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FINITE SIMPLE GROUPS WITH NARROW PRIME SPECTRUM

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Abstract. We find the nonabelian finite simple groups with order prime divisors not exceeding 1000. More generally, we determine the sets of nonabelian finite simple groups whose maximal order prime divisor is a fixed prime less than 1000. Our results are based on calculations in the computer algebra system GAP.

Keywords: Finite simple group, group order, prime divisor.

1. Introduction

In the study of arithmetical properties of finite simple groups, it is often desirable to know the groups whose order prime divisors are not too big. Given a finite group G, we denote by $\pi(G)$ the *prime spectrum* of G, i. e. the set of prime divisors of the order $|G|$. Let π be a finite set of primes. Since the orders of the simple groups are known, it is possible, in principle, to determine all simple groups G with $\pi(G) = \pi$. Although one can find these groups by hand, computer help usually facilitates the task and is less error-prone. The main result of this paper is the following assertion:

Theorem 1. *There are a total of* 1972 *isomorphism types of finite nonabelian simple groups* G *such that all prime divisors of* $|G|$ *do not exceed* 1000.

The groups from Theorem 1 are listed in Tables 1–4 of this paper. In naming simple groups, we mostly use the notation from [1].

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2. The algorithm

It was observed in [2, p. 51] that, given a finite set of primes π, there are only finitely many finite (nonabelian) simple groups G such that $\pi(G) \subseteq \pi$. The gist of the algorithm for finding these groups was also outlined in the proof of [2, Lemma 2]. One should have $G \in \mathcal{S} \cup \mathcal{A}_\pi \cup \mathcal{L}_\pi$, where $\mathcal{S}$ is the set of 26 sporadic groups, $\mathcal{A}_\pi$ is the set of the alternating groups A_n with $n \leqslant p-1$, where p is the smallest prime greater than all the primes in π, and $\mathcal{L}_\pi$ is the set of groups of Lie type of rank l defined over a field of order $q = p^k$, where $p \in \pi$, $k \leqslant t$, $l \leqslant \max\{8, t\}$, and

$$t = \max_{r \in \pi \setminus \{p\}} \operatorname{ord}_r p.$$

Hence, finding all such groups G reduces to checking the orders of finitely many groups. An implementation of this algorithm (with some technical improvements) was performed in the computer algebra system GAP [3]. Theorem 1 is the result of running the program for $\pi = \{2, 3, 5, \dots, 997\}$.

3. The tables

Given a prime p, we denote by $\mathfrak{S}_p$ the set of nonabelian finite simple groups G such that $p \in \pi(G) \subseteq \{2, 3, 5, \dots, p\}$. It is clear that the set of *all* nonabelian finite simple groups is the disjoint union of the finite sets $\mathfrak{S}_p$ for all primes p. Observe that the first two sets $\mathfrak{S}_2$ and $\mathfrak{S}_3$ are trivially empty, whereas the sets $\mathfrak{S}_p$, $p \geqslant 5$, are always nonempty because they contain some generic elements (see below). The groups from Theorem 1 thus constitute $\mathfrak{S}_5 \cup \ldots \cup \mathfrak{S}_{997}$.

In Table 1, we list the elements of $\mathfrak{S}_p$ for $p \leqslant 100$. The upper bound of 100 was chosen so as to include all the sporadic groups. The members of $\mathfrak{S}_p$ are ordered according to the size of their prime spectrum (same for Table 3). The number of groups in each set $\mathfrak{S}_p$ is given after the symbol "#". For each group, we also give the prime decomposition of the order (except for A_n, $n \geqslant 23$, whose order decompositions occupy too much space and are not too interesting).

The sets $\mathfrak{S}_p$ for larger primes p are described in a more abbreviated form. Denote by p' the smallest prime greater than p. Then $\mathfrak{S}_p$, $p \geqslant 5$, always contains the groups

$$L_2(p), A_p, A_{p+1}, \dots, A_{p'-1}$$

which we call the *generic* elements of $\mathfrak{S}_p$. The primes $p \in \{100, \dots, 1000\}$ such that $\mathfrak{S}_p$ contains no simple groups other than the generic ones are listed in Table 2. In this case, we have $|\mathfrak{S}_p| = p' - p + 1$. Incidentally, the only such primes p less than 100 are 59 and 89 as follows from Table 1. The non-generic elements of $\mathfrak{S}_p$ for the primes p not listed in Table 2 are given in Table 3.

Finally, in Table 4 we list all the groups from Theorem 1 collected in series so that the membership of a given simple group could be easily checked. Isomorphic groups like $L_2(7) \cong L_3(2)$ are all included. The parameters n, k are positive integers, p is a prime, and q is a prime power. Long sequences of consecutive primes or integers are abbreviated with an ellipsis.

The sequence $(a_1, a_2, \dots)$ of integers

$$0, 0, 3, 15, 10, 27, 18, 15, 14, \dots$$

defined by $a_n = |\mathfrak{S}_{p_n}|$, where p_n is the nth prime, appears in [4] as Sequence $A145179$.

Table 1: **Nonabelian simple groups G with $\pi(G) \subseteq \{1, 2, \ldots, 100\}$**

G	$\|G\|$	
$\mathfrak{S}_5$	$\pi(G) \subseteq \{2, 3, 5\}$	# 3
$A_5 \cong L_2(4) \cong L_2(5)$	$2^2{\cdot}3{\cdot}5$	
$A_6 \cong L_2(9)$	$2^3{\cdot}3^2{\cdot}5$	
$S_4(3) \cong U_4(2)$	$2^6{\cdot}3^4{\cdot}5$	
$\mathfrak{S}_7$	$7 \in \pi(G) \subseteq \{2, 3, 5, 7\}$	# 15
$L_2(7) \cong L_3(2)$	$2^3{\cdot}3{\cdot}7$	
$L_2(8)$	$2^3{\cdot}3^2{\cdot}7$	
$U_3(3)$	$2^5{\cdot}3^3{\cdot}7$	
A_7	$2^3{\cdot}3^2{\cdot}5{\cdot}7$	
$L_2(49)$	$2^4{\cdot}3{\cdot}5^2{\cdot}7^2$	
$U_3(5)$	$2^4{\cdot}3^2{\cdot}5^3{\cdot}7$	
$L_3(4)$	$2^6{\cdot}3^2{\cdot}5{\cdot}7$	
$A_8 \cong L_4(2)$	$2^6{\cdot}3^2{\cdot}5{\cdot}7$	
A_9	$2^6{\cdot}3^4{\cdot}5{\cdot}7$	
J_2	$2^7{\cdot}3^3{\cdot}5^2{\cdot}7$	
A_{10}	$2^7{\cdot}3^4{\cdot}5^2{\cdot}7$	
$U_4(3)$	$2^7{\cdot}3^6{\cdot}5{\cdot}7$	
$S_4(7)$	$2^8{\cdot}3^2{\cdot}5^2{\cdot}7^4$	
$S_6(2)$	$2^9{\cdot}3^4{\cdot}5{\cdot}7$	
$O_8^+(2)$	$2^{12}{\cdot}3^5{\cdot}5^2{\cdot}7$	
$\mathfrak{S}_{11}$	$11 \in \pi(G) \subseteq \{2, 3, 5, 7, 11\}$	# 10
$L_2(11)$	$2^2{\cdot}3{\cdot}5{\cdot}11$	
M_{11}	$2^4{\cdot}3^2{\cdot}5{\cdot}11$	
M_{12}	$2^6{\cdot}3^3{\cdot}5{\cdot}11$	
$U_5(2)$	$2^{10}{\cdot}3^5{\cdot}5{\cdot}11$	
M_{22}	$2^7{\cdot}3^2{\cdot}5{\cdot}7{\cdot}11$	
A_{11}	$2^7{\cdot}3^4{\cdot}5^2{\cdot}7{\cdot}11$	
McL	$2^7{\cdot}3^6{\cdot}5^3{\cdot}7{\cdot}11$	
HS	$2^9{\cdot}3^2{\cdot}5^3{\cdot}7{\cdot}11$	
A_{12}	$2^9{\cdot}3^5{\cdot}5^2{\cdot}7{\cdot}11$	
$U_6(2)$	$2^{15}{\cdot}3^6{\cdot}5{\cdot}7{\cdot}11$	
$\mathfrak{S}_{13}$	$13 \in \pi(G) \subseteq \{2, 3, 5, \ldots, 13\}$	# 27
$L_3(3)$	$2^4{\cdot}3^3{\cdot}13$	
$L_2(25)$	$2^3{\cdot}3{\cdot}5^2{\cdot}13$	
$U_3(4)$	$2^6{\cdot}3{\cdot}5^2{\cdot}13$	
$S_4(5)$	$2^6{\cdot}3^2{\cdot}5^4{\cdot}13$	
$L_4(3)$	$2^7{\cdot}3^6{\cdot}5{\cdot}13$	
${}^2F_4(2)'$	$2^{11}{\cdot}3^3{\cdot}5^2{\cdot}13$	
$L_2(13)$	$2^2{\cdot}3{\cdot}7{\cdot}13$	
$L_2(27)$	$2^2{\cdot}3^3{\cdot}7{\cdot}13$	
$G_2(3)$	$2^6{\cdot}3^6{\cdot}7{\cdot}13$	
${}^3D_4(2)$	$2^{12}{\cdot}3^4{\cdot}7^2{\cdot}13$	
$Sz(8)$	$2^6{\cdot}5{\cdot}7{\cdot}13$	
$L_2(64)$	$2^6{\cdot}3^2{\cdot}5{\cdot}7{\cdot}13$	
$U_4(5)$	$2^7{\cdot}3^4{\cdot}5^6{\cdot}7{\cdot}13$	
$L_3(9)$	$2^7{\cdot}3^6{\cdot}5{\cdot}7{\cdot}13$	
$S_6(3)$	$2^9{\cdot}3^9{\cdot}5{\cdot}7{\cdot}13$	
$O_7(3)$	$2^9{\cdot}3^9{\cdot}5{\cdot}7{\cdot}13$	
$G_2(4)$	$2^{12}{\cdot}3^3{\cdot}5^2{\cdot}7{\cdot}13$	
$S_4(8)$	$2^{12}{\cdot}3^4{\cdot}5{\cdot}7^2{\cdot}13$	
$O_8^+(3)$	$2^{12}{\cdot}3^{12}{\cdot}5^2{\cdot}7{\cdot}13$	
$L_5(3)$	$2^9{\cdot}3^{10}{\cdot}5{\cdot}11^2{\cdot}13$	

A_{13}	$2^9\cdot 3^5\cdot 5^2\cdot 7\cdot 11\cdot 13$	
A_{14}	$2^{10}\cdot 3^5\cdot 5^2\cdot 7^2\cdot 11\cdot 13$	
A_{15}	$2^{10}\cdot 3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13$	
$L_6(3)$	$2^{11}\cdot 3^{15}\cdot 5\cdot 7\cdot 11^2\cdot 13^2$	
Suz	$2^{13}\cdot 3^7\cdot 5^2\cdot 7\cdot 11\cdot 13$	
A_{16}	$2^{14}\cdot 3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13$	
Fi_{22}	$2^{17}\cdot 3^9\cdot 5^2\cdot 7\cdot 11\cdot 13$	
$\mathfrak{S}_{17}$	$17 \in \pi(G) \subseteq \{2,3,5,\dots,17\}$	# 18
$L_2(17)$	$2^4\cdot 3^2\cdot 17$	
$L_2(16)$	$2^4\cdot 3\cdot 5\cdot 17$	
$S_4(4)$	$2^8\cdot 3^2\cdot 5^2\cdot 17$	
He	$2^{10}\cdot 3^3\cdot 5^2\cdot 7^3\cdot 17$	
$O_8^-(2)$	$2^{12}\cdot 3^4\cdot 5\cdot 7\cdot 17$	
$L_4(4)$	$2^{12}\cdot 3^4\cdot 5^2\cdot 7\cdot 17$	
$S_8(2)$	$2^{16}\cdot 3^5\cdot 5^2\cdot 7\cdot 17$	
$U_4(4)$	$2^{12}\cdot 3^2\cdot 5^3\cdot 13\cdot 17$	
$U_3(17)$	$2^6\cdot 3^4\cdot 7\cdot 13\cdot 17^3$	
$O_{10}^-(2)$	$2^{20}\cdot 3^6\cdot 5^2\cdot 7\cdot 11\cdot 17$	
$L_2(13^2)$	$2^3\cdot 3\cdot 5\cdot 7\cdot 13^2\cdot 17$	
$S_4(13)$	$2^6\cdot 3^2\cdot 5\cdot 7^2\cdot 13^4\cdot 17$	
$L_3(16)$	$2^{12}\cdot 3^2\cdot 5^2\cdot 7\cdot 13\cdot 17$	
$S_6(4)$	$2^{18}\cdot 3^4\cdot 5^3\cdot 7\cdot 13\cdot 17$	
$O_8^+(4)$	$2^{24}\cdot 3^5\cdot 5^4\cdot 7\cdot 13\cdot 17^2$	
$F_4(2)$	$2^{24}\cdot 3^6\cdot 5^2\cdot 7^2\cdot 13\cdot 17$	
A_{17}	$2^{14}\cdot 3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17$	
A_{18}	$2^{15}\cdot 3^8\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17$	
$\mathfrak{S}_{19}$	$19 \in \pi(G) \subseteq \{2,3,5,\dots,19\}$	# 15
$L_2(19)$	$2^2\cdot 3^2\cdot 5\cdot 19$	
$L_3(7)$	$2^5\cdot 3^2\cdot 7^3\cdot 19$	
$U_3(8)$	$2^9\cdot 3^4\cdot 7\cdot 19$	
$U_3(19)$	$2^5\cdot 3^2\cdot 5^2\cdot 7^3\cdot 19^3$	
$L_4(7)$	$2^9\cdot 3^4\cdot 5^2\cdot 7^6\cdot 19$	
J_3	$2^7\cdot 3^5\cdot 5\cdot 17\cdot 19$	
J_1	$2^3\cdot 3\cdot 5\cdot 7\cdot 11\cdot 19$	
$L_3(11)$	$2^4\cdot 3\cdot 5^2\cdot 7\cdot 11^3\cdot 19$	
HN	$2^{14}\cdot 3^6\cdot 5^6\cdot 7\cdot 11\cdot 19$	
$U_4(8)$	$2^{18}\cdot 3^7\cdot 5\cdot 7^2\cdot 13\cdot 19$	
A_{19}	$2^{15}\cdot 3^8\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 19$	
A_{20}	$2^{17}\cdot 3^8\cdot 5^4\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 19$	
A_{21}	$2^{17}\cdot 3^9\cdot 5^4\cdot 7^3\cdot 11\cdot 13\cdot 17\cdot 19$	
A_{22}	$2^{18}\cdot 3^9\cdot 5^4\cdot 7^3\cdot 11^2\cdot 13\cdot 17\cdot 19$	
${}^2E_6(2)$	$2^{36}\cdot 3^9\cdot 5^2\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 19$	
$\mathfrak{S}_{23}$	$23 \in \pi(G) \subseteq \{2,3,5,\dots,23\}$	# 14
$L_2(23)$	$2^3\cdot 3\cdot 11\cdot 23$	
$U_3(23)$	$2^7\cdot 3^2\cdot 11\cdot 13^2\cdot 23^3$	
M_{23}	$2^7\cdot 3^2\cdot 5\cdot 7\cdot 11\cdot 23$	
M_{24}	$2^{10}\cdot 3^3\cdot 5\cdot 7\cdot 11\cdot 23$	
Co_3	$2^{10}\cdot 3^7\cdot 5^3\cdot 7\cdot 11\cdot 23$	
Co_2	$2^{18}\cdot 3^6\cdot 5^3\cdot 7\cdot 11\cdot 23$	
Co_1	$2^{21}\cdot 3^9\cdot 5^4\cdot 7^2\cdot 11\cdot 13\cdot 23$	
Fi_{23}	$2^{18}\cdot 3^{13}\cdot 5^2\cdot 7\cdot 11\cdot 13\cdot 17\cdot 23$	
$A_{23},\dots,A_{28}$	$\pi(G) = \{2,3,5,\dots,23\}$	
$\mathfrak{S}_{29}$	$29 \in \pi(G) \subseteq \{2,3,5,\dots,29\}$	# 8
$L_2(29)$	$2^2\cdot 3\cdot 5\cdot 7\cdot 29$	
$L_2(17^2)$	$2^5\cdot 3^2\cdot 5\cdot 17^2\cdot 29$	
$S_4(17)$	$2^{10}\cdot 3^4\cdot 5\cdot 17^4\cdot 29$	

Ru	$2^{14}\cdot 3^3\cdot 5^3\cdot 7\cdot 13\cdot 29$	
$U_4(17)$	$2^{11}\cdot 3^7\cdot 5\cdot 7\cdot 13\cdot 17^6\cdot 29$	
Fi'_{24}	$2^{21}\cdot 3^{16}\cdot 5^2\cdot 7^3\cdot 11\cdot 13\cdot 17\cdot 23\cdot 29$	
A_{29}, A_{30}	$\pi(G) = \{2,3,5,\dots,29\}$	
$\mathfrak{S}_{31}$	$31 \in \pi(G) \subseteq \{2,3,5,\dots,31\}$	# 28
$L_2(31)$	$2^5\cdot 3\cdot 5\cdot 31$	
$L_3(5)$	$2^5\cdot 3\cdot 5^3\cdot 31$	
$L_2(32)$	$2^5\cdot 3\cdot 11\cdot 31$	
$L_2(5^3)$	$2^2\cdot 3^2\cdot 5^3\cdot 7\cdot 31$	
$G_2(5)$	$2^6\cdot 3^3\cdot 5^6\cdot 7\cdot 31$	
$L_5(2)$	$2^{10}\cdot 3^2\cdot 5\cdot 7\cdot 31$	
$L_6(2)$	$2^{15}\cdot 3^4\cdot 5\cdot 7^2\cdot 31$	
$L_4(5)$	$2^7\cdot 3^2\cdot 5^6\cdot 13\cdot 31$	
$L_3(25)$	$2^7\cdot 3^2\cdot 5^6\cdot 7\cdot 13\cdot 31$	
$O_7(5)$	$2^9\cdot 3^4\cdot 5^9\cdot 7\cdot 13\cdot 31$	
$S_6(5)$	$2^9\cdot 3^4\cdot 5^9\cdot 7\cdot 13\cdot 31$	
$O_8^+(5)$	$2^{12}\cdot 3^5\cdot 5^{12}\cdot 7\cdot 13^2\cdot 31$	
$O_{10}^+(2)$	$2^{20}\cdot 3^5\cdot 5^2\cdot 7\cdot 17\cdot 31$	
$U_3(31)$	$2^{11}\cdot 3\cdot 5\cdot 7^2\cdot 19\cdot 31^3$	
$L_5(4)$	$2^{20}\cdot 3^5\cdot 5^2\cdot 7\cdot 11\cdot 17\cdot 31$	
$S_{10}(2)$	$2^{25}\cdot 3^6\cdot 5^2\cdot 7\cdot 11\cdot 17\cdot 31$	
$O_{12}^+(2)$	$2^{30}\cdot 3^8\cdot 5^2\cdot 7^2\cdot 11\cdot 17\cdot 31$	
ON	$2^9\cdot 3^4\cdot 5\cdot 7^3\cdot 11\cdot 19\cdot 31$	
Th	$2^{15}\cdot 3^{10}\cdot 5^3\cdot 7^2\cdot 13\cdot 19\cdot 31$	
$O_{12}^-(2)$	$2^{30}\cdot 3^6\cdot 5^3\cdot 7\cdot 11\cdot 13\cdot 17\cdot 31$	
$L_6(4)$	$2^{30}\cdot 3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 31$	
$S_{12}(2)$	$2^{36}\cdot 3^8\cdot 5^3\cdot 7^2\cdot 11\cdot 13\cdot 17\cdot 31$	
$A_{31},\dots,A_{36}$	$\pi(G) = \{2,3,5,\dots,31\}$	
$\mathfrak{S}_{37}$	$37 \in \pi(G) \subseteq \{2,3,5,\dots,37\}$	# 13
$L_2(37)$	$2^2\cdot 3^2\cdot 19\cdot 37$	
$U_3(11)$	$2^5\cdot 3^2\cdot 5\cdot 11^3\cdot 37$	
$L_2(31^2)$	$2^6\cdot 3\cdot 5\cdot 13\cdot 31^2\cdot 37$	
$S_4(31)$	$2^{12}\cdot 3^2\cdot 5^2\cdot 13\cdot 31^4\cdot 37$	
${}^2G_2(27)$	$2^3\cdot 3^9\cdot 7\cdot 13\cdot 19\cdot 37$	
$U_3(27)$	$2^5\cdot 3^9\cdot 7^2\cdot 13\cdot 19\cdot 37$	
$L_2(11^3)$	$2^2\cdot 3^2\cdot 5\cdot 7\cdot 11^3\cdot 19\cdot 37$	
$G_2(11)$	$2^6\cdot 3^3\cdot 5^2\cdot 7\cdot 11^6\cdot 19\cdot 37$	
$U_4(31)$	$2^{16}\cdot 3^2\cdot 5^2\cdot 7^2\cdot 13\cdot 19\cdot 31^6\cdot 37$	
$A_{37},\dots,A_{40}$	$\pi(G) = \{2,3,5,\dots,37\}$	
$\mathfrak{S}_{41}$	$41 \in \pi(G) \subseteq \{2,3,5,\dots,41\}$	# 17
$L_2(3^4)$	$2^4\cdot 3^4\cdot 5\cdot 41$	
$S_4(9)$	$2^8\cdot 3^8\cdot 5^2\cdot 41$	
$Sz(32)$	$2^{10}\cdot 5^2\cdot 31\cdot 41$	
$L_2(41)$	$2^3\cdot 3\cdot 5\cdot 7\cdot 41$	
$O_8^-(3)$	$2^{10}\cdot 3^{12}\cdot 5\cdot 7\cdot 13\cdot 41$	
$L_4(9)$	$2^{10}\cdot 3^{12}\cdot 5^2\cdot 7\cdot 13\cdot 41$	
$S_8(3)$	$2^{14}\cdot 3^{16}\cdot 5^2\cdot 7\cdot 13\cdot 41$	
$O_9(3)$	$2^{14}\cdot 3^{16}\cdot 5^2\cdot 7\cdot 13\cdot 41$	
$L_2(41^2)$	$2^4\cdot 3\cdot 5\cdot 7\cdot 29^2\cdot 41^2$	
$S_4(41)$	$2^8\cdot 3^2\cdot 5^2\cdot 7^2\cdot 29^2\cdot 41^4$	
$L_2(2^{10})$	$2^{10}\cdot 3\cdot 5^2\cdot 11\cdot 31\cdot 41$	
$S_4(32)$	$2^{20}\cdot 3^2\cdot 5^2\cdot 11^2\cdot 31^2\cdot 41$	
$U_5(4)$	$2^{20}\cdot 3^2\cdot 5^4\cdot 13\cdot 17\cdot 41$	
$O_{10}^+(3)$	$2^{15}\cdot 3^{20}\cdot 5^2\cdot 7\cdot 11^2\cdot 13\cdot 41$	
$U_6(4)$	$2^{30}\cdot 3^4\cdot 5^6\cdot 7\cdot 13^2\cdot 17\cdot 41$	
A_{41}, A_{42}	$\pi(G) = \{2,3,5,\dots,41\}$	

$\mathfrak{S}_{43}$	$43 \in \pi(G) \subseteq \{2,3,5,\dots,43\}$	# 22
$U_3(7)$	$2^7 \cdot 3 \cdot 7^3 \cdot 43$	
$U_4(7)$	$2^{10} \cdot 3^2 \cdot 5^2 \cdot 7^6 \cdot 43$	
$L_2(43)$	$2^2 \cdot 3 \cdot 7 \cdot 11 \cdot 43$	
$L_2(7^3)$	$2^3 \cdot 3^2 \cdot 7^3 \cdot 19 \cdot 43$	
$G_2(7)$	$2^8 \cdot 3^3 \cdot 7^6 \cdot 19 \cdot 43$	
$U_7(2)$	$2^{21} \cdot 3^8 \cdot 5 \cdot 7 \cdot 11 \cdot 43$	
$L_3(49)$	$2^9 \cdot 3^2 \cdot 5^2 \cdot 7^6 \cdot 19 \cdot 43$	
$S_6(7)$	$2^{12} \cdot 3^4 \cdot 5^2 \cdot 7^9 \cdot 19 \cdot 43$	
$O_7(7)$	$2^{12} \cdot 3^4 \cdot 5^2 \cdot 7^9 \cdot 19 \cdot 43$	
$O_8^+(7)$	$2^{16} \cdot 3^5 \cdot 5^4 \cdot 7^{12} \cdot 19 \cdot 43$	
$U_3(37)$	$2^4 \cdot 3^2 \cdot 19^2 \cdot 31 \cdot 37^3 \cdot 43$	
$U_8(2)$	$2^{28} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 17 \cdot 43$	
$L_2(43^2)$	$2^3 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 37 \cdot 43^2$	
$S_4(43)$	$2^6 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11^2 \cdot 37 \cdot 43^4$	
$U_9(2)$	$2^{36} \cdot 3^{11} \cdot 5^2 \cdot 7 \cdot 11 \cdot 17 \cdot 19 \cdot 43$	
$O_{14}^-(2)$	$2^{42} \cdot 3^9 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 31 \cdot 43$	
$U_{10}(2)$	$2^{45} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11^2 \cdot 17 \cdot 19 \cdot 31 \cdot 43$	
J_4	$2^{21} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11^3 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 43$	
$A_{43},\dots,A_{46}$	$\pi(G) = \{2,3,5,\dots,43\}$	
$\mathfrak{S}_{47}$	$47 \in \pi(G) \subseteq \{2,3,5,\dots,47\}$	# 10
$L_2(47)$	$2^4 \cdot 3 \cdot 23 \cdot 47$	
$L_2(47^2)$	$2^5 \cdot 3 \cdot 5 \cdot 13 \cdot 17 \cdot 23 \cdot 47^2$	
$S_4(47)$	$2^{10} \cdot 3^2 \cdot 5 \cdot 13 \cdot 17 \cdot 23^2 \cdot 47^4$	
B	$2^{41} \cdot 3^{13} \cdot 5^6 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47$	
$A_{47},\dots,A_{52}$	$\pi(G) = \{2,3,5,\dots,47\}$	
$\mathfrak{S}_{53}$	$53 \in \pi(G) \subseteq \{2,3,5,\dots,53\}$	# 10
$L_2(53)$	$2^2 \cdot 3^3 \cdot 13 \cdot 53$	
$L_2(23^2)$	$2^4 \cdot 3 \cdot 5 \cdot 11 \cdot 23^2 \cdot 53$	
$S_4(23)$	$2^8 \cdot 3^2 \cdot 5 \cdot 11^2 \cdot 23^4 \cdot 53$	
$U_4(23)$	$2^{10} \cdot 3^4 \cdot 5 \cdot 11^2 \cdot 13^2 \cdot 23^6 \cdot 53$	
$A_{53},\dots,A_{58}$	$\pi(G) = \{2,3,5,\dots,53\}$	
$\mathfrak{S}_{59}$	$59 \in \pi(G) \subseteq \{2,3,5,\dots,59\}$	# 3
$L_2(59)$	$2^2 \cdot 3 \cdot 5 \cdot 29 \cdot 59$	
A_{59}, A_{60}	$\pi(G) = \{2,3,5,\dots,59\}$	
$\mathfrak{S}_{61}$	$61 \in \pi(G) \subseteq \{2,3,5,\dots,61\}$	# 27
$L_2(3^5)$	$2^2 \cdot 3^5 \cdot 11^2 \cdot 61$	
$U_5(3)$	$2^{11} \cdot 3^{10} \cdot 5 \cdot 7 \cdot 61$	
$L_2(11^2)$	$2^3 \cdot 3 \cdot 5 \cdot 11^2 \cdot 61$	
$S_4(11)$	$2^6 \cdot 3^2 \cdot 5^2 \cdot 11^4 \cdot 61$	
$L_2(61)$	$2^2 \cdot 3 \cdot 5 \cdot 31 \cdot 61$	
$L_3(13)$	$2^5 \cdot 3^2 \cdot 7 \cdot 13^3 \cdot 61$	
$U_6(3)$	$2^{13} \cdot 3^{15} \cdot 5 \cdot 7^2 \cdot 13 \cdot 61$	
$U_4(11)$	$2^7 \cdot 3^4 \cdot 5^2 \cdot 11^6 \cdot 37 \cdot 61$	
$L_3(47)$	$2^6 \cdot 3 \cdot 23^2 \cdot 37 \cdot 47^3 \cdot 61$	
$L_4(11)$	$2^7 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11^6 \cdot 19 \cdot 61$	
$L_4(13)$	$2^7 \cdot 3^4 \cdot 5 \cdot 7^2 \cdot 13^6 \cdot 17 \cdot 61$	
$O_{10}^-(3)$	$2^{15} \cdot 3^{20} \cdot 5^2 \cdot 7 \cdot 13 \cdot 41 \cdot 61$	
$L_5(9)$	$2^{15} \cdot 3^{20} \cdot 5^2 \cdot 7 \cdot 11^2 \cdot 13 \cdot 41 \cdot 61$	
$S_{10}(3)$	$2^{17} \cdot 3^{25} \cdot 5^2 \cdot 7 \cdot 11^2 \cdot 13 \cdot 41 \cdot 61$	
$O_{11}(3)$	$2^{17} \cdot 3^{25} \cdot 5^2 \cdot 7 \cdot 11^2 \cdot 13 \cdot 41 \cdot 61$	
$O_{12}^+(3)$	$2^{19} \cdot 3^{30} \cdot 5^2 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 41 \cdot 61$	
$L_3(11^2)$	$2^7 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11^6 \cdot 19 \cdot 37 \cdot 61$	
$S_6(11)$	$2^9 \cdot 3^4 \cdot 5^3 \cdot 7 \cdot 11^9 \cdot 19 \cdot 37 \cdot 61$	
$O_7(11)$	$2^9 \cdot 3^4 \cdot 5^3 \cdot 7 \cdot 11^9 \cdot 19 \cdot 37 \cdot 61$	

$O_8^+(11)$	$2^{12}\cdot 3^5\cdot 5^4\cdot 7\cdot 11^{12}\cdot 19\cdot 37\cdot 61^2$	
$L_4(47)$	$2^{11}\cdot 3^2\cdot 5\cdot 13\cdot 17\cdot 23^3\cdot 37\cdot 47^6\cdot 61$	
$A_{61},\ldots,A_{66}$	$\pi(G)=\{2,3,5,\ldots,61\}$	
$\mathfrak{S}_{67}$	$67\in\pi(G)\subseteq\{2,3,5,\ldots,67\}$	# 11
$L_2(67)$	$2^2\cdot 3\cdot 11\cdot 17\cdot 67$	
$L_3(37)$	$2^5\cdot 3^4\cdot 7\cdot 19\cdot 37^3\cdot 67$	
$L_3(29)$	$2^5\cdot 3\cdot 5\cdot 7^2\cdot 13\cdot 29^3\cdot 67$	
$L_3(67)$	$2^4\cdot 3^2\cdot 7^2\cdot 11^2\cdot 17\cdot 31\cdot 67^3$	
Ly	$2^8\cdot 3^7\cdot 5^6\cdot 7\cdot 11\cdot 31\cdot 37\cdot 67$	
$L_2(37^3)$	$2^2\cdot 3^3\cdot 7\cdot 19\cdot 31\cdot 37^3\cdot 43\cdot 67$	
$G_2(37)$	$2^6\cdot 3^5\cdot 7\cdot 19^2\cdot 31\cdot 37^6\cdot 43\cdot 67$	
$A_{67},\ldots,A_{70}$	$\pi(G)=\{2,3,5,\ldots,67\}$	
$\mathfrak{S}_{71}$	$71\in\pi(G)\subseteq\{2,3,5,\ldots,71\}$	# 6
$L_2(71)$	$2^3\cdot 3^2\cdot 5\cdot 7\cdot 71$	
$L_5(5)$	$2^{11}\cdot 3^2\cdot 5^{10}\cdot 11\cdot 13\cdot 31\cdot 71$	
$L_6(5)$	$2^{13}\cdot 3^4\cdot 5^{15}\cdot 7\cdot 11\cdot 13\cdot 31^2\cdot 71$	
M	$2^{46}\cdot 3^{20}\cdot 5^9\cdot 7^6\cdot 11^2\cdot 13^3\cdot 17\cdot 19\cdot 23\cdot 29\cdot 31\cdot 41\cdot 47\cdot 59\cdot 71$	
A_{71},A_{72}	$\pi(G)=\{2,3,5,\ldots,71\}$	
$\mathfrak{S}_{73}$	$73\in\pi(G)\subseteq\{2,3,5,\ldots,73\}$	# 34
$U_3(9)$	$2^5\cdot 3^6\cdot 5^2\cdot 73$	
$L_3(8)$	$2^9\cdot 3^2\cdot 7^2\cdot 73$	
$L_2(73)$	$2^3\cdot 3^2\cdot 37\cdot 73$	
$U_4(9)$	$2^9\cdot 3^{12}\cdot 5^3\cdot 41\cdot 73$	
${}^3D_4(3)$	$2^6\cdot 3^{12}\cdot 7^2\cdot 13^2\cdot 73$	
$L_2(2^9)$	$2^9\cdot 3^3\cdot 7\cdot 19\cdot 73$	
$G_2(8)$	$2^{18}\cdot 3^5\cdot 7^2\cdot 19\cdot 73$	
$L_2(3^6)$	$2^3\cdot 3^6\cdot 5\cdot 7\cdot 13\cdot 73$	
$S_4(27)$	$2^6\cdot 3^{12}\cdot 5\cdot 7^2\cdot 13^2\cdot 73$	
$G_2(9)$	$2^8\cdot 3^{12}\cdot 5^2\cdot 7\cdot 13\cdot 73$	
$L_4(8)$	$2^{18}\cdot 3^4\cdot 5\cdot 7^3\cdot 13\cdot 73$	
$L_3(64)$	$2^{18}\cdot 3^4\cdot 5\cdot 7^2\cdot 13\cdot 19\cdot 73$	
$S_6(8)$	$2^{27}\cdot 3^7\cdot 5\cdot 7^3\cdot 13\cdot 19\cdot 73$	
$O_8^+(8)$	$2^{36}\cdot 3^9\cdot 5^2\cdot 7^4\cdot 13^2\cdot 19\cdot 73$	
$L_3(3^4)$	$2^9\cdot 3^{12}\cdot 5^2\cdot 7\cdot 13\cdot 41\cdot 73$	
$S_6(9)$	$2^{12}\cdot 3^{18}\cdot 5^3\cdot 7\cdot 13\cdot 41\cdot 73$	
$O_7(9)$	$2^{12}\cdot 3^{18}\cdot 5^3\cdot 7\cdot 13\cdot 41\cdot 73$	
$F_4(3)$	$2^{15}\cdot 3^{24}\cdot 5^2\cdot 7^2\cdot 13^2\cdot 41\cdot 73$	
$O_8^+(9)$	$2^{16}\cdot 3^{24}\cdot 5^4\cdot 7\cdot 13\cdot 41^2\cdot 73$	
$L_2(73^2)$	$2^4\cdot 3^2\cdot 5\cdot 13\cdot 37\cdot 41\cdot 73^2$	
$S_4(73)$	$2^8\cdot 3^4\cdot 5\cdot 13\cdot 37^2\cdot 41\cdot 73^4$	
$E_6(2)$	$2^{36}\cdot 3^6\cdot 5^2\cdot 7^3\cdot 13\cdot 17\cdot 31\cdot 73$	
$U_4(27)$	$2^7\cdot 3^{18}\cdot 5\cdot 7^3\cdot 13^2\cdot 19\cdot 37\cdot 73$	
$O_{12}^-(3)$	$2^{18}\cdot 3^{30}\cdot 5^3\cdot 7\cdot 11^2\cdot 13\cdot 41\cdot 61\cdot 73$	
$L_6(9)$	$2^{18}\cdot 3^{30}\cdot 5^3\cdot 7^2\cdot 11^2\cdot 13^2\cdot 41\cdot 61\cdot 73$	
$O_{13}(3)$	$2^{21}\cdot 3^{36}\cdot 5^3\cdot 7^2\cdot 11^2\cdot 13^2\cdot 41\cdot 61\cdot 73$	
$S_{12}(3)$	$2^{21}\cdot 3^{36}\cdot 5^3\cdot 7^2\cdot 11^2\cdot 13^2\cdot 41\cdot 61\cdot 73$	
${}^2E_6(3)$	$2^{19}\cdot 3^{36}\cdot 5^2\cdot 7^3\cdot 13^2\cdot 19\cdot 37\cdot 41\cdot 61\cdot 73$	
$A_{73},\ldots,A_{78}$	$\pi(G)=\{2,3,5,\ldots,73\}$	
$\mathfrak{S}_{79}$	$79\in\pi(G)\subseteq\{2,3,5,\ldots,79\}$	# 14
$L_2(79)$	$2^4\cdot 3\cdot 5\cdot 13\cdot 79$	
$L_3(23)$	$2^5\cdot 3\cdot 7\cdot 11^2\cdot 23^3\cdot 79$	
$L_3(79)$	$2^6\cdot 3^2\cdot 5\cdot 7^2\cdot 13^2\cdot 43\cdot 79^3$	
$L_2(23^3)$	$2^3\cdot 3^2\cdot 7\cdot 11\cdot 13^2\cdot 23^3\cdot 79$	
$G_2(23)$	$2^8\cdot 3^3\cdot 7\cdot 11^2\cdot 13^2\cdot 23^6\cdot 79$	
$L_4(23)$	$2^9\cdot 3^2\cdot 5\cdot 7\cdot 11^3\cdot 23^6\cdot 53\cdot 79$	
$L_3(23^2)$	$2^9\cdot 3^2\cdot 5\cdot 7\cdot 11^2\cdot 13^2\cdot 23^6\cdot 53\cdot 79$	

$S_6(23)$	$2^{12}{\cdot}3^4{\cdot}5{\cdot}7{\cdot}11^3{\cdot}13^2{\cdot}23^9{\cdot}53{\cdot}79$	
$O_7(23)$	$2^{12}{\cdot}3^4{\cdot}5{\cdot}7{\cdot}11^3{\cdot}13^2{\cdot}23^9{\cdot}53{\cdot}79$	
$O_8^+(23)$	$2^{16}{\cdot}3^5{\cdot}5^2{\cdot}7{\cdot}11^4{\cdot}13^2{\cdot}23^{12}{\cdot}53^2{\cdot}79$	
$A_{79},\dots,A_{82}$	$\pi(G)=\{2,3,5,\dots,79\}$	
$\mathfrak{S}_{83}$	$83\in\pi(G)\subseteq\{2,3,5,\dots,83\}$	# 9
$L_2(83)$	$2^2{\cdot}3{\cdot}7{\cdot}41{\cdot}83$	
$L_2(83^2)$	$2^3{\cdot}3{\cdot}5{\cdot}7{\cdot}13{\cdot}41{\cdot}53{\cdot}83^2$	
$S_4(83)$	$2^6{\cdot}3^2{\cdot}5{\cdot}7^2{\cdot}13{\cdot}41^2{\cdot}53{\cdot}83^4$	
$A_{83},\dots,A_{88}$	$\pi(G)=\{2,3,5,\dots,83\}$	
$\mathfrak{S}_{89}$	$89\in\pi(G)\subseteq\{2,3,5,\dots,89\}$	# 9
$L_2(89)$	$2^3{\cdot}3^2{\cdot}5{\cdot}11{\cdot}89$	
$A_{89},\dots,A_{96}$	$\pi(G)=\{2,3,5,\dots,89\}$	
$\mathfrak{S}_{97}$	$97\in\pi(G)\subseteq\{2,3,5,\dots,97\}$	# 6
$L_2(97)$	$2^5{\cdot}3{\cdot}7^2{\cdot}97$	
$L_3(61)$	$2^5{\cdot}3^2{\cdot}5^2{\cdot}13{\cdot}31{\cdot}61^3{\cdot}97$	
$A_{97},\dots,A_{100}$	$\pi(G)=\{2,3,5,\dots,97\}$	

Table 2: **Primes** $p\in\{100,\dots,1000\}$ **with generic** $\mathfrak{S}_p$

107, 131, 167, 197, 223, 227, 311, 317, 347, 359, 379,
383, 389, 397, 419, 461, 479, 503, 541, 569, 587, 617,
643, 647, 691, 761, 797, 827, 839, 887, 967, 977, 983

Table 3: **Non-generic groups** G **with** $100<p\in\pi(G)\subseteq\{2,3,5,\dots,p\}$

p	$\lvert\mathfrak{S}_p\rvert$	G
101	5	$U_3(101), U_5(17)$
103	14	$U_3(47), U_3(103), L_2(47^3), G_2(47), U_4(47), L_3(47^2), S_6(47), O_7(47), O_8^+(47)$
109	12	$U_3(64), {}^3D_4(8), Sz(2^9), {}^2F_4(8), L_2(2^{18}), G_2(64), S_4(2^9)$
113	16	$U_7(4)$
127	24	$L_2(2^7), L_3(19), Sz(2^7), L_2(19^3), G_2(19), L_7(2), L_8(2), L_2(2^{14}), S_4(2^7), L_3(107), L_9(2), O_{14}^+(2), L_{10}(2), L_7(4), S_{14}(2), O_{16}^+(2), L_{11}(2), E_7(2), L_{12}(2)$
137	12	$L_2(37^2), S_4(37), L_4(37), U_4(37), L_3(137), L_3(37^2), S_6(37), O_7(37), O_8^+(37)$
139	14	$U_3(97), U_3(43), U_4(43)$
149	4	$L_3(149)$
151	11	$L_3(32), L_4(32), L_5(8), L_6(8)$
157	17	$U_3(13), L_2(13^3), G_2(13), U_4(13), L_3(13^2), S_6(13), O_7(13), O_8^+(13), L_2(157^2), S_4(157)$
163	7	$U_3(59), L_3(163)$
173	11	$L_2(173^2), S_4(173), U_3(173), U_4(173)$
179	4	$U_3(179)$
181	25	$L_2(19^2), S_4(19), U_3(49), U_4(19), L_4(19), L_3(19^2), O_7(19), S_6(19), O_8^+(19), {}^3D_4(7), L_2(7^6), S_4(7^3), G_2(49), L_3(181)$
191	9	$U_5(7), U_6(7), L_3(191), L_2(191^2), S_4(191), L_4(191)$
193	15	$L_2(3^8), S_4(3^4), L_2(193^2), S_4(193), U_3(109), O_8^-(9), L_4(3^4), S_8(9), O_9(9), O_{10}^+(9)$
199	16	$U_3(107), L_2(107^3), G_2(107)$

211	20	$U_3(197), L_2(211^2), S_4(211), L_3(211), U_5(23), L_4(211), U_6(23)$
229	14	$L_2(107^2), S_4(107), L_3(229), U_4(107), L_4(107), L_3(107^2), S_6(107), O_7(107), O_8^+(107)$
233	11	$L_2(89^2), S_4(89), L_2(233^2), S_4(233)$
239	5	$L_2(239^2), S_4(239)$
241	25	$U_3(16), {}^3D_4(4), L_2(2^{12}), G_2(16), S_4(64), O_8^-(8), L_4(64), S_8(8), U_4(64), O_{10}^+(8), L_3(2^{12}), S_6(64), O_8^+(64), F_4(8)$
251	9	$L_2(251^2), S_4(251)$
257	51	$L_2(2^8), S_4(16), U_4(16), O_8^-(4), L_4(16), S_8(4), L_2(241^2), S_4(241), U_3(257), O_{10}^-(4), L_3(2^8), S_6(16), O_8^+(16), F_4(4), O_{10}^+(4), L_5(16), S_{10}(4), O_{12}^+(4), U_8(4), O_{12}^-(4), L_6(16), S_{12}(4), O_{16}^-(2), L_8(4), S_{16}(2), {}^2E_6(4), O_{18}^-(2), E_6(4), O_{18}^+(2), U_9(4), L_9(4), S_{18}(2), O_{20}^+(2), O_{14}^-(4), O_{14}^+(4), O_{20}^-(2), L_{10}(4), S_{20}(2), U_{10}(4), L_7(16), S_{14}(4), O_{16}^+(4), O_{22}^+(2), E_7(4)$
263	11	$U_3(263), L_3(263), L_2(263^3), G_2(263)$
269	4	$L_3(269)$
271	13	$U_3(29), {}^2G_2(3^5), U_3(3^5), L_2(29^3), G_2(29), U_5(27)$
277	6	$L_3(277)$
281	5	$L_2(53^2), S_4(53)$
283	13	$U_3(239), U_4(239)$
293	19	$U_3(293), L_2(293^2), S_4(293), U_4(293)$
307	16	$L_3(17), L_4(17), L_2(17^3), G_2(17), L_3(17^2), S_6(17), O_7(17), O_8^+(17), L_2(307^2), S_4(307), U_6(17)$
313	17	$L_2(5^4), S_4(25), L_3(313), O_8^-(5), L_4(25), S_8(5), O_9(5), L_2(313^2), S_4(313), O_{10}^+(5), L_4(313), {}^3D_4(29)$
331	32	$L_3(31), U_3(32), L_2(31^3), G_2(31), U_4(32), L_4(31), L_2(2^{15}), G_2(32), U_5(8), U_6(8), L_3(2^{10}), S_6(32), O_8^+(32), L_3(31^2), O_7(31), S_6(31), O_8^+(31), O_{10}^-(8), L_5(64), S_{10}(8), O_{12}^+(8), O_{12}^-(8), L_6(64), S_{12}(8), E_8(2)$
337	18	$L_3(2^7), L_2(337^2), S_4(337), L_4(2^7), L_7(8), L_8(8), O_{14}^+(8)$
349	6	$U_3(227)$
353	10	$L_2(311^2), S_4(311), U_3(353)$
367	10	$L_3(83), L_3(283), L_4(83)$
373	16	$U_3(89), L_2(269^2), S_4(269), U_4(89), L_3(373), U_3(373), L_4(269), L_2(373^3), G_2(373)$
401	11	$L_2(401^2), S_4(401)$
409	14	$L_3(53), L_4(53), {}^3D_4(49)$
421	29	$L_2(29^2), S_4(29), U_4(29), U_3(401), L_4(29), L_2(421^2), S_4(421), L_3(29^2), O_7(29), S_6(29), O_8^+(29), U_3(29^2), U_5(29), U_4(401), U_6(29), L_2(29^6), S_4(29^3), G_2(29^2)$
431	15	$U_3(431), L_2(431^2), S_4(431), L_3(431), U_4(431), L_2(431^3), G_2(431), L_4(431), L_3(431^2), S_6(431), O_7(431), O_8^+(431)$
433	11	$L_2(179^2), S_4(179), U_3(199), U_4(179)$
439	6	$L_3(439)$
443	9	$L_2(443^2), S_4(443)$
449	12	$L_2(67^2), S_4(67), L_4(67)$
457	8	$L_2(109^2), S_4(109), U_4(109)$
463	7	$L_2(463^2), S_4(463)$
467	17	$L_2(467^2), S_4(467), U_3(467), U_4(467)$
487	7	$U_3(233), U_4(233)$
491	10	$U_3(491)$
499	11	$L_3(499), L_3(139), L_3(359), L_2(499^2), S_4(499), L_4(499)$
509	15	$L_2(509^2), S_4(509)$
521	19	$U_5(5), L_2(5^5), U_6(5), O_{10}^-(5), L_3(521), U_3(521), U_7(5), L_5(25), S_{10}(5), O_{11}(5), O_{12}^+(5), L_2(43^4), S_4(43^2), U_8(5), L_2(521^3), G_2(521)$
523	24	$U_3(61), U_3(463), L_2(61^3), G_2(61), U_4(463)$
547	18	$U_3(41), U_7(3), U_4(41), U_8(3), O_{14}^-(3), U_9(3), U_{10}(3)$

557	12	$L_2(439^2), S_4(439), L_2(557^2), S_4(557), L_4(439)$
563	8	$U_3(563)$
571	17	$L_3(109), L_4(109), L_3(461), L_2(109^3), G_2(109), L_3(571), L_3(109^2), S_6(109), O_7(109), O_8^+(109)$
577	13	$L_2(577^2), S_4(577)$
593	11	$L_2(593^2), S_4(593), U_3(593), U_4(593)$
599	5	$L_2(599^2), S_4(599)$
601	27	$U_3(25), {}^3D_4(5), U_4(25), L_2(5^6), S_4(5^3), G_2(25), U_3(577), L_3(5^4), S_6(25), O_7(25), F_4(5), O_8^+(25), L_2(601^2), S_4(601), U_4(577), O_{12}^-(5), L_6(25), O_{13}(5), S_{12}(5), O_{14}^-(5)$
607	16	$U_3(211), U_3(397), U_4(211), L_2(211^3), G_2(211), L_3(211^2), S_6(211), O_7(211), O_8^+(211)$
613	6	$L_3(547)$
619	15	$U_3(367), U_3(619)$
631	25	$L_3(43), L_3(587), L_4(43), L_3(631), L_2(43^3), G_2(43), L_3(43^2), S_6(43), O_7(43), O_8^+(43), O_8^-(43), L_4(43^2), O_9(43), S_8(43)$
641	6	$L_2(487^2), S_4(487), U_3(641)$
653	11	$L_2(149^2), S_4(149), L_3(653), L_4(149)$
659	5	$L_2(659^2), S_4(659)$
661	15	$U_{11}(3), U_{12}(3)$
673	14	$U_3(2^8), {}^3D_4(16), L_2(2^{24}), G_2(2^8), S_4(2^{12}), O_8^-(64), L_4(2^{12}), S_8(64), O_{10}^+(64)$
677	8	$U_3(677)$
683	19	$L_2(2^{11}), U_{11}(2), U_{12}(2), O_{22}^-(2), L_{11}(4), S_{22}(2), O_{24}^+(2), O_{24}^-(2), L_{12}(4), S_{24}(2)$
701	11	$L_2(701^2), S_4(701)$
709	16	$L_3(227), L_2(613^2), S_4(613), L_2(227^3), G_2(227)$
719	10	$U_3(719)$
727	8	$L_3(281)$
733	12	$L_2(353^2), S_4(353), L_3(307), L_4(307), U_4(353)$
739	7	$U_3(419), U_3(739)$
743	11	$L_2(743^2), S_4(743)$
751	9	$U_3(73), U_4(73)$
757	17	$L_3(27), L_4(27), L_2(3^9), G_2(27), L_2(757^2), S_4(757), E_6(3), L_3(3^6), S_6(27), O_7(27), O_8^+(27), U_6(27)$
769	12	$U_3(19^2), {}^3D_4(19), L_2(19^6), S_4(19^3), G_2(19^2), U_3(409), U_3(3^8)$
773	18	$L_2(317^2), S_4(317), U_3(773)$
787	16	$L_3(787), L_3(379), L_2(787^2), S_4(787), L_4(787)$
809	7	$L_2(491^2), S_4(491), L_3(809), U_4(491)$
811	16	$U_3(131), L_2(811^2), S_4(811), L_3(811), L_4(811)$
821	7	$U_3(821), L_3(821), L_2(821^3), G_2(821)$
823	6	$L_3(823)$
829	18	$L_3(5^3), L_4(5^3), L_2(829^2), S_4(829), U_3(829), E_6(5), U_4(829)$
853	7	$L_2(853^2), S_4(853)$
857	7	$L_2(857^2), S_4(857), U_3(857), U_4(857)$
859	8	$U_3(599), U_3(859), U_4(599)$
863	19	$U_3(863), L_2(863^2), S_4(863), U_4(863)$
877	10	$L_2(151^2), S_4(151), U_3(283), L_2(283^3), G_2(283)$
881	4	$U_3(881)$
883	7	$L_3(337), L_4(337)$
907	6	$U_3(523)$
911	14	$L_5(19), L_6(19), U_7(7), L_2(911^2), S_4(911)$
919	23	$U_3(53), U_4(53), L_2(53^3), G_2(53), L_3(53^2), S_6(53), O_7(53), O_8^+(53), L_2(919^2), S_4(919), L_3(919), L_4(919)$
929	10	$U_3(929)$
937	6	$U_3(13^3)$
941	11	$L_2(97^2), S_4(97), U_4(97), U_3(941)$

947	8	$L_3(947)$
953	16	$U_3(953)$
971	8	$L_3(971)$
991	10	$L_3(113), L_3(877), L_3(991)$
997	14	$L_3(997)$

Table 4: **The simple groups G with $\pi(G) \subseteq \{1, 2, \ldots, 1000\}$**

Series	Parameter	Values
Sporadic	all	—
A_n	n	$5, \ldots, 1008$
$L_2(p)$	p	$5, \ldots, 997$
$L_2(p^2)$	p	$2, \ldots, 53, 67, 73, 83, 89, 97, 107, 109, 149, 151, 157, 173, 179, 191, 193, 211, 233, \ldots, 251, 269, 293, \ldots, 317, 337, 353, 401, 421, 431, 439, 443, 463, 467, 487, 491, 499, 509, 557, 577, 593, 599, 601, 613, 659, 701, 743, 757, 787, 811, 829, 853, 857, 863, 911, 919$
$L_2(p^3)$	p	$2, \ldots, 37, 43, 47, 53, 61, 107, 109, 211, 227, 263, 283, 373, 431, 521, 821$
$L_2(p^k)$, $k \geqslant 4$	p^k	$2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}, 2^{11}, 2^{12}, 2^{14}, 2^{15}, 2^{18}, 2^{24}, 3^4, 3^5, 3^6, 3^8, 3^9, 5^4, 5^5, 5^6, 7^6, 19^6, 29^6, 43^4$
$L_3(p)$	p	$2, \ldots, 37, 43, 47, 53, 61, 67, 79, 83, 107, 109, 113, 137, 139, 149, 163, 181, 191, 211, 227, 229, 263, 269, 277, 281, 283, 307, 313, 337, 359, 373, 379, 431, 439, 461, 499, 521, 547, 571, 587, 631, 653, 787, 809, \ldots, 823, 877, 919, 947, 971, 991, 997$
$L_3(p^2)$	p	$2, \ldots, 37, 43, 47, 53, 107, 109, 211, 431$
$L_3(p^k)$, $k \geqslant 3$	p^k	$2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^{10}, 2^{12}, 3^3, 3^4, 3^6, 5^3, 5^4$
$L_4(p)$	p	$2, \ldots, 37, 43, 47, 53, 67, 83, 107, 109, 149, 191, 211, 269, 307, 313, 337, 431, 439, 499, 787, 811, 919$
$L_4(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^{12}, 3^2, 3^3, 3^4, 5^2, 5^3, 43^2$
$L_n(q)$, $n = 5, 6$	q	$2, 2^2, 2^3, 2^4, 2^6, 3, 3^2, 5, 5^2, 19$
$L_n(q)$, $n \geqslant 7$	n, q	$n = 7, q = 2, 2^2, 2^3, 2^4$; $n = 8, q = 2, 2^2, 2^3$; $n = 9, q = 2, 2^2$; $n = 10, q = 2, 2^2$; $n = 11, q = 2, 2^2$; $n = 12, q = 2, 2^2$
$U_3(p)$	p	$3, \ldots, 61, 73, 89, \ldots, 109, 131, 173, 179, 197, 199, 211, 227, 233, 239, 257, 263, 283, 293, 353, 367, 373, 397, \ldots, 419, 431, 463, 467, 491, 521, 523, 563, 577, 593, 599, 619, 641, 677, 719, 739, 773, 821, 829, 857, 859, 863, 881, 929, 941, 953$
$U_3(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 2^8, 3^2, 3^3, 3^5, 3^8, 5^2, 7^2, 13^3, 19^2, 29^2$
$U_4(p)$	p	$2, \ldots, 53, 73, 89, 97, 107, 109, 173, 179, 211, 233, 239, 293, 353, 401, 431, 463, 467, 491, 577, 593, 599, 829, 857, 863$
$U_4(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 3^2, 3^3, 5^2$
$U_n(q)$, $n = 5, 6$	q	$2, 2^2, 2^3, 3, 3^3, 5, 7, 17, 23, 29$
$U_n(q)$, $n \geqslant 7$	n, q	$n = 7, q = 2, 2^2, 3, 5, 7$; $n = 8, q = 2, 2^2, 3, 5$; $n = 9, q = 2, 2^2, 3$; $n = 10, q = 2, 2^2, 3$; $n = 11, q = 2, 3$; $n = 12, q = 2, 3$
$S_4(p)$	p	$3, \ldots, 53, 67, 73, 83, 89, 97, 107, 109, 149, 151, 157, 173, 179, 191, 193, 211, 233, \ldots, 251, 269, 293, \ldots, 317, 337, 353, 401, 421, 431, 439, 443, 463, 467, 487, 491, 499, 509, 557, 577, 593, 599, 601, 613, 659, 701, 743, 757, 787, 811, 829, 853, 857, 863, 911, 919$
$S_4(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^9, 2^{12}, 3^2, 3^3, 3^4, 5^2, 5^3, 7^3, 19^3, 29^3, 43^2$
$S_6(p)$	p	$2, \ldots, 37, 43, 47, 53, 107, 109, 211, 431$
$S_6(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 3^2, 3^3, 5^2$
$S_8(q)$	q	$2, 2^2, 2^3, 2^6, 3, 3^2, 5, 43$
$S_n(q)$, $n \geqslant 10$	n, q	$n = 10, q = 2, 2^2, 2^3, 3, 5$; $n = 12, q = 2, 2^2, 2^3, 3, 5$; $n = 14, q = 2, 2^2$; $n = 16, q = 2$; $n = 18, q = 2$; $n = 20, q = 2$; $n = 22, q = 2$; $n = 24, q = 2$
$O_7(p)$	p	$3, \ldots, 37, 43, 47, 53, 107, 109, 211, 431$

$O_7(p^k)$, $k \geqslant 2$	p^k	$3^2, 3^3, 5^2$
$O_n(q)$, $n \geqslant 9$	n, q	$n = 9, q = 3, 3^2, 5, 43$; $n = 11, q = 3, 5$; $n = 13, q = 3, 5$
$O_8^+(p)$	p	$2, \dots, 37, 43, 47, 53, 107, 109, 211, 431$
$O_8^+(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 3^2, 3^3, 5^2$
$O_n^+(q)$, $n \geqslant 10$	n, q	$n = 10, q = 2, 2^2, 2^3, 2^6, 3, 3^2, 5$; $n = 12, q = 2, 2^2, 2^3, 3, 5$; $n = 14, q = 2, 2^2, 2^3$; $n = 16, q = 2, 2^2$; $n = 18, q = 2$; $n = 20, q = 2$; $n = 22, q = 2$; $n = 24, q = 2$
$O_8^-(q)$	q	$2, 2^2, 2^3, 2^6, 3, 3^2, 5, 43$
$O_n^-(q)$, $n \geqslant 10$	n, q	$n = 10, q = 2, 2^2, 2^3, 3, 5$; $n = 12, q = 2, 2^2, 2^3, 3, 5$; $n = 14, q = 2, 2^2, 3, 5$; $n = 16, q = 2$; $n = 18, q = 2$; $n = 20, q = 2$; $n = 22, q = 2$; $n = 24, q = 2$
$G_2(p)$	p	$3, \dots, 37, 43, 47, 53, 61, 107, 109, 211, 227, 263, 283, 373, 431, 521, 821$
$G_2(p^k)$, $k \geqslant 2$	p^k	$2^2, 2^3, 2^4, 2^5, 2^6, 2^8, 3^2, 3^3, 5^2, 7^2, 19^2, 29^2$
$F_4(q)$	q	$2, 2^2, 2^3, 3, 5$
$E_6(q)$	q	$2, 2^2, 3, 5$
$E_7(q)$	q	$2, 2^2$
$E_8(q)$	q	2
${}^3D_4(q)$	q	$2, 2^2, 2^3, 2^4, 3, 5, 7, 7^2, 19, 29$
${}^2E_6(q)$	q	$2, 2^2, 3$
$Sz(2^k)$	k	$3, 5, 7, 9$
${}^2G_2(3^k)$	k	$3, 5$
${}^2F_4(2^k)$, $k \geqslant 3$	k	3
${}^2F_4(2)'$	—	—

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