t \partial \tau}\Big) + \frac{1}{\omega^4}\Big(\frac{\partial^2 v_4(x,t,\tau)}{\partial t^2} - \frac{\partial^2 v_4(x,t,\tau)}{\partial x^2}\Big),$$
$$w(x) = \frac{1}{\omega^4} v_4(x,0,0),$$
$$r(x) = \frac{1}{\omega^4}\frac{\partial v_4(x,t,\tau)}{\partial t}\Big|_{t,\tau=0}.$$

From this we obtain the following asymptotic equalities

$$z(x,t,\tau) = O(\omega^{-3}), \omega \to \infty, \text{ uniformly with respect to } (x,t,\tau),\ (x,t,\tau) \in \Omega, |x| \le M,$$
$$w(x) = O(\omega^{-4}), \omega \to \infty, \text{ uniformly with respect to } x,\ |x| \le M,$$
$$r(x) = O(\omega^{-4}), \omega \to \infty, \text{ uniformly with respect to } x,\ |x| \le M.$$

Therefore,

$$\| u_\omega(x,t) - \overset{2}{u_\omega}(x,t) \|_{C(\Pi_M)}$$
$$\le \| u_\omega(x,t) - \overset{3}{\hat{u}_\omega}(x,t) \|_{C(\Pi_M)} + \| \overset{3}{\hat{u}_\omega}(x,t) - \overset{2}{u_\omega}(x,t) \|_{C(\Pi_M)}$$
$$= O(\omega^{-3}) + O(\omega^{-3}) = O(\omega^{-3}), \omega \to \infty$$

And the statement of the theorem follows from the obtained estimates. □

2. The inverse problem

Let T and Π be the same as in the previous section. On the strip Π consider the following Cauchy problem

$$\begin{cases} u_{tt} - u_{xx} = f(x,t) r(t,\omega t) \\ u|_{t=0} = 0 \\ u_t|_{t=0} = 0 \end{cases} \quad (2.1)$$

We make some assumptions regarding functions $f(x,t)$ and $r(t,\tau)$: $f(x,t)$ is defined on Π and is, along with its derivative with respect to x is four times continuously differentiable with respect to (x,t), while $r(t,\tau)$ is defined and continuous on the set $Q = \{(t,\tau) : t \in [0,T], \tau \in [0;\infty)\}$ and can be presented as follows: $r(t,\tau) = r_0(t) + r_1(t,\tau)$, where $r_0(t)$ is continuous on the segment $[0;T]$ and $r_1(t,\tau)$ is four times differentiable by t and is, along with all of these derivatives, continuous on Q and is 2π-periodic in τ with its mean value equal to zero, i.e.

$$< r_1(t,\tau) >= 0$$

For the sake of brevity we will call the function $r(t,\tau)$, which satisfies the aforementioned conditions the function of the (I) class. In this section, f and r are assumed to be known and unkown respectively.

Consider the following predetermined objects: point x_0, such that $f(x_0,t) \ne 0$, $t \in [0,T]$; the continuous and two times continuously differentiable on the segment

$[0,T]$ function $q(t)$, such that $q(0) = q'(0) = 0$; function $\chi(t,\tau)$, continuous on Q and 2π-periodic in τ with its mean value equal to zero. Moreover, $\chi(t,\tau)$ is two times continuously differentiable with respect to τ and $\chi''_{\tau^2}(t,\tau)$ is four times differentiable by t and is, along with all of these derivatives, continuous. Consider two additional functions

$$\phi(t) = \tilde{u}_1(x_0,t),$$
$$\psi(t) = \tilde{u}_2(x_0,t),$$

where $\tilde{u}_1(x,t), \tilde{u}_2(x,t)$ are the solutions of problems (1.7)-(1.9), such that $v_2(x,t,\tau) = \frac{f(x,t)\chi(t,\tau)}{f(x_0,t)}$.

Definition. The problem of finding an (I) class function $r(t,\tau)$, such that the solution for (2.1) on the segment $(x_0,t), t \in [0,T]$, satisfies the following condition

$$\Big|\Big|u_\omega(x_0,t) - q(t) - \frac{1}{\omega}\phi(t) - \frac{1}{\omega^2}(\psi(t) + \chi(t,\omega t))\Big|\Big|_{C([0,T])} = O(\omega^{-3}), \omega \to \infty, \quad (2.3)$$

is called the inverse problem.

Theorem 2. *The inverse problem is uniquely solvable.*

Доказательство. By Theorem 1, the solution of the Cauchy problem (2.1) satisfies the following condition:

$$\Big|\Big|u_\omega(x,t) - u_0(x,t) - \frac{1}{\omega}u_1(x,t) - \frac{1}{\omega^2}(u_2(x,t) + v_2(x,t,\omega t))\Big|\Big|_{C(\Pi_M)} = O(\omega^{-3}), \omega \to \infty,$$

where u_i, v_i are the same as in the previous section. From this, while considering (2.3) we derive that

$$u_0(x_0,t) + \frac{1}{\omega}u_1(x_0,t) + \frac{1}{\omega^2}(u_2(x_0,t) + v_2(x_0,t,\omega t)) =$$
$$= q(t) + \frac{1}{\omega}\phi(t) + \frac{1}{\omega^2}(\psi(t) + \chi(t,\omega t)) + O(\omega^{-3}) \quad (2.4)$$

Equate the coefficients of $\omega^0, \omega^{-1}, \omega^{-2}$ in (2.4). From this, while considering the averaging operation with respect to $\tau, \tau = \omega t$ we get that

$$u_0(x_0,t) = q(t); \quad (2.5)$$
$$u_1(x_0,t) = \phi(t); \quad (2.6)$$
$$u_2(x_0,t) = \psi(t); \quad (2.7)$$
$$v_2(x_0,t,\tau) = \chi(t,\tau). \quad (2.8)$$

From the previous section we obtain that

$$u_0(x_0,t) = \frac{1}{2}\int_0^t r_0(s)\int_{x_0-(t-s)}^{x_0+(t-s)} f(\xi,s)d\xi ds.$$

From this and (2.5), it follows that the equality

$$q(t) = \frac{1}{2}\int_0^t r_0(s)\int_{x_0-(t-s)}^{x_0+(t-s)} f(\xi,s)d\xi ds.$$

holds. By differentiating it twice with respect to t we obtain the Volterra equation of the second kind:

$$q''(t) = r_0(t)f(x_0,t) + \frac{1}{2}\int_0^t r_0(s)[f'_x(x_0+(t-s),s) - f'_x(x_0-(t-s),s)]ds,$$

which implies the existense and uniquness of the continuious solution r_0. Now, by differentiating (2.8) twice with respect to τ and considering (1.6) we obtain that

$$f(x_0, t)r_1(t, \tau) = \frac{\partial^2 \chi(t, \tau)}{\partial \tau^2}$$

Therefore, the function $r_1(t, \tau)$ can also be uniquely determined. Due to conditions, imposed on functions $q(t)$ and $\chi(t, \tau)$, we get that the resulting function $r(t, \tau) = r_0(t) + r_1(t, \tau)$ is an (I) class function.

It can be easily shown that the solution of the Cauchy problem (2.1) with the obtained function $r(t, \tau)$ satisfies the asymptotic formula (2.3). Therefore, the theorem is proved.

□

References

[1] P.V. Babich, V.B. Levenshtam, *Direct and inverse asymptotic problems high-frequency terms*, Asymptotic Anal., **97**:3-4 (2016), 329–336. Zbl 1342.35387

[2] P.V. Babich, V.B. Levenshtam, S.P. Prika, *Recovery of a rapidly oscillating source in the heat equation from solution asymptotics*, Comput. Math. Math. Phys., **57**:12 (2017), 1908–1918. Zbl 6864318

[3] P.V. Babich, V.B. Levenshtam, *Recovery of a rapidly oscillating absolute term in the multidimensional hyperbolic equation*, Math. Notes, **104**:4 (2018), 489–497. Zbl 7035826

[4] V.B. Levenshtam, Parabolic equations with large parameters. Inverse problems, Math Notes, **107**:3 (2020), 452–463.

[5] P.V. Babich, V.B. Levenshtam, *Inverse problem in the multidimensional hyperbolic equation with rapidly oscillating absolute term*, in A.G.Kusraev, Z.D. Totieva edt., *Operator Theory and Differential Equations*, 7–25, Birkhäuser, Cham, 2021.

[6] M.M. Lavret'ev, K.G. Reznitskaya, V.G. Yakhno, *One-dimensional inverse problems of mathematical physics*, Nauka, Novosibirsk, 1982. Zbl 0526.35001

[7] V.G. Romanov, *Inverse problems of mathematical physics*, Nauka, Moscow, 1984. Zbl 0576.35001

[8] A.M. Denisov, *Elements of the theory of inverse problems*, Inverse and Ill-Posed Problems Series, VSP, Utrecht, 1999. Zbl 0940.35003

[9] S.I. Kabanikhin, *Inverse and ill-posed problems. Theory and applications*, Inverse and Ill-Posed Problems Series, **55**, de Gruyter, Berlin, 2012. Zbl 1247.65077

[10] A.I. Prilepko, D.G. Orlovsky, I.A. Vasin, *Methods for solving inverse problems in mathematical physics*, Pure and Applied Mathematics, Marcel Dekker, **231**, Marcel Dekker, New York, 2000. Zbl 0947.35173

[11] A.O. Vatulyan, *Inverse problems in mechanics of deformable solids*, Phizmatlit, Moscow, 2007.

Ella Viktorovna Korablina
Steklov Mathematical Institute of the Russian Academy of Sciences,
8, Gubkin str.,
Moscow, 119991, Russia
Email address: ellakorablina1998@gmail.com

Valery Borisovich Levenshtam
Steklov Mathematical Institute of the Russian Academy of Sciences,
8, Gubkin str.,
Moscow, 119991, Russia
Email address: vlevenshtam@yandex.ru