S©MR

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ
Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru
ISSN 1813-3304

Vol. 21, No. 1, pp. 266–270 (2024)
https://doi.org/10.33048/semi.2024.21.019

УДК 512.54
MSC 20-03, 01A70

VICTOR DANILOVICH MAZUROV (ON HIS 80-TH BIRTHDAY)

YU. L. ERSHOV, S. S. GONCHAROV, A. V. VASIL'EV, M. A. GRECHKOSEEVA, P. S. KOLESNIKOV, N. S. ROMANOVSKII, A. N. RYASKIN, E. I. KHUKHRO

Communicated by Yu.L. Trakhinin

Ershov Yu. L., Goncharov S.S., Vasil'ev A.V., Grechkoseeva M.A., Kolesnikov P.S., Romanovskii N.S., Ryaskin A.N., Khukhro E.I., Victor Danilovich Mazurov (on his 80-th birthday).

This is an authorized English version of an article in Russian which appeared in Algebra i Logika **62**, no. 1 (2023), i–v.

Received March, 14, 2024, Published April, 5, 2024.

Abstract: On the occasion of his 80th birthday, we give an outline of the recent works of Victor Danilovich Mazurov.

Keywords: group theory, algebra, biography.

Last year marked the 80-th birthday of the corresponding member of the Russian Academy of Sciences Victor Danilovich Mazurov. Mazurov is a world-wide known mathematician, a recognized authority in the area of group theory and its applications, creator of the renowned Novosibirsk school of finite and periodic groups.

A biographical paper to the 70-th birthday of Victor Danilovich appeared in "Siberian Electronic Mathematical Reports" [1]. Therefore here we discuss his new achievements. Note that just over these 10 years he published about 40 papers in prestigious scientific journals. We try to very briefly touch upon the most interesting results of this period.

The main direction of Mazurov's research in this decade became the study of periodic groups with various finiteness conditions. In 2013–2015, together with his perennial co-author D. V. Lytkina and with the participation of his student A. S. Mamontov and the Italian colleague E. Jabara, Mazurov obtained a number of results on local finiteness of periodic $\{2,3\}$-groups with various natural restrictions on the element orders. These results, in particular, were used to show that if the orders of elements of a group do not exceed 6, then the group either is locally finite or is a 5-group. We also highlight the classification of 2-groups in which every finite subgroup is generated by two elements obtained by Mazurov jointly with Lytkina in 2016. Another striking result of the same period obtained jointly with A. Yu. Ol'shanskii and A. I. Sozutov is a negative answer to A. V. Borovik's question on possibility of extending the classical Baer–Suzuki theorem from finite groups to periodic groups. The papers listed above completed the cycle "Periodic groups with given element orders", for which Victor Danilovich was awarded the A. I. Mal'cev Prize of the Russian Academy of Sciences in 2018.

At the end of the XX-th century A. K. Shlëpkin introduced into scientific discourse the notion of a saturating set, which naturally generalizes the notion of a local system of subgroups. A group G is said to be *saturated with groups in a set* $\mathfrak{M}$ (and the set $\mathfrak{M}$ itself is said to be *saturating*) if every finite subgroup of G is contained in a subgroup isomorphic to some group in $\mathfrak{M}$. By the famous theorem of Belyaev–Borovik–Hartley–Schute–Thomas, a locally finite group saturated with finite simple groups of Lie type of bounded rank is itself a group of Lie type over a locally finite field. Shlëpkin formulated a bold conjecture that a similar result was valid for arbitrary periodic groups. As a result of long-term studies, he, together with colleagues and students, managed to prove the validity of this conjecture for groups of rank 1. Note that working with groups saturated with finite simple groups of Lie rank 1 is made easier by the fact that groups of rank 1 have a 2-transitive permutational representation. Passing from groups saturated with groups of

rank 1 to groups saturated with groups of higher ranks required application of essentially new methods. Starting from 2017, Mazurov and Lytkina proved this conjecture for several series of simple groups of Lie type of rank higher than 1. Maybe the most impressive result here was achieved in 2021: they managed to prove Shlëpkin's conjecture for a saturating set consisting of orthogonal groups of an arbitrary (!) odd dimension. The methods developed in these papers can be rightly called revolutionary.

Since the very beginning of his scientific career, Victor Danilovich was interested in groups *freely* acting on other groups (as this happens for the action of the complement of a finite Frobenius group on its kernel). In his papers of this period written in co-authorship with Lytkina, A. Kh. Zhurtov and several Chinese colleagues, Mazurov considered various generalizations of the notion of Frobeniusness for finite and infinite groups. For example, infinite Frobenius groups in which the complement is generated by elements of order 3 were classified. Non-soluble finite generalized Frobenius groups were described. This description, in particular, implies the solubility of finite generalized Frobenius groups with kernel of odd index.

Mazurov was also re-visiting the well-known problem of recognizability of finite groups by the spectrum, in the solution of which for simple groups he played such an important part. He was especially interested in the structure of finite groups that are isospectral to non-recognizable simple groups. Quite recently, jointly with A.V. Vasil'ev, M.A. Grechkoseeva, W.J. Shi and N. Yang, he prepared a survey on these problems, in which, along with a detailed description of the main achievements, a number of new open problems were posed, some of which were included in the new 20-th edition of "Kourovka Notebook".

We would like to describe in more detail the role of Victor Danilovich in the development of this famous collection of unsolved questions in group theory. Mazurov's name for the first time appeared on the pages of the 2-nd edition of "Kourovka Notebook" in 1966. He posed one question therein, about simple groups whose Sylow p-subgroups are cyclic for all odd p, which was solved 12 years later by M. Aschbacher; Mazurov himself solved two problems from this edition. In total, over the entire lifespan of the collection, he (sometimes with co-authors) solved 15 problems, and posed 55 questions (of which 31 remain open). Starting from the 5-th edition of 1975, in the time of rapid development of the theory of finite groups, Mazurov became one of the editors of "Kourovka Notebook". Since the 11-th edition of 1990 Victor Danilovich becomes the Editor-in-Chief of "Kourovka Notebook". Jointly with his student E. I. Khukhro, Victor Danilovich did a lot of work on improving the interaction with the authors, on reinstating several unjustly deleted problems, on attracting a broad range of Russian and foreign specialists to posing new problems. At present, under the leadership of Mazurov, "Kourovka Notebook" acquired an even greater weight among researchers all over the world and continues to serve as an exceptionally useful factor of scientific

development, by providing a unique opportunity for communication of specialists in various areas of group theory. The authors of problems in "Kourovka Notebook" are more than 500 researchers from all over the world; it is a broad international platform for advancing new ideas and trends in group theory. Attracting early career researchers, "Kourovka Notebook" increases the objectivity of assessing the work of specialists, introducing elements of healthy mathematical competition on international stage.

The attractiveness of Victor Danilovich as a scientist and a person facilitated the creation of an outstanding, without exaggeration, scientific school. This school currently consists of 42 people (students and students of students of Mazurov), including 27 Candidates of Science (PhD) and 15 Doctors of Science (DSc). Many years of his experience of teaching at the Mechanics-Mathematics Faculty of the Novosibirsk State University are reflected in the textbook "Higher Algebra", which he wrote in co-authorship with Vasil'ev and Lytkina and which was published in 2020 by the Institute of Mathematics of the Siberian Division of Russian Academy of Sciences. Mazurov is a regular member of programme committees of international mathematical conferences, a member of editorial boards of a number of well-known mathematical journals, a member of the Dissertation Council at the Institute of Mathematics of the Siberian Division of Russian Academy of Sciences, one of the supervisors of the seminars "Algebra and Logic" and "Group Theory" of the Institute of Mathematics of the Siberian Division of Russian Academy of Sciences and Mechanics-Mathematics Faculty of the Novosibirsk State University. Quite recently, already in 2023, to the long list of his achievements and distinctions, one more high state award was added — Order of Honour.

We wish Victor Danilovich new scientific achievements, creative and human longevity, happiness and health for him and his loved ones!

References

[1] *A. V. Vasil'ev, E. P. Vdovin, E. I. Khukhro*, Victor Danilovich Mazurov (on his 70-th birthday), Sib. Electron. Math. Rep., **10** (2013), A.7–A.12.

Yuri Leonidovich Ershov
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: ershov@math.nsc.ru

Sergey Savostyanovich Goncharov
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: s.s.goncharov@math.nsc.ru

Andrey Victorovich Vasil'ev
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: vasand@math.nsc.ru

Mariya Aleksandrovna Grechkoseeva
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: gma@math.nsc.ru

Pavel Sergeyevich Kolesnikov
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: pavelsk@math.nsc.ru

Nikolay Semenovich Romanovskii
Sobolev Institute of Mathematics,
pr. Koptyuga, 4,
630090, Novosibirsk, Russia
Email address: rmnvski@math.nsc.ru

Aleksandr Nikolayevich Ryaskin
Novosibirsk State University,
Pirogova St., 2,
630090, Novosibirsk, Russia
Email address: a.riaskin@g.nsu.ru

Evgeniy Ivanovich Khukhro
Charlotte Scott Research Centre for Algebra,
University of Lincoln, U.K.
Email address: khukhro@yahoo.co.uk