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RESTORATION OF IMAGES CORRUPTED BY STRIPE INTERFERENCE USING RADON DOMAIN FILTERING

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Abstract. The article deals with the problem of removing noise that has some anisotropy in a certain direction, in images received as a result of remote sensing. Such interference can occur with satellite imagery of the surface of Earth and planets due to the peculiarities of the imaging equipment. In the article, the method of removing such noise in the Radon space is considered, using its singular value decomposition. The use of this approach provides significant advantages over spatial filtering methods when pixel brightness values are used and can be a noticeable loss of useful information in the form of a blurring of borders. When using filtering in the Radon space to remove periodic noise predominantly only interference is removed, since only a small part of the Radon projections corresponds to noise. The numerical experiments on real-world images demonstrate the efficiency of the techniques proposed.

Keywords: image restoration, stripe interference, Radon transform, ridge functions.

1. Introduction

Image enhancement approaches fall into two broad categories: spatial domain processing methods (spatial methods) and transform domain processing methods (Fourier tansform, or frequency methods, Laplace, Radon and other transforms) [1], [2], [3], [4], [5], [6]. The term spatial domain refers to the image plane as such,

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and this category includes approaches based on direct manipulation of image pixels. Methods of processing in the frequency domain are based on the modification of the signal generated by applying the Fourier transform to the image. Drawback of the spatial filtering is often a blurring of the contours, and decrease in sharpness. Frequency same filtering, in case of interference with a periodic component, allows exert a more concentrated influence, at a minimum affecting the useful information.

Intermittent noise is usually caused by electrical or electromechanical interference during image acquisition (the so-called "drift of dark currents"). In this case the image is usually heavily distorted by spatial sinusoidal noise of various frequencies. Such noise can be significantly reduced by frequency filtering in the region of the Fourier transform [1]. The Fourier transform of a sinusoid in its purest form is a pair conjugate pulses located at centrally symmetrical points of the frequency areas that correspond to the frequencies of the sine wave. If the amplitudes of sinusoidal waves in the spatial domain are large enough, it can be expected that pairs will be visible in the spectrum of the image pulses, one for each sine wave in the original image. Therefore, periodic noise can be analyzed and filtered quite effectively using frequency methods. The main idea is that in the Fourier spectrum, periodic noise looks like a concentrated spike energy at the position corresponding to the frequencies of the periodic interference. An approach comes down to using a selective filter (notch, bandpass and narrowband), capable of isolating noise.

In this article we consider an alternative approach, and pay basic attention to the Radon transform, its singular value decomposition (SVD) into sums of ridge functions and its application to removal of strip interference.The Radon transform maps an image into a one-dimensional integral projection, which makes it possible to calculate the convolution and correlation of two images [3], linear and nonlinear filtering, compression and encoding of information [4] in devices designed to process one-dimensional signals. Estimates show that the use of modern elements of optoelectronics allows such image processing systems to compete successfully with other similar devices.

The motivation of using the Radon transform in this paper is an observation that a sequence of strips is a ridge function. We need to extract a single ridge function (or few neighbors of it) from the distorted image. Compared to the frequency approach, projections of Radon space still retain their connection with spatial geometry. The mathematical relation of the Fourier and Radon transforms is known [5], [6]. Important theorem, Central Slice Theorem relates Radon and Fourier spaces : the 1 -D Fourier transformation of a projection yields a "slice"of the 2 -D Fourier transform of the object along a central line.

The paper structure is as follows. In Section 2, we revisit the methods relevant to the image reconstruction using the ridge functions. In Section 3, we present a scheme for the background and strip interference (single ridge function) separation. In Section 4, we illustrate the results of extrating the strip interference in a real-world images of remote sensing. We present our conclusion in Section 5.

2. The Radon transform and ridge functions

Definition 1. *The Radon transform R of the function f with a support in the form of the unit disk D is defined as the set of its integrals along a line with direction α*

and distance s from the origin

$$R_\alpha[f](s) = \int_{-\sqrt{1-s^2}}^{\sqrt{1-s^2}} f(s\cos\alpha - t\sin\alpha, s\sin\alpha + t\cos\alpha)\,dt. \tag{1}$$

We denote a single projection (1) as $p(\alpha, s) = p_\alpha(s)$ and refer to it as a complete projection if it is known for all points in the interval $s \in [-1, 1]$. In practical tomographic reconstructions, only a limited number of projections and a finite number of samples per projection are available. Let us recall the definition of the ridge functions [7], [8], [9], [10], [11]. [12], [13].

Definition 2. *A ridge function $h(x, y)$ on D is understood as a function of the form $h(x, y) = h(x\cos\alpha + y\sin\alpha)$.*

If a set of directions of the projections is the n-tuple $\omega = (\omega_1, ..., \omega_n) \in [0, \pi)^n$, then we denote the corresponding set of complete projections by

$$R_\omega[f] = (R_{\omega_1}[f], \ldots, R_{\omega_n}[f]). \tag{2}$$

Tomographic scanners, recording a sufficient number of projections with equally distributed direction angles, provide the reconstruction of the function f from the data $R_\omega[f]$ in the form of the filtered back-projection (FBP) approximation that reads as

$$f(x, y) \approx \sum_{i=1}^{n} r_i(x\cos\omega_i + y\sin\omega_i). \tag{3}$$

Here, the function r_i is defined as $r_i(s) = \int_{-\infty}^{\infty} p(\omega_i, t)k(s-t)dt$. For each r_i the same kernel k is used [6]. Formula (3) expresses the approximation of the function f as superposition of the ridge functions r_i with the directions ω_i uniformly distributed in the interval $[0, \pi)$. However, in the case of an arbitrary set of the directions $\omega = (\omega_1, ..., \omega_n) \in [0, \pi)^n$, different convolution kernels k -s have to be used for different directions. It is known that generally the FBP approximation is not accurate in the case of a small $(n < 10)$ number of projections.

The alternative approach providing a better accuracy in case of small n is based on the following approach by Logan and Shepp [7]. The representation they use is similar to (3)

$$H(x, y) = \sum_{i=1}^{n} h_{\omega_i}(x\cos\omega_i + y\sin\omega_i), \tag{4}$$

which satisfies the Radon transform - based constraints

$$R_{\omega_i}[f] = R_{\omega_i}[H], \ i = 1, \ldots, n. \tag{5}$$

The minimum norm solution to problem (4) - (5) has the following form

$$h_{\omega_i}(s) = \frac{1}{\pi}\sum_{k=1}^{\infty}\sum_{j=1}^{n} \eta_{ij}^{(k)} U_{k-1}(s) \int_{-1}^{1} p_{\omega_j}(t) U_{k-1}(t)\,dt, \tag{6}$$

where $p_{\omega_j}(t) = R_{\omega_j}[f](t)$, $U_{k-1}(t) = \frac{\sin(k \arccos t)}{\sin(\arccos t)}$, $k = 1, 2, \ldots$ are the Chebyshev polynomials of the second kind, $\eta_{ij}^{(k)}$ are entries of the matrix Λ_k^+ (generalized inverse), $\Lambda_k = (\lambda_{ij}^{(k)})$, $i, j = 1, \ldots, n$, $\lambda_{ij}^{(k)} = \frac{\sin\left(k(\omega_i - \omega_j)\right)}{k\sin(\omega_i - \omega_j)}$.

Equation (6) provides us with the singular value decomposition (SVD) solution of the Radon transform inversion task. It can be shown that the norm of H can be calculated using the projections $p_i \equiv p_{\omega_i}$

$$\|H\|^2 = \frac{1}{\pi}\sum_{k=1}^{\infty}\sum_{i,j=1}^{n}\eta_{ij}^{(k)}\int_{-1}^{1} p_{\omega_i}(s)U_{k-1}(s)\,ds\int_{-1}^{1} p_{\omega_j}(t)U_{k-1}(t)\,dt. \tag{7}$$

The directions ω can be arbitrary, and for the generalized pseudo inversion of the matrices Λ_k we use the **pinv** function of the Matlab package. The integrals $\int p_{\omega_j}(t)U_{k-1}(t)dt$ in (6) and (7) can be calculated using the Gauss-Chebyshev quadrature formula

$$\int_{-1}^{1}\frac{g(t)}{\sqrt{1-t^2}}dt \cong \frac{\pi}{m}\sum_{l=1}^{m} g(t_l), \tag{8}$$

where $t_l = \cos(\frac{2l-1}{2m}\pi), l = 1, \ldots, m$ are sampling points of a function g. Then formula (6) becomes

$$h_{\omega_i}(s) = \frac{1}{m}\sum_{k=1}^{\infty}\sum_{j=1}^{n}\eta_{ij}^{(k)}U_{k-1}(s)\sum_{l=1}^{m} p_j(t_l)\sin(k\frac{2l-1}{2m}\pi). \tag{9}$$

The truncation parameter m in series (9) is used for regularization of the SVD decomposition [14]. We will use an algorithm (6) for computing the superposition of the ridge functions H_ω from an arbitrary set of complete projections R_ω.

3. An image model with background and stripes

Let us consider an additive model [15], [16], [17] of image formation (distortion) in the form of the equation

$$z(x, y) = u(x, y) + v(x, y), \tag{10}$$

where z is the observed image; u is the desired useful image, as a rule, with many details, and v is a certain interfering structure, which is known to be well approximated by the sum of a small number of the ridge functions in n directions $\omega = (\omega_1, \ldots, \omega_n)$. The task is to restore u from the data of z. Due to the additivity of model (10) and the linearity of the Radon transform, the expansion of the functions z, u, v into the superposition of the ridge functions in the directions ω results in the equality

$$Z_\omega = U_\omega + V_\omega. \tag{11}$$

Here, Z_ω, U_ω, V_ω are the minimum norm solutions obtained by reconstructing the images z, u, v from the data sets $R_\omega[z]$, $R_\omega[u]$, $R_\omega[v]$, respectively.

Since it is known that the distortion v is well approximated by the ridge functions in the directions ω, we have $v \approx V_\omega$. Then subtracting equation (10) from (11), we approximately obtain

$$z(x, y) - Z_\omega(x, y) = u(x, y) - U_\omega(x, y). \tag{12}$$

We hypothesize that the difference $u(x, y) - U_\omega(x, y)$ on the right-hand side of (12) is approximately similar to u with a shifted average value. Converting the image $z(x, y) - Z_\omega(x, y)$ to the range of values of a visualizing device, we synthesize an image to be essentially free from distortions caused by the presence of the ridge function v.

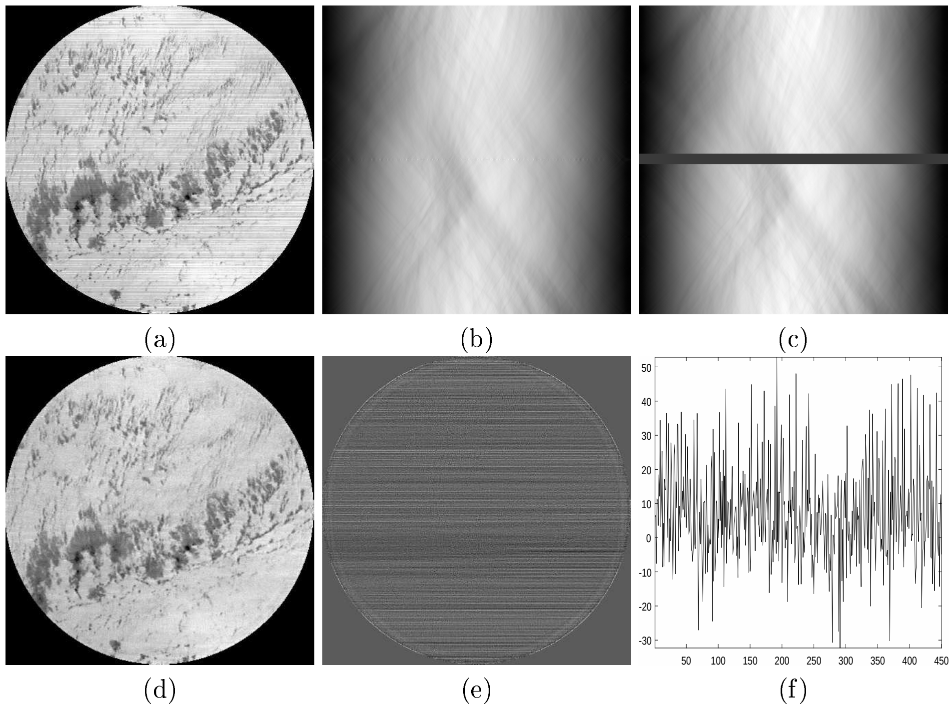

FIG. 1. (a) The test image $z = u + v$ is the sum of the remote sensing image u distorted with parallel stripes v; (b) The Radon transform, or sinogram; (c) The sinogram with 15 projections filtered out (horizontal black strip of 15 samples width, or projections directions angle range of $2\gamma \approx 6°$); (d) The result of the SVD-based approximation Z_ω of z in the directions ω without filtered out projections; (e) The difference $z - Z_\omega$ of images (a) and (d); (f) The profile of the sinusoidal distortion v (central column of image (e)).

In this paper $n = 1$ and direction ω_1 is known in advance. However, when restoring a useful image, we filter out more than one ridge function, but several close adjacent ones, in the range $[\omega_1 - \gamma, \omega_1 + \gamma]$, using small γ as a parameter.

4. The stripe inteference extraction: a real-world examples

To illustrate the task of computing the SVD reconstruction for the real-world problem of removing strip interference in an image, we use (publicly available from the open datasets MODIS [18] and Hyperion [19]) test remote sensing images "Example 1" (Fig. 1 (a)) and "Example 2" (Fig. 2 (a)), respectively, as the distorted images. They are denoted as z and treated as a sum of useful unknown images u that are superimposed by the stripe distortion v, and the image $z = u + v$ is image (10) observed by the vision system. The ridge distortions v are unknown. However, their directions are known in advance, they constitute a beam of parallel horizontal lines resembling dark and light scratches. Images "Example 1" and "Example 2" are both of 450×450 size.

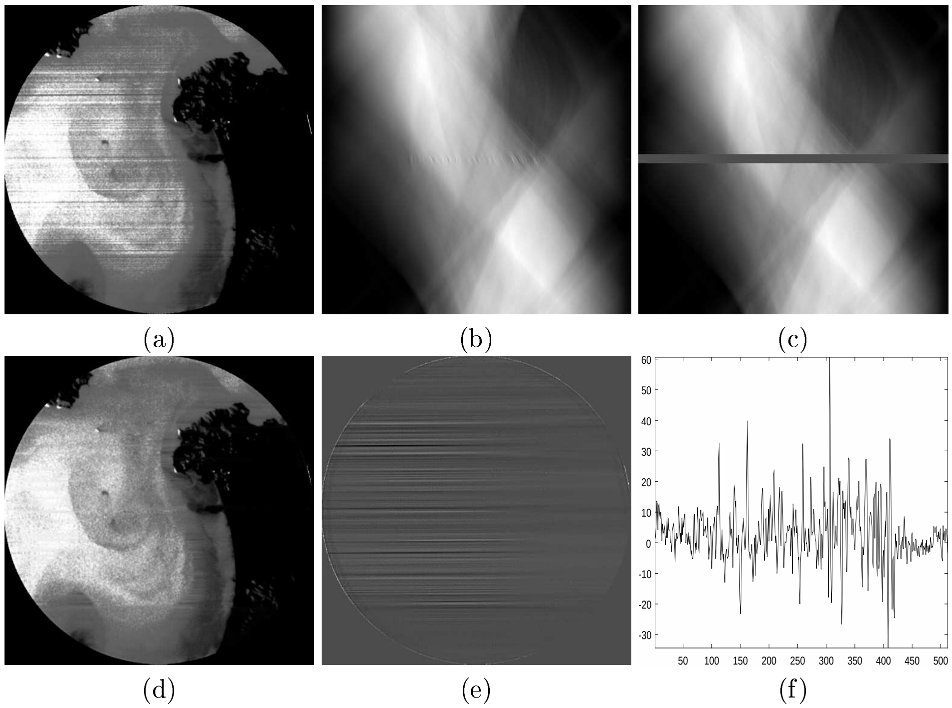

FIG. 2. (a) The test image $z = u + v$ is the sum of the Hyperion satellite image u distorted with parallel stripes v; (b) The Radon transform; (c) The sinogram with 25 projections filtered out (horizontal black strip of 25 samples width, or projections directions angle range of $2\gamma \approx 9°$); (d) The result of the SVD-based approximation Z_ω of z in the directions ω without filtered out projections; (e) The difference $z - Z_\omega$ of images (a) and (d); (f) The profile of the distortion v (central column of image (e)).

It is easy to show that a ridge function with a known direction can be uniquely reconstructed from its projection taken in the same direction. This property is used by us to extract the ridge function from the additive model. To do this, we generate the Radon transform for a sufficiently large number of projections, then we calculate the reconstruction based on the singular value decomposition, by excluding the ridge function (or group of adjacent projections) responsible for the distortion from the data set. If this ridge function is not excluded, the reconstruction will contain distortion and this is almost the original noisy image. This is the filtration in the Radon space - noise components are filtered out. The distorted image z is limited to a circular domain within $n \times n$ image, $n = 450$. For the image z, n projections are numerically generated, evenly distributed in the range $[0, \pi)$, that is, with a discreteness of $180°/n$. Each projection has $n = 450$ samples, so the Radon transform, or a sinogram, is the $n \times n$ image. The projections are arranged line by line in succession from top to bottom, counts ("detectors") make up the horizontal axis (Fig. 1 (b) and Fig. 2 (b). In Fig. 2 (the upper and bottom rows) we repeat computations with a similar methodology of numerical experiments. The reconstruction, or a minimum norm solution Z_α is shown in (Fig. 2 (d)), and the difference between the image z and its ridge approximation Z_α can be seen in (Fig. 2 (e)). We did not use additional image enhancement tricks in the numerical examples.

5. Conclusion

We consider the task of removing the stripe interference in terms of the Radon integral transform, where the approach based on the ridge functions allow the use of anisotropy information in the enhancement of noisy images. We have performed computational experiments to illustrate the efficiency of the SVD algorithm for the Radon transform as applied to the method of filtering in the Radon space of an image to supress sinusoidal noise. The experimental results in the visual inspection of real-world images, show a benefit in decomposition of a distorted image into a sum of a ridge function and useful informative image.

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