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AN ALGORITHM OF FINDING PLANAR SURFACES IN THREE-MANIFOLDS

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This paper is devoted to the question: does there exist an algorithm to decide whether or not a given 3-manifold contains a proper essential *planar surface*? By a *planar surface* we mean a punctured disc.

There is an algorithm, due to W. Jaco, to decide whether a 3-manifold admits a proper essential disc, i.e., whether it is boundary reducible. A close result, an algorithm allow us to say whether a manifold contains a proper essential disc with a given boundary, was obtained by W. Haken in 60-th. In 1998 W. Jaco, H. Rubinstein and E. Sedgwick described an algorithm to decide whether or not a given link-manifold contains a proper essential planar surface (a link-manifold is a compact orientable 3-manifold whose boundary consists of tori) [1]. We generalize this result to manifolds with arbitrary boundaries.

A *slope* on the boundary of a 3-manifold M is the isotopy class of a finite set of disjoint simple closed curves $\{\alpha_1, \dots, \alpha_n\}$ in ∂M which are nontrivial and pairwise nonparallel. We say that the boundary of a proper surface F has a slope $\alpha = \{\alpha_1, \dots, \alpha_n\}$ if the boundary components of F are each parallel to one of the curves $\alpha_1, \dots, \alpha_n$.

Theorem 1. *There is an algorithm to decide if a compact orientable irreducible and boundary irreducible 3-manifold M contains an essential planar surface $F \subset M$ such that ∂F has a given slope α.*

The proof of this theorem is based on the Haken theory of normal surfaces in a 3-manifold with a boundary pattern, see [2].

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The algorithm in Theorem 1 takes as the input a given slope on the boundary of a manifold. At the same time we are interested in another question: does M contain some planar surface, with arbitrary boundary?

Theorem 2. *There is an algorithm to decide if a compact orientable irreducible and boundary irreducible 3-manifold M contains an essential planar surface $F \subset M$.*

The proof of this theorem consists of two steps:

(1) Construct a set A of slopes in ∂M such that if a given manifold M contains a planar slope (one belonging to a planar surface) then also one of the slopes in A must be planar;
(2) For each slope $\alpha \in A$ check if M contains an essential planar surface whose boundary has a slope α.

The algorithm in Theorem 1 allows us to do the checking in the last item. For the first item define the notion of the *length* of a curve γ (denoted by $l(\gamma)$) as the number of intersection points of γ with the edges of the triangulation.

Lemma. *(W. Jaco, H. Rubinstein and E. Sedgwick). Let M be an anannular irreducible and boundary irreducible 3-manifold with a fixed triangulation T. Then there exists a constant $C = C(M,T)$ such that for any proper essential planar surface $F \subset M$ the length of the shortest boundary curve of ∂F is at most C.*

Using this lemma, we can find at least one curve for each slope in A.

A certain particular triangulation of manifold allows us to find length estimates for the other curves of planar slopes in A.

References

[1] Jaco, W. and Sedgwick, E., *Decision problems in the space of Dehn fillings*, arXiv:math.GT/9811031.

[2] Matveev, S., *Algoritmic Topology and Classification of 3-Manifolds*, Springer-Verlag, Berlin, Heidelberg, 2003.

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