S©MR ISSN 1813-3304

СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ

Siberian Electronic Mathematical Reports

http://semr.math.nsc.ru

Том 5, стр. 68–74 (2008) УДК 512.54
MSC 20C20, 20D06, 20D60

EXCEPTIONAL ACTION OF THE SIMPLE GROUPS $\mathrm{L}_4(q)$ IN THE DEFINING CHARACTERISTIC

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Abstract. We find a counterexample to Problem 14.60 from the Kourovka notebook by giving an example of a group $\mathrm{PSL}_4(q)$ which is not recognizable among its covers from spectrum. Namely, we show that such a group has a module in the defining characteristic with the property that the element order set of the corresponding semidirect product equals that of the group itself.

1. Introduction

If a group H is a homomorphic image of a finite group G then we call G a *cover* for H, or say that G *covers* H. The Kourovka notebook includes the following [1, Problem 14.60]:

Problem 1. *Suppose that G is a proper cover for the finite simple group $L = \mathrm{L}_n(q)$, $n \geq 3$. Is it true that G contains an element whose order is distinct from the order of every element of L?*

This problem is related to the recognition of finite groups by spectrum. Recall that the *spectrum* $\omega(H)$ of a finite group H is the set of its elements orders. We say that H is *recognizable (by spectrum) among its covers* if, for every finite group G covering H, the equality of the spectra $\omega(G) = \omega(H)$ implies the isomorphism $G \cong H$. Thus, Problem 1 asks if every simple group $\mathrm{L}_n(q)$, $n \geq 3$, is recognizable among its covers.

Zavarnitsine, A.V., Exceptional action of the simple groups $\mathrm{L}_4(q)$ in the defining characteristic.

Supported by RFBR, grants 05-01-00797, 06-01-39001, and 08-01-00322; SB RAS, grant N29 for junior scientists and Integration Project 2006.1.2.

Received March, 13, 2008, published March, 25, 2008.

If follows from [2] that the question in Problem 1 is answered in the affirmative, unless $n = 4$ and q is an odd nonprime. It turns out that the groups $\mathrm{L}_4(q)$ exhibit a certain exceptional behavior when acting in the defining characteristic. Our main result is as follows:

Theorem 1. *The group $L = \mathrm{L}_4(13^{24})$ has an absolutely irreducible 96-dimensional module W over a field of characteristic 13 such that $\omega(W \leftthreetimes L) = \omega(L)$. In particular, L is not recognizable by spectrum among its covers.*

Observe that so far there have been only two known examples of nonabelian simple groups that are not recognizable among their covers, viz. $\mathrm{U}_3(3)$ and $\mathrm{U}_3(7)$. Theorem 1 gives a new example of this kind. We conjecture that there are infinitely many groups $\mathrm{L}_4(q)$ which are not recognizable among their covers. We do not know if there is such a group of the form $\mathrm{U}_4(q)$.

2. Auxiliary results

In what follows, we denote $\mathrm{L}_n^+(q) = \mathrm{PSL}_n(q)$ and $\mathrm{L}_n^-(q) = \mathrm{PSU}_n(q)$. A similar convention is used for $\mathrm{SL}_n^\varepsilon(q)$, where $\varepsilon \in \{+, -\}$. Given a finite group G, denote by $\mathrm{Z}(G)$ the center of G and by $\mu(G)$ the numbers in $\omega(G)$ that are maximal with respect to divisibility. Observe that $\omega(G)$ is uniquely restored from $\mu(G)$.

Lemma 1. *Let q be a power of an* odd *prime p and let $\varepsilon \in \{+, -\}$. Denote $d = \gcd(4, q - \varepsilon 1)$. Then $\mu(\mathrm{L}_4^\varepsilon(q))$ contains the following (and only the following) numbers:*

(i) $\dfrac{q^4 - 1}{d(q - \varepsilon 1)}$, $\dfrac{q^3 - \varepsilon 1}{d}$, $\dfrac{p(q^2 - 1)}{d}$, $q^2 - 1$;
(ii) $p(q - \varepsilon 1)$, *if and only if* $d = 4$;
(iii) 9, *if and only if* $p = 3$.

Proof. The claim can be proved by considering the Jordan canonical form of the matrices in $S = \mathrm{SL}_4^\varepsilon(q)$ and determining the intersection of cyclic subgroups of S with the center $\mathrm{Z}(S)$. See also [3]. □

Let r be a prime, G a finite group, and $g \in G$. We say that g is an element of *r-maximal order* if $r\,|g| \notin \omega(G)$. We denote by $\nu_r(G)$ the *minimal*-by-divisibility r-maximal element orders of G. Clearly, $\nu_r(G) \subseteq \omega(G)$.

Lemma 2. *Suppose that a finite group G acts on a vector space W over a field of characteristic r. Then $\omega(W \leftthreetimes G) = \omega(G)$ if and only if there is no $t \in \nu_r(G)$ such that $W \leftthreetimes G$ contains an element of order rt.*

Proof. If $\omega(W \leftthreetimes G) = \omega(G)$ then $rt \notin \omega(G)$ for any r-maximal order t. Conversely, if $\omega(W \leftthreetimes G) \neq \omega(G)$ then we may choose a minimal-by-divisibility element a of $\omega(W \leftthreetimes G) \setminus \omega(G)$. Then we necessarily have $a = rt$ for some $t \in \omega(G)$. Clearly, t is an r-maximal order and in fact $t \in \nu_r(G)$ by the choice of a. □

Lemma 2 shows that the element orders in $\nu_r(G)$ are *r-critical* in the sense that the equality $\omega(W \leftthreetimes G) = \omega(G)$ can be detected by analyzing only the action on W of the elements of G of r-critical orders.

Let $t > 1$ and n be natural numbers and let $\varepsilon \in \{+, -\}$. If there exists a prime that divides $t^n - (\varepsilon 1)^n$ and does not divide $t^i - (\varepsilon 1)^i$ for $1 \leqslant i < n$, then we denote this prime by $t_{[\varepsilon n]}$ and call it a *primitive divisor* of $t^n - (\varepsilon 1)^n$. In general, a primitive

divisor need neither exist nor be unique. The following lemma is a generalization of the well-known Zsigmondy's theorem:

Lemma 3. *Let $t, n > 1$ be natural numbers and $\varepsilon \in \{+, -\}$. There exists a primitive divisor $t_{[\varepsilon n]}$ of $t^n - (\varepsilon 1)^n$, except in the following cases:*

(*i*) $\varepsilon = +$, $n = 6$, $t = 2$;
(*ii*) $\varepsilon = +$, $n = 2$, *and* $t = 2^l - 1$ *for some* $l \geq 2$;
(*iii*) $\varepsilon = -$, $n = 3$, $t = 2$;
(*iv*) $\varepsilon = -$, $n = 2$, *and* $t = 2^l + 1$ *for some* $l \geq 0$.

Proof. See [4, Lemma 5]. □

Given a natural number n and a prime r, we denote by n_r the *r-part* of n, i.e. the largest power of r dividing n, and set $n_{r'} = n/n_r$. Also, we denote by $\pi(n)$ the set of prime divisors of n.

The following lemma describes the p-critical element orders of $\mathrm{L}_4^\varepsilon(q)$ in odd characteristic:

Lemma 4. *Let q be a power of an odd prime p. The set $\nu_p(\mathrm{L}_4^\varepsilon(q))$ contains the following (and only the following) numbers:*

(*i*) $q_{[\varepsilon 4]}$, $q_{[\varepsilon 3]}$, $(q^2-1)_2$;
(*ii*) $q_{[\varepsilon 2]}(q-\varepsilon 1)_2$, if and only if $3 < q \equiv \varepsilon 1 \pmod 4$;
(*iii*) $3(q-\varepsilon 1)_3$, if and only if $q \equiv \varepsilon 1 \pmod 3$;
(*iv*) p, if and only if $p > 3$;
(*v*) $3r$, where $r \in \pi(3(q^2-1))$, if and only if $p = 3$.

Proof. Denote $L = \mathrm{L}_4^\varepsilon(q)$. We will go through the elements a of $\mu(L)$ given in Lemma 1 and determine which divisors t of a belong to $\nu_p(L)$.

Let $a = \dfrac{q^4-1}{d(q-\varepsilon 1)} = a_1 a_2$, where $a_1 = \dfrac{q^2+1}{2}$ and $a_2 = \dfrac{2(q+\varepsilon 1)}{d}$ are coprime. Suppose that $t|a$ and $t \in \nu_p(L)$. Then $t = t_1 t_2$ for uniquely determined divisors t_1 and t_2 of a_1 and a_2. Since $pa_2 \in \omega(L)$, it follows that $t_1 \neq 1$ (otherwise, $pt \in \omega(L)$, a contradiction). Hence, $t_2 = 1$ by the minimality of t. But any nontrivial divisor of a_1 is p-maximal; thus, t is a prime divisor of a_1, i.e. is of the form $q_{[\varepsilon 4]}$.

Let $a = \dfrac{q^3-\varepsilon 1}{d} = a_1 a_2$, where $a_1 = \dfrac{q^2+\varepsilon q+1}{\gcd(3, q-\varepsilon 1)}$ and $a_2 = \dfrac{\gcd(3, q-\varepsilon 1)(q-\varepsilon 1)}{d}$ are coprime. As above, we have $t = t_1 t_2$. If $t_1 \neq 1$ then the minimal-by-divisibility elements of this form are the prime divisors of a_1, i.e. $t = q_{[\varepsilon 3]}$. Suppose that $t_1 = 1$. If $q \not\equiv \varepsilon 1 \pmod 3$ then $a_2 = (q-\varepsilon 1)/d$ and $pa_2 \in \omega(L)$, which implies $pt \in \omega(L)$, a contradiction. Let $q \equiv \varepsilon 1 \pmod 3$. Then $a_2 = a_{2,1}a_{2,2}$, where $a_{2,1} = 3(q-\varepsilon 1)_3$ and $a_{2,2} = (q-\varepsilon 1)_{3'}/d$ are coprime, and we have the corresponding decomposition $t_2 = t_{2,1}t_{2,2}$ with $t_{2,i}|a_{2,i}$, $i = 1, 2$. If $t_{2,1} < a_{2,1}$ then $t|(q-\varepsilon 1)/d$, but $p(q-\varepsilon 1)/d \in \omega(L)$ which yields a contradiction. Therefore, $t_{2,1} = a_{2,1}$. This number, however, is a p-maximal order and so the minimal-by-divisibility number in this case is $3(q-\varepsilon 1)_3$.

Let $a = q^2 - 1 = a_1 a_2$, where $a_1 = (q^2-1)_2$ and $a_2 = (q^2-1)_{2'}$ are coprime, and let $t = t_1 t_2$ as above. If $t_1 = a_1$ then this number is p-maximal and hence the minimal number in this case is $(q^2-1)_2$. If $t_1 \leq a_1/d$ then $pt \in \omega(L)$, a contradiction. Suppose that $t_1 = a_1/2$ and $d = 4$. Then $t_1 = (q-\varepsilon 1)_2$. We write $a_2 = a_{2,1}a_{2,2}$, where $a_{2,1} = (q-\varepsilon 1)_{2'}$ and $a_{2,2} = (q+\varepsilon 1)_{2'}$ are coprime and, correspondingly, $t_2 = t_{2,1}t_{2,2}$. If $t_{2,2} = 1$ then $pt \in \omega(L)$, a contradiction. Otherwise, the number t is

p-maximal it $t_{2,2}$ equal to any prime divisor of $(q+\varepsilon 1)_{2'}$ (which exist unless $q=3$ and have the form $q_{[\varepsilon 2]}$).

Finally, let $p|a$. Then, clearly, $p|t$. Since p is a p-maximal number if and only if $p^2 \notin \omega(L)$, i.e. if $p>3$, we have $p \in \nu_p(L)$ in this case. Let $p=3$. Then 9 is a maximal order and so $3r \in \nu_p(L)$ for every prime divisor r of $3(q^2-1)$. □

Lemma 5. *Let $q = 13^{24}$. All the possible values of $q_{[+n]}$ for $n = 2,3,4$ are as follows:*

(*i*) $q_{[+2]} \in \{1009, 407865361, 659481276875569\}$;
(*ii*) $q_{[+3]} \in \{19, 37, 73, 271, 937, 4177, 181297, 428041, 1471069, 1609669, 9818892432332713\}$;
(*iii*) $q_{[+4]} \in \{97, 2657, 88993, 441281, 283763713, 127028743393, 403791981344275297\}$.

Proof. The primitive divisors $q_{[+n]}$ can be obtained by analyzing the prime factorization of q^n-1, see [5]. □

In the following lemmas, we denote

$$J_n = \begin{pmatrix} 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix},$$

where the right-hand side is the unipotent Jordan block of size n over a field of characteristic p.

Lemma 6. *We have*

(*i*) $|J_n| = p^m$, *where m is the unique integer such that $p^{m-1} < n \le p^m$;*
(*ii*) *The minimal polynomial of J_n is $(x-1)^n$.*

Proof. Straightforward. □

Lemma 7. *Suppose that a cyclic group $C = \langle c \mid c^{p^m} = 1\rangle$ acts on a vector space W over a field of characteristic p. Then $p^{m+1} \in \omega(W \leftthreetimes C)$ if and only if the Jordan canonical form of c on W contains the block J_{p^m}.*

Proof. It is sufficient to consider element orders in the coset cW of $W \leftthreetimes C$ (see Lemma 9 in [8]). An element $cw \in cW$ has order p^{m+1} if and only if

$$1 \neq (cw)^{p^m} = w(1 + c + \ldots + c^{p^m-1}),$$

i. e. the polynomial $f(x) = 1 + x + \ldots + x^{p^m-1}$ does not annihilate c. However, in characteristic p, we have

$$f(x) = \frac{x^{p^m}-1}{x-1} = \frac{(x-1)^{p^m}}{x-1} = (x-1)^{p^m-1}.$$

From Lemma 6 it follows that $f(c)=0$ if all Jordan blocks of c have size at most p^m-1 and $f(c)\neq 0$ if there is a Jordan block of c of size p^m. The claim follows. □

For square matrices A and B, we write $A \simeq B$ if A and B are similar. The symbol $\wedge^2 A$ denotes the exterior square of the matrix A. (We are interested in $\wedge^2 A$ to be defined only up to similarity.)

Lemma 8. *Let the ground field have characteristic $p=13$. Then we have*

(*i*) $J_i \otimes J_j \simeq J_{j-i+1} \oplus J_{j-i+3} \oplus J_{j-i+5} \oplus \ldots \oplus J_{j+i-1}$ *for* $i = 2, \ldots, 5$, $j = i, i+1, \ldots, 7$;
(*ii*) $\wedge^2 J_2 \simeq J_1$, $\wedge^2 J_3 \simeq J_3$, $\wedge^2 J_4 \simeq J_1 \oplus J_5$,
(*iii*) $\wedge^2(J_i \oplus J_j) \simeq \wedge^2 J_i \oplus (J_i \otimes J_j) \oplus \wedge^2 J_j$ *for all* i, j.

Proof. Assertions (*i*) and (*ii*) follow by a direct calculation. Assertion (*iii*) is well known and holds for arbitrary matrices. □

3. Proof of main result

We will denote by $\mathbb{F}_q$ a finite field of $q = p^m$ elements. Let $G = \mathrm{SL}_4(q)$, and let W be an FG-module, where F is a field of characteristic p. Denote by W^{ρ^i}, $i = 0, 1, \ldots$, the the twisting of W by the ith power of the Frobenius map ρ which corresponds to the automorphism $x \mapsto x^p$ of F. The symbol $\wedge^i W$, $i = 0, 1, \ldots$, denotes the ith exterior power of W.

Proposition 1. *Let* $q = 13^{24}$ *and let* V *be the natural* 4*-dimensional module for* $\mathrm{SL}_4(q)$ *over* $\mathbb{F}_q$. *Denote*

$$W = V^{\rho^5} \otimes V^{\rho^{11}} \otimes (\wedge^2 V).$$

Then W, *when viewed as an (absolutely irreducible* 96*-dimensional) module for* $L = \mathrm{L}_4(q)$, *satisfies* $\omega(W \leftthreetimes L) = \omega(L)$.

Proof. Firstly, observe that V is the restriction to $S = \mathrm{SL}_4(q)$ of the natural module for the linear algebraic group $\mathrm{SL}_4(F)$, where F is the algebraic closure of $\mathbb{F}_q$, which is a (rational finite dimensional) irreducible module for this group of highest weight ω_1. Then W is such a restriction of the irreducible module of highest weight $(p^5 + p^{11})\omega_1 + \omega_2$, where $p = 13$, and so is absolutely irreducible as a module for S by the Steinberg Twisted Tensor Product Theorem [6, Theorem 41].

Secondly, the representation that corresponds to W sends the central elements of S to the identity matrix and so W indeed can be viewed as a module for L.

Assume to the contrary that there is $a \in \omega(W \leftthreetimes L) \setminus \omega(L)$. By Lemma 2, $a = 13t$ for some $t \in \nu_{13}(L)$. We will go through all values in $\nu_{13}(L)$ which are given in Lemma 4.

Let $t = q_{[+4]}$. Every element $g \in L$ of order t is the image of such an element of S with characteristic values $\theta, \theta^q, \theta^{q^2}, \theta^{q^3}$, where $\theta \in F^\times$ is of order t. Hence, the characteristic values of g in W have the form

$$\theta^{p^5 q^i} \cdot \theta^{p^{11} q^j} \cdot \theta^{q^k + q^l},$$

where $0 \le i, j, k, l \le 3$ and $k \ne l$. Therefore, in order to arrive at a contradiction in this case, we need to show that the congruence

$$13^{5+24i} + 13^{11+24j} + 13^{24k} + 13^{24l} \equiv 0 \pmod{t},$$

is not solvable for the indicated i, j, k, l and the values of t as in Lemma 5(*iii*). This can be verified directly (or using a computer).

Let $t = q_{[+3]}$ or $t = 3(q-1)_3 = 27$. Then $t | (q^3 - 1)/3$ and the characteristic values of a preimage in S of $g \in L$ of order t are $\theta, \theta^q, \theta^{q^2}, \theta^{-(1+q+q^2)}$, where $\theta \in F^\times$ is of order t. As above this gives the congruence

$$13^5 a_i + 13^{11} a_j + a_k + a_l \equiv 0 \pmod{t}, \tag{1}$$

where $a_i, a_j, a_k, a_l \in \{1, 13^{24}, 13^{48}, -(1+13^{24}+13^{48})\}$, $a_k \neq a_l$, which can be shown to have no solutions for the values of t given in Lemma 5(ii) or for $t = 3(q-1)_3 = 27$.

Let $t = q_{[+2]}(q-1)_2 = 32q_{[+2]}$ or $t = (q^2-1)_2 = 64$, respectively. Note that $t|(q^2-1)$. Since $\mathrm{Z}(S) = 4$ and t is a 2-maximal element order in $\omega(S)$ (which follows by considering the orders of the maximal tori of S, see [7, Theorem 2.1]) any preimage in S of an element $g \in L$ of order t has order t. We consider the possible cyclic subgroups $\langle c\rangle$ of S of order t that intersect trivially with $\mathrm{Z}(S)$. For some characteristic value $\theta_1 \in \mathbb{F}_{q^2}$ of the generator c of such a subgroup, we have either $q_{[+2]} \mid |\theta_1|$ or $|\theta_1| = 64$, respectively. Hence, there are two possibilities for the set of characteristic values of c:

(i) $\{\theta_1, \theta_1^q, \theta_2, \theta_2^q\}$, where $\theta_2 \in \mathbb{F}_{q^2} \setminus \mathbb{F}_q$ and $(\theta_1\theta_2)^{q+1} = 1$;
(ii) $\{\theta_1, \theta_1^q, \theta_2, (\theta_1^{q+1}\theta_2)^{-1}\}$, where $\theta_2 \in \mathbb{F}_q$.

We show that case (i) is impossible. Indeed, assume to the contrary that (i) holds. Then $q_{[+2]} \mid |\theta_2|$ or $|\theta_2| = 64$, respectively. However, in the latter case, we have $c^{32} = \mathrm{diag}\{-1,-1,-1,-1\} \in \mathrm{Z}(S)$, a contradiction, while in the former case we have either $|\theta_2| = t$ or $|\theta_2| < t$. If $|\theta_2| = t$ then again $c^{t/2} = \mathrm{diag}\{-1,-1,-1,-1\} \in \mathrm{Z}(S)$, which is impossible. If $|\theta_2| < t$ then $\theta_2 = \theta_1^{2m}$ for some m and we have

$$1 = (\theta_1\theta_2)^{q+1} = \theta_1^{(2m+1)(q+1)}.$$

This contradicts the fact that the 2-part of $|\theta_1|$ is $32 > 2 = ((2m+1)(q+1))_2$.

Therefore, the characteristic values of c are as in (ii). Note that in this case we must have $|\theta_1| = t$ and there is $0 \leq m < 32$ such that $\theta_2 = \theta_1^{mt/32}$. We determine whether $\langle c\rangle$ intersects $\mathrm{Z}(S)$ trivially, i. e. whether $c^{t/2} \neq \mathrm{diag}\{-1,-1,-1,-1\}$. We have $(\theta_1)^{t/2} = -1$ and so $c^{t/2} = \mathrm{diag}\{-1,-1,(-1)^{mt/32},(-1)^{mt/32}\}$. If $t = 64$ then $(-1)^{mt/32} = 1$ for any m. If $t = 32q_{[+2]}$ then $(-1)^{mt/32} = 1$ exactly when m is even.

Summarizing, as above we have to show that the congruence (1) does not hold, where $a_k \neq a_l$, and a_i, a_j, a_k, a_l are taken from the set $\{1, 13^{24}, 2m, -(1+13^{24}+2m)\}$ with a fixed m satisfying $0 \leq m < 32$ if $t = 64$, and from the set $\{1, 13^{24}, q_{[+2]}m, -(1+13^{24}+q_{[+2]}m)\}$ with a fixed *even* m satisfying $0 \leq m < 32$ if $t = 32q_{[+2]}$ and $q_{[+2]}$ is as given in Lemma 5(i). This can be shown by a direct verification.

Observe that we have shown so far that L contains no semisimple element of p-maximal order that centralizes in W a nonzero vector. We now show the same for unipotent elements.

Suppose, finally, that $t = p = 13$. A preimage of order t in S of an element $g \in L$ of order t has (unipotent) Jordan blocks J_i of size $i \leq 4$ in its action on the natural module V. The action of the field automorphism ρ preserves the Jordan structure, hence the Jordan blocks of g have size at most 7 on $V^{\rho^5} \otimes V^{\rho^{11}}$ and at most 5 on $\wedge^2 V$ by Lemma 8. Thus, the Jordan blocks of g on W have size at most 11 again by Lemma 8(i). However, in order for there to exist an element of order p^2 in $W \leftthreetimes L$, some element of L of order p must have a Jordan block of size $p = 13$ by Lemma 7. This contradiction shows that $W \leftthreetimes L$ does not have elements of order 13^2 and the proof is complete. □

Theorem 1 is now a direct consequence of Proposition 1.

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