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KANTOROVICH'S PATH AND MEMES

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Abstract. This is an overview of the worldline, contribution, and memes of Leonid Kantorovich (1912–1986).

Keywords: functional analysis, computational mathematics, mathematical economics.

KANTOROVICH'S LIFE ITINERARY

Leonid Vitalievich Kantorovich was born in the family of a venereologist at St. Petersburg on January 19, 1912 (January 6, according to the old Russian style). The boy's talent revealed itself very early. In 1926, just at the age of 14, he entered St. Petersburg (then Leningrad) State University (SPSU). After graduation from SPSU in 1930, Kantorovich started teaching, combining it with intensive scientific

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research. Already in 1932 he became a full professor at the Leningrad Institute of Civil Engineering and an assistant professor at SPSU.

From 1934 Kantorovich was a full professor at his *alma mater*. His close connection with SPSU and the Leningrad Department of the Steklov Mathematical Institute of the Academy of Sciences lasted until his transition to Novosibirsk on the staff of the Institute of Mathematics of the Siberian Division of the Academy of Sciences of the USSR (now, the Sobolev Institute) at the end of the 1950s.

In 1957 Kantorovich was invited to join the newly founded Siberian Division of the Academy of Sciences of the USSR. He agreed and soon became a corresponding member of the Division of Economics in the first elections to the Siberian Division. Since then his major publications were devoted to economics, with the exception of the celebrated course of functional analysis [1].

The 1960s became the decade of his recognition. In 1964 he was elected a full member of the Division of Mathematics of the Academy of Sciences of the USSR, and in 1965 he was awarded the Lenin Prize. In these years he vigorously propounded and maintained his views of interplay between mathematics and economics and exerted great efforts to instill the ideas and methods of modern science into the top economic management of the Soviet Union, which was almost in vain.

At the beginning of the 1970s Kantorovich left Novosibirsk for Moscow where he was still engaged in economic analysis, not ceasing his efforts to influence the everyday economic practice and decision making in the national economy. These years witnessed a considerable mathematical Renaissance of Kantorovich. Although he never resumed proving theorems, his impact on the mathematical life of this country increased sharply due to the sweeping changes in the Moscow academic life on the eve of Gorbi's "perestroĭka." Cancer terminated his path in science on April 7, 1986. He was buried at Novodevichy Cemetery in Moscow.

KANTOROVICH'S CONTRIBUTION

The main achievements in mathematics belong to the Leningrad period of Kantorovich's life.

Kantorovich started his scientific research in rather abstract and sophisticated sections of mathematics such as descriptive set theory, approximation theory, and functional analysis. It should be stressed that at the beginning of the 1930s these areas were most topical, prestigious, and difficult. Kantorovich's fundamental contribution to theoretic mathematics, now indisputable and universally acknowledged, consists in his pioneering works in the above-mentioned areas. Note also that in the "mathematical" years of his career he was primarily famous for his research into the approximate methods of analysis, the ancient euphemism for the computational mathematics of today.

The first works of Kantorovich on computational mathematics were published in 1933. He suggested some approaches to approximate solution of the problem of finding a conformal mapping between domains. These methods used the idea of embedding the original domains into some one-parameter family of domains. Expanding in a parameter, Kantorovich found out new explicit formulas for approximate calculation of conformal mappings between multiply-connected domains.

Kantorovich paid much attention to direct variational methods. He suggested an original method for approximate solution of second order elliptic equations which was based on reduction of the initial problem to minimization of a functional over some functions of one variable. This technique is now called reduction to ordinary differential equations.

The variational method was developed in his subsequent works under the influence of other questions. For instance, his collocation method was suggested in an article about calculations for a beam on an elastic surface.

A few promising ideas were proposed by Kantorovich in the theory of mechanical quadratures which formed a basis for some numerical methods of solution of a general integral equation with a singularity.

This period of his research into applied mathematics was crowned with a joint book with V. I. Krylov *Methods for Approximate Solution of Partial Differential Equations* whose further expanded editions appeared under the title *Approximate Methods of Higher Analysis* [2] .

Functional analysis occupies a specific place in the scientific legacy of Kantorovich. He has been listed among the classics of the theoretic sections of this area of research as one of the founders of ordered vector spaces. Also, he contributed much to making functional analysis a natural language of computational mathematics. His article "Functional analysis and applied mathematics" in *Russian Mathematical Surveys* (1948) made a record in the personal file of Kantorovich as well as in the history of mathematics in this country. Kantorovich wrote in the introduction to this article [3]:

> *...there is now a tradition of viewing functional analysis as a purely theoretic discipline far removed from direct applications, a discipline which cannot deal with practical questions. This article is an attempt to break with this tradition, at least to a certain extend, and to reveal the relationship between functional analysis and questions of applied mathematics, an attempt to show that functional analysis can be useful to mathematicians dealing with practical applications.*
> *Namely, we would try to show that the ideas and methods of functional analysis can be readily used to construct and analyze effective practical algorithms for solving mathematical problems with the same success as they were used for the theoretic studies of the problems.*

The mathematical ideas of this article remain classical by now: The method of finite-dimensional approximations, estimation of the inverse operator, and, last but not least, the Newton–Kantorovich method are well known to the majority of the persons recently educated in mathematics.

The general theory by Kantorovich for analysis and solution of functional equations bases on variation of "data" (operators and spaces) and provides not only estimates for the rate of convergence but also proofs of the very fact of convergence.

As an instance of incarnation of the idea of unity of functional analysis and computational mathematics Kantorovich suggested at the end of the 1940s that the Mechanics and Mathematics Department of SPSU began to prepare specialists in the area of computational mathematics for the first time in this country. He prolonged this line in Novosibirsk State University where he founded the chair of computational mathematics which delivered graduate courses in functional analysis in the years when Kantorovich held the chair.

It should be emphasized that Kantorovich tied the progress of linear programming as an area of applied mathematics with the general demand for improving the functional-analytical techniques pertinent to optimization: the theory of topological vector spaces, convex analysis, the theory of extremal problems, etc. Several major sections of functional analysis (in particular, nonlinear functional analysis) underwent drastic changes under the impetus of new applications.

In 1935 Kantorovich made his major mathematical discovery—he defined K-spaces, i. e., vector lattices whose every nonempty order bounded subset had an infimum and supremum. The K-spaces have provided the natural framework for developing the theory of linear inequalities which was a practically uncharted area of research those days. The concept of inequality is obviously relevant to approximate calculations where we are always interested in various estimates of the accuracy of results. Another challenging source of interest in linear inequalities was the stock of problems of economics. The language of partial comparison is rather natural in dealing with what is reasonable and optimal in human behavior when means and opportunities are scarce. Finally, the concept of linear inequality is inseparable from the key idea of a convex set. Functional analysis implies the existence of nontrivial continuous linear functional over the space under consideration, while the presence of a functional of this type amounts to the existence of nonempty proper open convex subset of the ambient space. Moreover, each convex set is generically the solution set of an appropriate system of simultaneous linear inequalities.

At the end of the 1940s Kantorovich formulated and explicated the thesis of interdependence between functional analysis and applied mathematics. He distinguished the three techniques: the Cauchy method of majorants also called domination, the method of finite-dimensional approximations, and the Lagrange multiplier method for the new optimization problems motivated by economics. Kantorovich based his study of the Banach space versions of the Newton method on domination in general ordered vector spaces. Approximation of infinite-dimensional spaces and operators by their finite-dimensional analogs must be considered alongside the marvelous universal understanding of computational mathematics as the science of finite approximations to general (not necessarily metrizable) compacta.The novelty of the extremal problems arising in social sciences is connected with the presence of multidimensional contradictory utility functions. This raises the major problem of agreeing conflicting aims. The corresponding techniques may be viewed as an instance of scalarization of vector-valued targets.

From the end of the 1930s the research of Kantorovich acquired new traits in his audacious breakthrough to economics. Kantorovich's booklet *Mathematical Methods in the Organization and Planning of Production* which appeared in 1939 is a material evidence of the birth of linear programming. Linear programming is a technique of maximizing a linear functional over the positive solutions of a system of linear inequalities. It is no wonder that the discovery of linear programming was immediate after the foundation of the theory of K-spaces.

The economic works of Kantorovich were hardly visible at the surface of the scientific information flow in the 1940s. But the problems of economics prevailed in his creative studies. During the Second World War he completed the first version of his book *The Best Use of Economic Resources* [4] which led to the Nobel Prize awarded to him and T. Koopmans in 1975. The pioneering ideas of Kantorovich were legalized and introduced in the national economy.

The Council of Ministers of the USSR issued the top secret Directive No. 1990–774ss/op in 1948 which ordered "to organize in the span of two weeks a group for computations with the staff up to 15 employees in the Leningrad Department of the Mathematical Institute of the Academy of Sciences of the USSR and to appoint Professor Kantorovich the head of the group." That was how Kantorovich was enlisted in the squad of participants of the project of producing an H-bomb.

The impressive diversity of the areas of research rests upon not only the personal traits of Kantorovich but also his methodological views. He always emphasized the innate integrity of his scientific research as well as mutual penetration and synthesis of the methods and techniques he used in solving the most diverse theoretic and applied problems of mathematics and economics. The characteristic feature of Kantorovich's contribution is his orientation to the most topical and difficult problems of mathematics and economics of his epoch.

The classical research of the Newton method led to acknowledging Kantorovich as a top figure in computations. Kantorovich's research was based on the general scheme of domination in K-spaces. These days the development of domination proceeds within Boolean valued analysis. The modern technique of mathematical modeling opened an opportunity to demonstrate that the principal properties of lattice normed spaces represent the Boolean valued interpretations of the relevant properties of classical normed spaces. The most important interrelations here are as follows: Each Banach space inside a Boolean valued model becomes a universally complete Banach–Kantorovich space in result of the external deciphering of constituents. Moreover, each lattice normed space may be realized as a dense subspace of some Banach space in an appropriate Boolean valued model.

Boolean valued analysis enables us to expand the range of applicability of K-spaces and more general modules for studying extensional equations. Many promising possibilities are open by the new method of hyperapproximation which rests on the ideas of infinitesimal analysis. The classical discretization approximates an infinite-dimensional space with the aid of finite-dimensional subspaces. Arguing within nonstandard set theory we may approximate an infinite-dimensional vector space with external finite-dimensional spaces. Undoubtedly, the dimensions of these hyperapproximations are given as actually infinite numbers. Infinitesimal methods also provide new schemes for hyperapproximation of general compact spaces. As an approximation to a compact space we may take an arbitrary internal subset containing all standard elements of the space under approximation. Hyperapproximation of the present day stems from Kantorovich's ideas of discretization.

It was in the 1930s that Kantorovich became engrossed in the practical problems of decision making. Inspired by the ideas of functional analysis and order, Kantorovich attacked these problems in the spirit of searching for an optimal solution. Kantorovich was among the first scientists who formulated optimality conditions for rather general extremal problems. Classical remains his approach to the theory of optimal transport whose center is occupied by the Monge–Kantorovich problem. Another particularity of the extremal problems stemming from praxis consists in the presence of numerous conflicting ends and interests which are to be harmonized. In fact, we encounter the instances of multicriteria optimization whose characteristic feature is a vector-valued target. Seeking for an optimal solution in these circumstances, we must take into account various contradictory preferences which combine into a sole compound aim. Furthermore, it is impossible as a rule to distinguish some particular scalar target and ignore the rest of the targets without distorting the original statement of the problem under study.

The specific difficulties of practical problems and the necessity of reducing them to numerical calculations led Kantorovich to pondering over the nature of the reals.

He viewed the members of his K-spaces as generalized numbers, developing the ideas that are now collected around the concept of scalarization. In the most general sense, scalarization is reduction to numbers. Since a number is a measure of quantity; therefore, the idea of scalarization is of importance to mathematics in general. Kantorovich's studies on scalarization were primarily connected with the problems of economics he was interested in from the very first days of his creative path in science.

Kantorovich's Siberian and Moscow periods were tied primarily with economics. Mathematics studies the forms of reasoning. The subject of economics is the circumstances of human behavior. Mathematics is abstract and substantive, and the professional decisions of mathematicians do not interfere with the life routine of individuals. Economics is concrete and declarative, and the practical exercises of economists change the life of individuals substantially. The aim of mathematics consists in impeccable truths and methods for acquiring them. The aim of economics is the well-being of an individual and the way of achieving it. Mathematics never intervenes into the private life of an individual. Economics touches his purse and bag. Immense is the list of striking differences between mathematics and economics.

Mathematical economics is an innovation of the twentieth century. It is then when the understanding appeared that the problems of economics need a completely new mathematical technique.

G. Cantor, the creator of set theory, remarked as far back as in 1883 that "the essence of mathematics lies entirely in its freedom." The freedom of mathematics does not reduce to the absence of exogenous restriction on the objects and methods of research. The freedom of mathematics reveals itself mostly in the new intellectual tools for conquering the ambient universe which are provided by mathematics for liberation of humans by widening the frontiers of their independence. Mathematization of economics is the unavoidable stage of the journey of the mankind into the realm of freedom.

The nineteenth century is marked with the first attempts at applying mathematical methods to economics in the research by A. Cournot, K. Marx, W. Jevons, L. Walras, and his successor in Lausanne University V. Pareto. J. von Neumann and Kantorovich, mathematicians of the first caliber, addressed the economic problems in the twentieth century. The former developed game theory, making it an apparatus for the study of economic behavior. The latter invented linear programming for decision making in the problems of best use of limited resources. These contributions of von Neumann and Kantorovich occupy an exceptional place in science. They demonstrated that modern mathematics opens up broad opportunities for economic analysis of practical problems. Economics has been drifted closer to mathematics. Still remaining a humanitarian science, it mathematizes rapidly, demonstrating high self-criticism and an extraordinary ability of objective thinking.

The principal discovery of Kantorovich at the junction of mathematics and economics is linear programming which is now studied by hundreds of thousands of people throughout the world. The term signifies the colossal area of science which is allotted to linear optimization models. In other words, linear programming is the science of the theoretical and numerical analysis of the problems in which we seek for an optimal (i. e., maximum or minimum) value of some system of indices of a process whose behavior is described by simultaneous linear inequalities. It is worth observing that to an optimal plan of every linear program there corresponds some optimal prices. The interdependence of optimal solutions and optimal prices is the crux of the economic discovery of Kantorovich

The present-day research has corroborated that the ideas of linear programming are immanent in the theory of K-spaces. It was demonstrated that the validity of one of the various statements of the duality principle of linear programming in an abstract mathematical structure implies with necessity that the structure under consideration is in fact a K-space. Kantorovich's heuristic principle declares that the elements of K-spaces are instances of reals. It was discovered at the end od the twentieth century that this principle is in full agreement with the modern technologies of nonstandard set theory in Boolean valued analysis. The progress of the latter has demonstrated the fundamental importance of the so-called universally complete K-spaces. Each of these spaces turns out to present one of the possible noble models of the real axis and so such a space plays a similar key role in mathematics. The K-spaces provide new models of the reals, thus earning their eternal immortality. Kantorovich's heuristics has received brilliant corroboration, which proves the integrity of science and inevitability of interpenetration of mathematics and economics.

KANTOROVICH'S MEMES

Kantorovich's life itinerary is not a series of parades and decorations but a path of the perennial war against inertia, stagnation, ignorance, hatred, and misunderstanding. The epoch of the USSR in Russia was the time of collective triumphs, personal tragedies, bright victories, and grim cannibalism. The main moral loss of the Soviet society was the refusal from universal humanism. The excesses of the collectivist eschatology had never bypassed science. Kantorovich encountered many abominations in mathematics and economics. Careerism prevailed and flourished, and the main symptoms of careerism was antisemitism aggravated by intolerance to any form of dissidence. Antisemitism had never vanished in Tsarist Russia, since Russia was never a secular state. Some attempts at secularizing the society were made after the October revolution but soon them annihilated completely. The same fate was doomed to many other Utopian if not ephemeral dreams of Russian intelligentsia. The freedom of conscience and scientific outlook could not opposed Stalinism. All-Union Communist Party (Bolsheviks) had attained the generic traits of a totalitarian sect which did not disappear in the USSR Communist Party after the dethronement of the personality cult. The domestic antisemitism was covertly appraised and even inspired by the party bonzes. Soon it became a rather effective mechanism of building a career in the years of the Jewish exodus from the country.

The negative processes did not bypass Kantorovich. The theses of his coworkers and students were hampered or flunked, some obstacles awaited the publication of his books, his articles were procrastinated and his proposals were dillydallied. The utmost disaster was his short-term hospitalization in a lunatic asylum after his brave but vain attack against the pseudo-scientific attempt of the notorious "machine deciphering of Maya script." Last year G.G. Ershova included the materials about Kantorovich's Don Quixote attack against the haters of Yu.V. Knorozov in her book [5].

In the times of "victorious and developed Socialism" the abomination often wore the cassocks of the "orthodox pops of Marxism" who tried to disavow Kantorovich's economic ideas as well as their author. Mathematization of economics by Kantorovich deprived from the vanish of would-be professionalism all his opponents who could not cope with the challenges of the new realities. The unacceptability of Kantorovich's conception for the top stratum of Soviet economists was due to the total lack of understanding of the role of optimal prices which was characteristic of the profaners of the Marxist theory of labor value. The novelty of Kantorovich's ideas for "anti-Soviet" economists consisted in the fact that prices in Kantorovich's theory are formed during the choice of an optimal plan of production rather than by a market. The market for Kantorovich is a mechanism of experimental determination of the optimal prices of production. Kantorovich was a greater scientist than any enlisted "Marxist."

The contradistinction between the brilliant achievements and the instances of poor adaptation to the practical seamy side of life is listed among the dramatic enigmas by Kantorovich. His life became a fabulous and puzzling humanitarian phenomenon. Kantorovich's introversion, obvious in personal communications, was inexplicably accompanied by outright public extroversion. The absence of any orator's abilities neighbored his deep logic and special mastery in polemics. His innate freedom and self-sufficiency coexisted with the purposeful and indefatigable endurance that reached the power of a "wolf grip" in the case of necessity. Kantorovich's freedom can hardly bewilder anyone as stemming from his essence, the gift of mathematics. His kindness and mildness were inborn. The tenacity and tremendous force of penetration were the acquired traits that he selected and cultivated conscientiously for the sake of rationality.

Kantorovich's life is a turnpike of a scientist and citizen whose contributions are tied up with the fates of his next of kin and with service to his homeland irrespective of the prevalent ideological obstinacy. This lesson is of utmost import these days. Attempts at slandering and silencing Kantorovich's life and legacy are doomed to vanish. Pygmies can never hide a giant. The genius of rationality in science, Kantorovich was ingeniously rational in choosing his world-line and path in science. He bequeathed us an exemplar of the best use of personal resources in the presence of restrictive internal and external constraints.

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