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TEMPORAL LOCALIZATION OF MOVING SOURCES IN HOMOGENEOUS MEDIA USING THE TIME REVERSAL MIRROR

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Abstract: The study is devoted to solving the problem of recovering the trajectory of a seismic source moving underground. The Time Reversal Mirror (TRM) method, which is based on the principle of reversibility of wave processes in media without attenuation, is proposed as a solution.

In this paper, different approaches to source trajectory recovery and visualisation are investigated. The algorithms are tested on synthetic data. The results show that TRM is an effective approach to accurately reconstruct the trajectory of a moving source, even when the observational data are incomplete.

Keywords: wave propagation, seismic source, numerical modeling, Time Reversal Mirror.

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1 Introduction

Understanding the Earth's interior is essential to understanding natural phenomena such as earthquakes, volcanic eruptions and tectonic plate movements. In addition, knowledge of the Earth's interior is crucial to gaining access to valuable resources such as minerals, oil and gas, which are vital to various industries. A number of techniques are used to study the Earth's internal structure, including electrical, seismic, gravity and magnetic methods. Using a combination of these techniques, researchers can analyse the composition, structure and evolution of the Earth, allowing them to make informed decisions about resource extraction, natural hazard prevention and environmental protection.

Seismic methods are widely used in geology and geophysics to study the properties and structure of the Earth's subsurface. These methods involve generating seismic waves that propagate through a medium and then analysing their reflection, refraction or scattering to produce images of the subsurface using specialised interpretation techniques. However, interpretation of seismic data can be complex due to differences in rock type, porosity and fluid content in the subsurface. To gain a complete understanding of the subsurface structure, it is necessary to accurately identify the sources of seismic waves generated in heterogeneous media under specific conditions. These sources can be of different types and can be classified as natural or anthropogenic.

The natural sources of seismic waves are earthquakes, volcanic eruptions and landslides. These events create waves that propagate through the Earth's crust and mantle, providing valuable information about the structure and composition of the planet.

Anthropogenic sources of seismic waves include explosions, drilling and underground nuclear testing. These activities generate waves that can be used for scientific purposes, such as exploring the Earth's interior in search of natural resources or monitoring the effects of human activities on the environment.

An example of an artificial source of seismic waves is hydraulic fracturing, or "fracking", which involves injecting water and chemicals into underground rock formations to release natural gas or oil. The pressure created by this process can cause small earthquakes that can be detected and studied using seismometers.

Another example is the use of controlled explosions in mining. These explosions generate seismic waves that can be used to map the location and size of mineral deposits.

Determining the location of a seismic source in a heterogeneous geological environment is therefore an important issue with major implications for emergency response, earthquake hazard assessment, natural resource exploration, urban planning and infrastructure development.

There are a number of methods that can be used to locate the source of seismic waves. Here are some of them:

1. Seismic arrays: Seismic arrays consist of multiple sensors (seismometers) placed over a wide area. By analysing the arrival times and amplitudes of seismic waves at different sensors, it is possible to determine the location and characteristics of the seismic source. Seismic arrays are often used in earthquake seismology to locate the epicentre and magnitude of earthquakes [1].

2. Waveform inversion: Waveform inversion is a method that uses numerical simulations of seismic waves to match observed waveforms with synthetic ones. By adjusting the parameters of the simulation (e.g. location, size and shape of the source) it is possible to find the best match between observed and synthetic waveforms. Waveform inversion is often used in exploration geophysics to image subsurface structures [2].

3. Back-projection: Back-projection is a technique that uses the recorded seismic waves to reconstruct the propagation path of the waves back to their source. By analysing the direction and timing of the reconstructed paths, it is possible to determine the location and characteristics of the seismic source. Back-projection is often used in earthquake seismology to study the rupture process of earthquakes [3].

Reconstructing the location of moving sources in the subsurface is a more difficult problem for geophysicists to solve. This is because the geological medium is complex and heterogeneous, with different rock properties and structures that can affect the propagation of seismic waves. In addition, the movement of a source can be considered as the superposition of a number of independent sources operating with a time delay. This means that it is not a single source that needs to be recovered, but a series of sources localised in space and time.

2 Mathematical Statement

Elastic wave propagation in an isotropic inhomogeneous two-dimensional medium is described by a system of partial differential equations written in the first-order velocity-stress formulation [4]:

$$
\begin{aligned}
&\rho \frac{\partial v_x}{\partial t} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \\
&\rho \frac{\partial v_z}{\partial t} = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \\
&\frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} + F_{xx} \\
&\frac{\partial \tau_{zz}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} + F_{zz} \\
&\frac{\partial \tau_{xz}}{\partial t} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial z} \right) + F_{xz}
\end{aligned}
\tag{1}
$$

Initial conditions:

$$\vec{v}|_{t=0} = 0, \qquad \vec{\tau}|_{t=0} = 0. \tag{2}$$

Boundary conditions at the free surface:

$$\tau_{zz} = 0, \qquad \tau_{xz} = 0, \tag{3}$$

where $\vec{v} = (v_x, v_z)$ are displacement velocity components, $\vec{\tau} = (\tau_{xx}, \tau_{zz}, \tau_{xz})$ are stress tensor components, ρ is the density and λ, μ are the Lame coefficients. The Lame parameters are expressed in terms of the longitudinal V_p and shear V_s wave velocities as follows

$$\lambda = \rho \left(V_p^2 - 2V_s^2\right), \qquad \mu = \rho V_s^2. \tag{4}$$

The functions F_{xx}, F_{zz}, F_{xz} on the right hand side of the equations (1) allow us to specify different kinds of point sources, both stationary and moving in the medium. In particular, if

$$F_{xx} = F_{zz} = f(t) \cdot \delta(x - x_0, z - z_0), \tag{5}$$

we obtain a stationary volumetric source with the time wavelet $f(t)$ centered at the source point (x_0, z_0) by the Dirac delta function $\delta(x - x_0, z - z_0)$. If

$$F_{xx} = F_{zz} = f(t) \cdot \delta(x - p_x(x,t), z - p_z(z,t)), \tag{6}$$

then we get a volumetric source with time wavelet $f(t)$ moving in the medium along the trajectory $(p_x(x,t), p_z(z,t))$.

In our modeling, we use the Ricker wavelet

$$f(t) = (1 - 2\pi^2 \ f_0^2(t - t_0)^2) exp(-\pi^2 \ f_0^2(t - t_0)^2), \tag{7}$$

where f_0 is the dominant frequency and t_0 is the time delay of the signal pulse.

3 Time Reversal Mirror Technique

The Time Reversal Mirror Method (TRM) is a powerful tool in geophysics for solving problems of imaging and characterising subsurface structures. First introduced by Fink [5] and further developed in subsequent years [6, 7, 8, 9], the TRM method is based on the principle of time reversibility in media without attenuation, which states that the wave processes are invariant under time reversal.

The TRM method involves sending a signal into the subsurface and recording the reflected response from underground structures. These recorded signals are then inverted in time and sent back into the subsurface. Through this process, the retransmitted signals converge at the original source, forming a focused beam that can be used to visualise and describe subsurface structures.

The advantage of TRM lies in its ability to be used in media where the study of geological structures is challenging due to low resolution or poor signal-to-noise ratio, as well as in complex environments where the subsurface is not well understood, such as the study of igneous rocks [10] or human tissue [12, 11].

The TRM method has been successfully applied in several geophysical applications, including seismic, electromagnetic and acoustic imaging. In seismic applications, the TRM has been used to image structures in complex geological media such as salt domes and shale formations [13]. In electromagnetic imaging, the TRM has been used to visualise groundwater resources and mineral deposits [14, 15].

In addition to visualisation, several studies have demonstrated the use of TRM for source localisation in various contexts. For example, in underwater acoustics, TRM is used to determine the location of an underwater source by measuring the time delay of echoes [16]. Similarly, in wireless communications, TRM is used to locate a transmitter using multipath reflections [17].

The TRM process consists of the following steps:

1. Data acquisition. Data is acquired using a variety of methods including seismic, ground penetrating radar and hydroacoustics. The data collected in the form of seismograms provide indirect information about the structure, composition and properties of the material under investigation.

2. Time reversal: This method involves reversing the time direction of the recorded seismograms. By reversing the time direction, the wave field can be traced back to its source.

3. Reconstruction and interpretation. This method involves using mathematical algorithms to create a 3D model of an underground structure, or creating a visual representation of the data using computer software.

However, the ability to accurately locate a moving source is still important for many applications.

4 Time Reversed Wavefield Focusing Techniques

Using TRM, it is easy to find a point source with a known time of occurrence. However, if the exact time of occurrence is unknown, there are multiple sources, or the source is moving, additional methods are required.

Special techniques are used to improve the resolution. We use the time reversal method in sequential mode for a set of sources uniformly distributed along a horizontal line on a free surface. The results from each source are then summed. These methods aim to focus on the location of the sources:

1. **Total elastic energy of TRM**. In order to attenuate the local extrema of the wave fields and to enhance the coherent component of the total wave field, we implement the stacking of the elastic energy density at each time step, following the methodology proposed in [18]. This means that at each computational time t^k, over the entire computational domain, the total energy density E for all previous time steps t^m is determined:

$$E_{sum}(x_i, z_j, t^k) = \sum_{t^m \le t^k} E(x_i, z_j, t^m), \tag{8}$$

where elastic energy density at the time t^m computed by the following relations:

$$E(x_i, z_j, t^m) = \tau_{\mathrm{xx}}(x_i, z_j, t^m)\varepsilon_{\mathrm{xx}}(x_i, z_j, t^m) + \\ + \tau_{\mathrm{zz}}(x_i, z_j, t^m)\varepsilon_{\mathrm{zz}}(x_i, z_j, t^m) + 2\tau_{\mathrm{xz}}(x_i, z_j, t^m)\varepsilon_{\mathrm{xz}}(x_i, z_j, t^m) \tag{9}$$

Here τ and ε are the stress and the strain components, respectively.

2. **Maximum Absolute Pressure Value (MAPV)** was also used to enhance spatial focusing:

$$MAPV(x, z) = \max_{t \in T} |E(x, z, t)|, \tag{10}$$

where T is the computation time of the backward seismic wave propagation.

3. **Peak Average Power Ratio (PAPR)** is an improved version of MAPV:

$$PAPR(x, z) = \frac{E^2_{\mathrm{max}}(x, z)}{E^2_{\mathrm{total}}(x, z)/T}, \tag{11}$$

where

$$E_{\mathrm{max}} = \max_{t \in T} |E(x, z, t)|, \qquad E^2_{\mathrm{total}}(x, z) = \sum_{t=0}^{T} E^2(x, z, t). \tag{12}$$

Like MAPV, it detects peaks, but the average field value beyond the peaks is significantly lower than in MAPV.

In the next section, we present the results of numerical experiments demonstrating the performance of the TRM in recovering the location of a moving source.

5 Numerical simulations

A series of numerical simulations were carried out to validate the algorithm. The process began with the computation of a forward problem involving the modelling of wave propagation from a moving source. During the simulation, the outgoing waves were recorded by an array of receivers located along the upper boundary of the medium. The inverse problem was then solved to determine the trajectory of the moving source. At this stage, the receivers acted as sources, emitting the recorded waves in reverse time order, from the last moment to the first. Methods such as elastic energy summation and calculation of PAPR and MAPV coefficients were used to focus the waves at the location of the source trajectory.

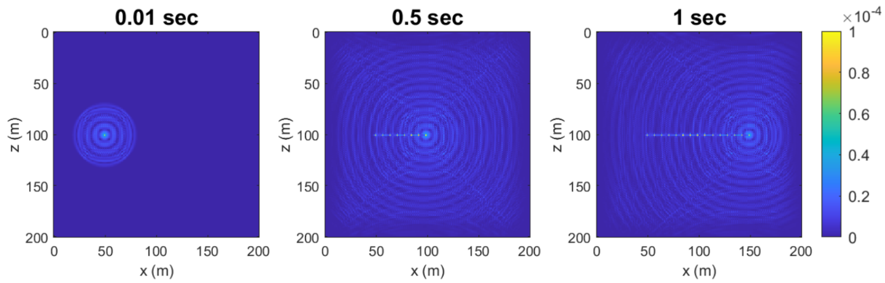

Figure 1. Wavefield snapshots generated by a linearly moving source in a homogeneous medium at different times.

Below we have described the process of modelling the reconstruction of the source trajectory in a homogeneous medium.

The numerical method we use to simulate a symmetric system of first-order evolutionary equations in the velocity-stress formulation (1)-(3) is based on a computationally efficient and widely used approach for this type of equations based on a finite difference scheme on staggered grids [4, 19, 20]. Following Levander [20] we develop a fourth order accurate scheme on a staggered grid in space and a second order accurate scheme in time.

The model parameters are: longitudinal wave velocity $V_p = 3000\ m/s$, shear wave velocity $V_s = V_p/\sqrt{3}$, density $\rho = 2000\ kg/m^3$, domain size $200 \times 200\ m$, and finite difference grid step $1\ m$.

The modelling was carried out for two types of source motion: uniform rectilinear motion and motion following a sinusoidal trajectory. In order to generate a uniform rectilinear horizontal motion, it is necessary to configure the functions $p_x(x,t)$ and $p_z(z,t)$ from the equation (6) as follows:

$$p_x(x,t) = V_{source}t, \qquad p_z(z,t) = z_0, \tag{13}$$

where the source motion velocity V_{source} is $100\ m/s$ in this example.

For the sinusoidal trajectory, the functions should be defined as follows:

$$p_x(x,t) = V_{source}t, \qquad p_z(z,t) = Asin(2\pi ft), \tag{14}$$

where the amplitude of the sinusoidal motion A is $20\ m$, the frequency of the sinusoidal motion f is $5\ Hz$, and the velocity of the source motion V_{source} is $300\ m/s$.

The modelling of wave propagation from a linearly moving source is shown in Figure 1. In addition, the modelling of wave propagation for a source moving along a sinusoidal trajectory is shown in Figure 2. As can be seen from the wavefield snapshots, the waves are not reflected from the boundaries of the computational domain, but are completely suppressed by the use of the PML technique.

The arrangement of the receivers that record the waves emitted by a moving source is shown in Figure 3 and Figure 4. The receivers are equally

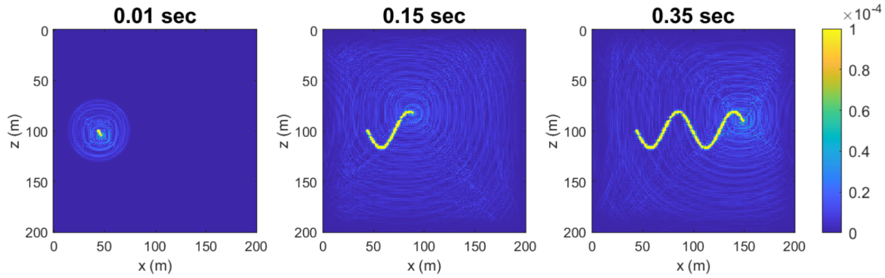

Figure 2. Wavefield snapshots generated by a sinusoidal moving source in a homogeneous medium at different times.

spaced along a horizontal line. In addition, the images show the residual trajectories left by the source.

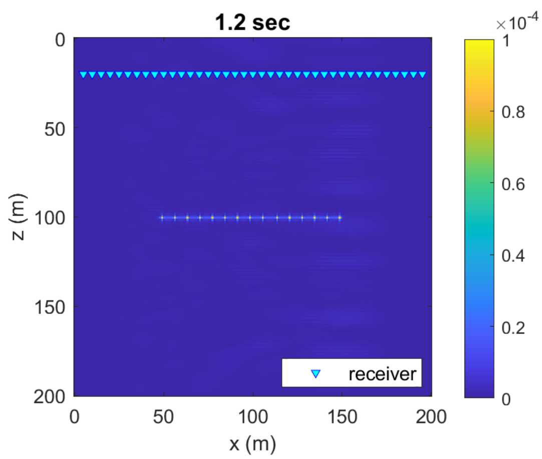

Figure 3. Scheme of uniformly located receivers and trajectory of a rectilinearly moving source in the experiment.

After modelling the forward problem, in which the wave field is registered by receivers, we proceed to solving the inverse problem. In this step, inverted traces are emitted from the receivers and propagate into the medium. According to the TRM, the waves should converge at the location of the original source position. An example of modelling the inverse problem is shown in Figure 5.

Let us now consider the results of the source trajectory reconstruction. Figure 6 presents the reconstruction results using the PAPR method. The true trajectory of the source movement is indicated in red. As inferred from the graph, the method replicates the true trajectory with high precision.

Figure 7 shows the results of other reconstruction methods. It can be seen that the MAPV method also recovers the trajectory efficiently, but its

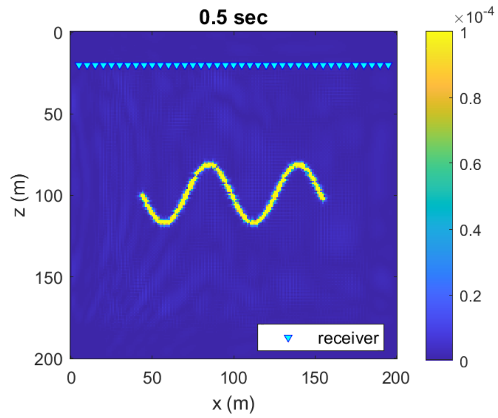

Figure 4. Scheme of uniformly located receivers and trajectory of a sinusoidal moving source in the experiment.

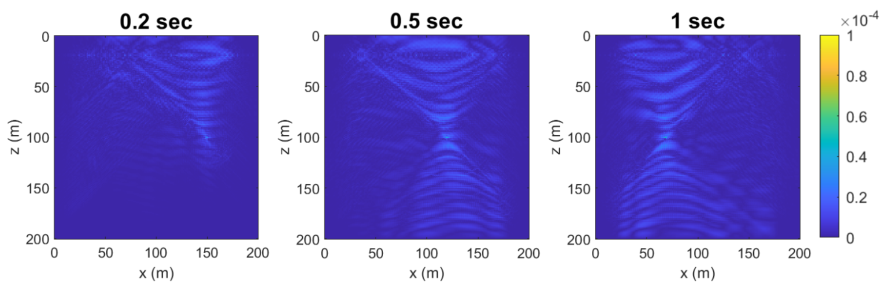

Figure 5. Wavefield snapshots generated by a linear moving source in a homogeneous medium at different times when modelling the inverse problem.

snapshot contains noise that is not represented in the PAPR method. As for the total elastic energy method, the snap contains additional lines to the source trajectory line, making it difficult to identify the true trajectory.

Now consider the results of reconstructing the sinusoidal trajectory of the moving source. Figure 8 shows the result of reconstruction using the PAPR method, with the true trajectory marked in red for ease of comparison. Figure 9, shows the results obtained by other methods. From their comparison we can conclude that the PAPR method gives the most accurate information about the true trajectory. The MAPV method has minor artefacts, and the total elastic energy method, although it allows the trajectory to be extracted, introduces significant noise into the wavefield snapshot.

6 Conclusion

One of the approaches investigated in this paper to solve the problem of recovering the trajectory of a seismic source moving underground is the

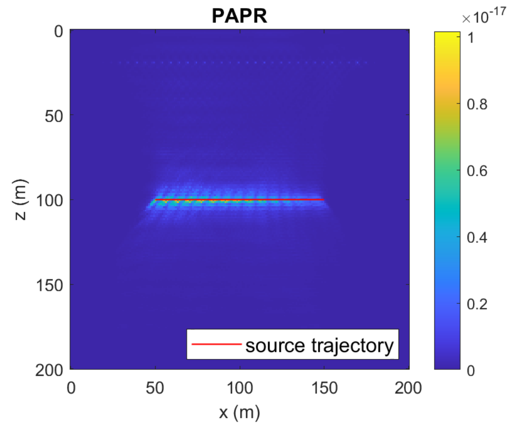

Figure 6. Snapshot of the resulting wavefield for a homogeneous model at the final time instant with the linear moving source acquired using the PAPR coefficient.

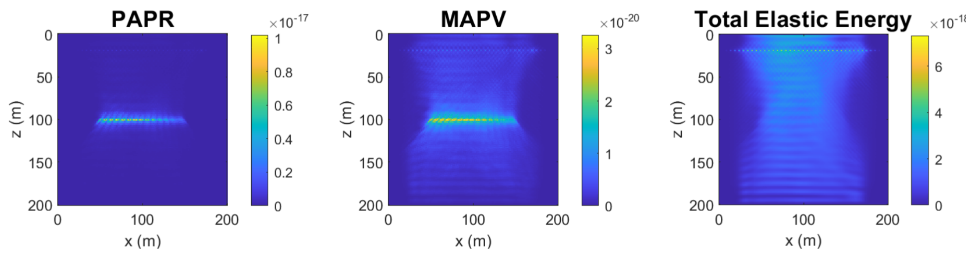

Figure 7. Snapshots of the resulting wavefield for a homogeneous model with the linear moving source acquired using the PAPR coefficient, MAPV coefficient, and total elastic energy.

Time Reversal Mirror method. This method is based on the principle of reversibility of wave processes in media without attenuation.

The study showed that the combination of the TRM method with the PAPR approach is the most effective for reconstructing and visualizing the trajectory of a seismic source moving underground. In addition, the results showed that the PAPR method provided the most accurate information about the true trajectory, making it the preferred method in this study. Despite slightly worse performance, the MAPV method still provided valuable information with minor artefacts. In contrast, the total elastic energy method was not as efficient as the other two methods and introduced significant noise into the wavefield image.

By investigating different methods and comparing their results, this study highlights the importance of using a combination of approaches to ensure the accuracy and reliability of seismic source trajectory reconstruction and

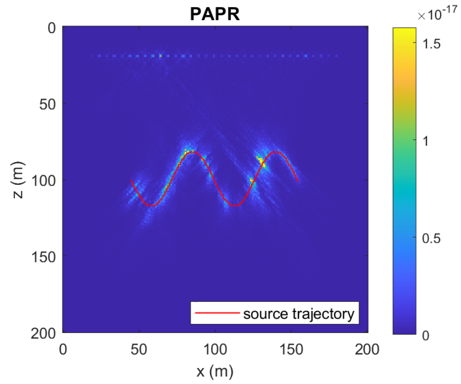

Figure 8. Snapshot of the resulting wavefield for a homogeneous model at the final time instant with the sinusoidal moving source acquired using the PAPR coefficient.

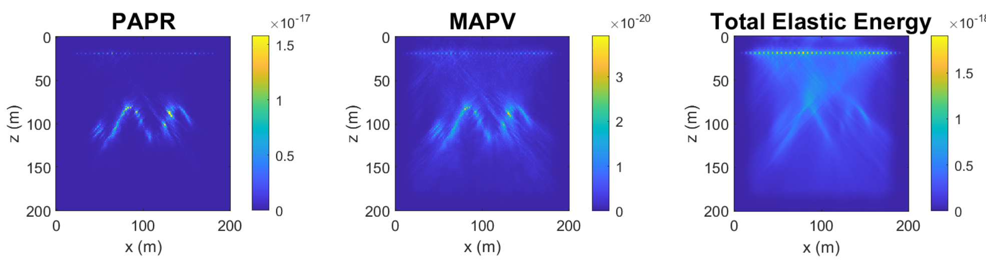

Figure 9. Snapshots of the resulting wavefield for a homogeneous model with the sinusoidal moving source acquired using the PAPR coefficient, MAPV coefficient, and total elastic energy.

visualization. Further research could focus on improving these methods and exploring new techniques to improve the quality of the resulting images of subsurface seismic sources.

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