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REPRESENTATIONS OF (1, 1)-KNOTS

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A knot K in a 3-manifold N^3 is called a $(1,1)$-*knot* if there exists a Heegaard splitting of genus one $(N^3, K) = (H, A) \cup_\psi (H', A')$, where H and H' are solid tori, $A \subset H$ and $A' \subset H'$ are properly embedded trivial arcs and $\psi : (\partial H', \partial A') \to (\partial H, \partial A)$ is an attaching homeomorphism. Obviously, N^3 turns out to be a lens space (possibly $\mathbf{S}^3$). In particular, the family of $(1,1)$-knots contains all torus knots and all two-bridge knots in $\mathbf{S}^3$. The topological properties of $(1,1)$-knots have recently been studied in several papers from different points of view (see references in [2]).

We develop two different representations of $(1,1)$-knots and study the connections between them.

The first representation is algebraic: every $(1,1)$-knot is represented by an element of the pure mapping class group of the twice punctured torus $PMCG_2(T)$, where $T = \partial H$ (see [1, 2]).

Proposition 1. [2] *The kernel of the natural homomorphism* $\Omega : PMCG_2(T) \to MCG(T) \cong SL(2, \mathbb{Z})$ *is a free group of rank two, and there is a surjective map* $\Theta_{p,q}$ *from* $\ker \Omega$ *to the class of all* $(1,1)$*-knots in a fixed lens space* $L(p,q)$*, sending the identity element to the trivial knot in* $L(p,q)$*.*

This type of representation has been explicitly obtained for two-bridge knots and torus knots.

The second representation is parametric: using the results of [4] and [3], every $(1,1)$-knot can be represented by a 4-tuple (a, b, c, r) of integer parameters.

Proposition 2. [3] *A* $(1,1)$*-knot* $(L(p,q), K) = (H, A) \cup_\psi (H', A')$*, is completely determined, up to equivalence, by the isotopy class in* $\partial H - \partial A$ *of the curve* $\psi(\beta')$*, where* β' *is the boundary of a meridian disk of* H' *not intersecting* A'*. As a consequence,* K *can be represented by four non-negative integers* a, b, c, r*.*

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This representation has been obtained for two-bridge knots, torus knots and several $(1,1)$-knots (not belonging in $\mathbf{S}^3$) related to Seifert manifolds.

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