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TRANSPARENT ORE EXTENSIONS OVER WEAK σ-RIGID RINGS

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Abstract. Recall that a Noetherian ring R is said to be a *Transparent ring* if there exist irreducible ideals I_j, $1 \leq j \leq n$ such that $\cap_{j=1}^{n} I_j = 0$ and each R/I_j has a right Artinian quotient ring. Let R be a commutative Noetherian ring, which is also an algebra over $\mathbb{Q}$ (the field of rational numbers); σ an automorphism of R and δ a σ-derivation of R. Also let R be a weak σ-rigid ring (i.e. $a\sigma(a) \in N(R)$ if and only if $a \in N(R)$, where $N(R)$ the set of nilpotent elements of R). Then we prove that $R[x;\sigma,\delta]$ is a *Transparent ring*.

Keywords: Automorphism, σ-derivation, weak σ-rigid ring, quotient ring, transparent ring.

1. Introduction

A ring R always means an associative ring with identity $1 \neq 0$. The field of complex numbers, the field of rational numbers and the set of positive integers are denoted by $\mathbb{C}$, $\mathbb{Q}$ and $\mathbb{N}$ respectively unless otherwise stated. The set of prime ideals of R is denoted by $Spec(R)$. The set of minimal prime ideals of R is denoted by $Min.Spec(R)$. Prime radical and the set of nilpotent elements of R are denoted by $P(R)$ and $N(R)$ respectively. The set of associated prime ideals of R (viewed as a right R-module over itself) is denoted by $Ass(R_R)$. The notion of the quotient ring of a ring appears in chapter (9) of Goodearl and Warfield [8].

Transparent Ore extensions over weak σ-rigid rings.

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This article concerns the study of Ore extensions. Now let R be a ring, σ an automorphism of R and δ a σ-derivation of R. Recall that δ is an additive map $\delta : R \to R$ such that $\delta(ab) = \delta(a)\sigma(b) + a\delta(b)$, for all $a, b \in R$.

Example 1. Let σ be an automorphism of a ring R and $\delta : R \to R$ any map. Let $\phi : R \to M_2(R)$ be defined by
$\phi(r) = \begin{pmatrix} \sigma(r) & 0 \\ \delta(r) & r \end{pmatrix}$, for all $r \in R$. Then ϕ is a ring homomorphism if and only if δ is a σ-derivation of R.

Recall that the Ore extension $R[x;\sigma,\delta]$ is the usual polynomial ring with coefficients in R, in which multiplication is subject to the relation $ax = x\sigma(a)+\delta(a)$ for all $a \in R$. We take any $f(x) \in R[x;\sigma,\delta]$ to be of the form $f(x) = \sum_{i=0}^{n} x^i a_i$ as follows in McConnell and Robson [13]. We denote $R[x;\sigma,\delta]$ by $O(R)$. If I is an ideal of R such that $\sigma(I) = I$ and $\delta(I) \subseteq I$, then $O(I)$ denotes $I[x;\sigma,\delta]$, which is an ideal of $O(R)$.

In case $\delta = 0$, we denote $R[x;\sigma]$ by $S(R)$. If J is an ideal of R such that $\sigma(J) = J$, then $S(J)$ denotes $J[x;\sigma]$, which is an ideal of $S(R)$. In case σ is the identity map, δ is just called a derivation of R, and we denote $R[x;\delta]$ by $D(R)$. If K is an ideal of R such that $\delta(K) \subseteq K$, then $D(K)$ denotes $K[x;\delta]$, which is an ideal of $D(R)$. Ore-extensions including skew-polynomial rings $S(R)$ and differential operator rings $D(R)$ have been of interest to many authors. For example [2–8, 12, 13].

In Blair and Small [7], it is shown that if R is embeddable in a right Artinian ring and if characteristic of R is zero, then the differential operator ring $R[x;\delta]$ embeds in a right Artinian ring, where δ is a derivation of R. It is also shown in [7] that if R is a commutative Noetherian ring and σ is an automorphism of R, then the skew-polynomial ring $R[x;\sigma]$ embeds in an Artinian ring.

In Theorem (2.11) of Bhat [4], it is proved that if R is a ring which is an order in a right Artinian ring. Then $O(R)$ is an order in a right Artinian ring.

In this paper the above mentioned properties have been studied with emphasis on primary decomposition of the Ore extension $O(R)$, where R is a commutative Noetherian $\mathbb{Q}$-algebra. We actually discuss a stronger type of primary decomposition (known as Transparency) of a right Noetherian ring.

We recall the following.

Definition 1. A ring R is said to be an *irreducible ring* if the intersection of any two non-zero ideals of R is non-zero. An ideal I of R is called *irreducible* if $I = J \cap K$ implies that either $J = I$ or $K = I$. Note that if I is an irreducible ideal of R, then R/I is an irreducible ring.

Proposition 1. *Let R be a Noetherian ring. Then there exist irreducible ideals I_j, $1 \leq j \leq n$ of R such that $\cap_{j=1}^{n} I_j = 0$.*

Proof: The proof is obvious and we leave the details to the reader. □

Definition 2. (Definition 1.2 of [6]) A Noetherian ring R is said to be a *Transparent ring* if there exist irreducible ideals I_j, $1 \leq j \leq n$ such that $\cap_{j=1}^{n} I_j = 0$ and each R/I_j has a right Artinian quotient ring.

It can be easily seen that a Noetherian integral domain is a *Transparent ring*, a commutative Noetherian ring is a *Transparent ring* and so is a Noetherian ring having an Artinian quotient ring. A fully bounded Noetherian ring is also a *Transparent ring*.

Remark 1. This type of decomposition was actually introduced by the author in [2]. Such a ring was called a *decomposable ring*, but in order to distinguish between one more definition of a *decomposable ring* given below and pointed out by the referee of one of the papers of the author, we now call such a ring a *Transparent ring.*

Definition 3. Let R be a ring. An R-module M is said to be decomposable if $M \simeq M_1 \oplus M_2$ where M_1 and M_2 are non zero R-modules. A ring R is called a *decomposable ring* if it is a direct sum of two rings (Hazewinkel and Kirichenko [9]).

The following results have been proved in Bhat [2] towards the transparency of Ore extensions.

Theorem 1. *Let R be a commutative Noetherian ring and σ an automorphism of R. Then $S(R)$ is decomposable.*

Theorem 2. *Let R be a commutative Noetherian ring which is also an algebra over $\mathbb{Q}$ and δ a derivation of R. Then $D(R)$ is decomposable.*

The following result has been proved in Bhat [6].

Theorem 3. *Let R be a commutative Noetherian $\mathbb{Q}$-algebra, σ an automorphism of R. Then there exists an integer $m \geq 1$ such that the skew-polynomial ring $R[x;\alpha,\delta]$ is a transparent ring, where $\sigma^m = \alpha$ and δ is an α-derivation of R such that $\alpha(\delta(a)) = \delta(\alpha(a))$, for all $a \in R$.*

In this paper we investigate the *Transparent ring* property for $O(R) = R[x;\sigma,\delta]$. Before we state the main result, we have the following.

1.1. **Weak σ-rigid rings.** Let R be a ring and σ be an endomorphism of R. Recall that in [6], σ is called a rigid endomorphism if $a\sigma(a) = 0$ implies a $= 0$ for $a \in R$, and R is called a σ-rigid ring.

Example 2. Let $R = \mathbb{C}$, and $\sigma : R \to R$ be the map defined by $\sigma(a + ib) = a - ib$, a, $b \in \mathbb{R}$. Then it can be seen that R is a σ-rigid ring.

Definition 4. (Ouyang [14]) Let R be a ring and σ an endomorphism of R such that $a\sigma(a) \in N(R)$ if and only if $a \in N(R)$ for $a \in R$. Then R is called a *weak σ-rigid ring.*

Example 3. (Example (2.1) of Ouyang [14]) Let σ be an endomorphism of a ring R such that R is a σ-rigid ring. Let

$$A = \left\{ \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix} \mid \text{a, b, c, d} \in R \right\}$$

be a subring of $T_3(R)$, the ring of upper triangular matrices over R. Now σ can be extended to an endomorphism $\overline{\sigma}$ of A by $\overline{\sigma}((a_{ij})) = (\sigma(a_{ij}))$. Then it can be seen that A is a weak $\overline{\sigma}$-rigid ring.

We now state the main results of this paper in the form of the following Theorems which will be proved in the next section.

Theorem 4. *Let R be a commutative Noetherian weak σ-rigid ring, which is also an algebra over $\mathbb{Q}$ (σ an automorphism of R). Let δ be a σ-derivation of R. Then $R[x;\sigma,\delta]$ is a Transparent ring.*

Theorem 5. *Let R be a semiprime Noetherian weak σ-rigid ring, which is also an algebra over $\mathbb{Q}$, (σ an automorphism of R). Let δ be a σ-derivation of R. Then R is a Transparent ring and $R[x;\sigma,\delta]$ is also a Transparent ring.*

2. Proof of the main Theorems

Towards the proof of the above Theorems, we require the following.

Recall that an ideal I of a ring R is said to be completely semiprime if $a^2 \in I$ implies that $a \in I$.

We now have the following Theorem.

Theorem 6. *Let R be a Noetherian ring such that $N(R)$ is an ideal of R. Let σ be an automorphism of R. Then R is a weak σ-rigid ring implies that $N(R)$ is completely semiprime.*

Proof: First of all we show that $\sigma(N(R)) = N(R)$. We have $\sigma(N(R)) \subseteq N(R)$ as $\sigma(N(R))$ is a nilpotent ideal of R. Now for any $n \in N(R)$, there exists $a \in R$ such that $n = \sigma(a)$. So $I = \sigma^{-1}(N(R)) = \{a \in R \text{ such that } \sigma(a) = n \in N(R)\}$ is an ideal of R. Now I is nilpotent, therefore $I \subseteq N(R)$, which implies that $N(R) \subseteq \sigma(N(R))$. Hence $\sigma(N(R)) = N(R)$.

Now let R be a weak σ-rigid ring. We will show that $N(R)$ is completely semiprime. Let $a \in R$ be such that $a^2 \in N(R)$. Then $a\sigma(a)\sigma(a\sigma(a)) = a\sigma(a)\sigma(a)\sigma^2(a) \in \sigma(N(R)) = N(R)$. Therefore $a\sigma(a) \in N(R)$ and hence $a \in N(R)$. So $N(R)$ is completely semiprime. □

Corollary 1. *Let R be a commutative Noetherian ring. Let σ be an automorphism of R. Then R is a weak σ-rigid ring implies that $N(R)$ is completely semiprime.*

2.1. 2-primal rings.

Recall that a ring R is 2-primal if and only if $N(R) = P(R)$ if and only if the prime radical is a completely semiprime ideal. We note that a reduced is 2-primal and a commutative Noetherian ring is also 2-primal. Minimal prime ideals of 2-primal rings have been discussed by Kim and Kwak in [10]. 2-primal near rings have been discussed by Argac and Groenewald in [1].

2-primal rings have been studied in recent years and are being treated by authors for different structures. In [12], Greg Marks discusses the 2-primal property of $R[x;\sigma,\delta]$, where R is a local ring, σ an automorphism of R and δ a σ-derivation of R. In Greg Marks [12], it has been shown that for a local ring R with a nilpotent maximal ideal, the Ore extension $R[x;\sigma,\delta]$ will or will not be 2-primal depending on the δ-stability of the maximal ideal of R. In the case where $R[x;\sigma,\delta]$ is 2-primal, it will satisfy an even stronger condition; in the case where $R[x;\sigma,\delta]$ is not 2-primal, it will fail to satisfy an even weaker condition.

Proposition 2. *Let R be a 2-primal right Noetherian ring which is also an algebra over $\mathbb{Q}$. Let σ be an automorphism of R such that R is a weak σ-rigid ring and δ a σ-derivation of R. Then $\sigma(U) = U$ and $\delta(U) \subseteq U$ for all $U \in Min.Spec(R)$.*

Proof: Let R be 2-primal weak σ-rigid ring. Then $N(R) = P(R)$, i.e. $P(R)$ is completely semiprime.

We next show that $\sigma(U) = U$ for all $U \in Min.Spec(R)$. Let $U = U_1$ be a minimal prime ideal of R. Now Theorem (2.4) of Goodearl and Warfield [8] implies that $Min.Spec(R)$ is finite. Let $U_2, U_3, ..., U_n$ be the other minimal primes of R. Suppose that $\sigma(U) \neq U$. Then $\sigma(U)$ is also a minimal prime ideal of R. Renumber so that $\sigma(U) = U_n$. Let $a \in \cap_{i=1}^{n-1} U_i$. Then $\sigma(a) \in U_n$, and so $a\sigma(a) \in \cap_{i=1}^{n} U_i = P(R)$. Now $P(R)$ is completely semiprime implies that $a \in P(R)$, and thus $\cap_{i=1}^{n-1} U_i \subseteq U_n$, which implies that $U_i \subseteq U_n$ for some $i \neq n$, which is impossible. Hence $\sigma(U) = U$.

Let now $T = \{a \in U \mid \text{such that } \delta^k(a) \in U \text{ for all integers } k \geq 1\}$. First of all, we will show that T is an ideal of R. Let $a, b \in T$. Then $\delta^k(a) \in U$ and $\delta^k(b) \in U$ for all integers $k \geq 1\}$. Now $\delta^k(a-b) = \delta^k(a) - \delta^k(b) \in U$ for all $k \geq 1$. Therefore $a - b \in T$. Therefore T is a δ-invariant ideal of R.

We will now show that $T \in Spec(R)$. Suppose $T \notin Spec(R)$. Let $a \notin T$, $b \notin T$ be such that $aRb \subseteq T$. Let t, s be least such that $\delta^t(a) \notin U$ and $\delta^s(b) \notin U$, i.e. $\delta^m(a) \in U$ and $\delta^k(b) \in U$ for $m < t$ and $k < s$.

Now there exists $c \in R$ such that $\delta^t(a)c\sigma^t(\delta^s(b)) \notin U$. Let $d = \sigma^{-t}(c)$. Now $\delta^{t+s}(adb) \in U$ as $aRb \subseteq T$. This implies on simplification that $\delta^t(a)\sigma^t(d)\sigma^t(\delta^s(b)) + u \in U$, where u is sum of terms involving $\delta^l(a)$ or $\delta^m(b)$, where $l < t$ and $m < s$. Therefore by assumption $u \in U$ which implies that $\delta^t(a)\sigma^t(d)\sigma^t(\delta^s(b)) \in U$. This is a contradiction. Therefore, our supposition must be wrong. Hence $T \in Spec(R)$. Now $T \subseteq U$, so $T = U$ as $U \in Min.Spec(R)$. Hence $\delta(U) \subseteq U$. □

Lemma 1. *Let R be a right Noetherian ring which is also an algebra over* $\mathbb{Q}$. *Let* σ *be an automorphism of R such that R is a weak* σ*-rigid ring and* δ *a* σ*-derivation of R. Then*

(1) *If U is a minimal prime ideal of R, then O(U) is a minimal prime ideal of of O(R) and* $O(U) \cap R = U$.
(2) *If P is a minimal prime ideal of O(R), then* $P \cap R$ *is a minimal prime ideal of R.*

Proof: (1) Let U be a minimal prime ideal of R. Then by Proposition (2) $\sigma(U) = U$ and $\delta(U) \subseteq U$. Now on the same lines as in Theorem (2.22) of Goodearl and Warfield [8] we have $O(U) \in Spec(O(R))$. Suppose $L \subset O(U)$ be a minimal prime ideal of $O(R)$. Then $L \cap R \subset U$ is a prime ideal of R, a contradiction. Therefore $O(U) \in Min.Spec(O(R))$. Now it is easy to see that $O(U) \cap R = U$.

(2) We note that $x \notin P$ for any prime ideal P of $O(R)$ as it is not a zero divisor. Now the proof follows on the same lines as in Theorem (2.22) of Goodearl and Warfield [8] using Lemma (2.1) and Lemma (2.2) of Bhat [3] and Proposition (2). □

Theorem 7. *(Hilbert Basis Theorem) Let R be a right/left Noetherian ring. Let* σ *and* δ *be as usual. Then the ore extension* $O(R) = R[x;\sigma,\delta]$ *is right/left Noetherian.*

Proof: See Theorem (2.6) of Goodearl and Warfield [8]. □

Proposition 3. *Let R be a Noetherian ring having an Artinian quotient ring. Then R is a Transparent ring.*

Proof: See Lemma (2.8) of Bhat [6]. □

Definition 5. Let P be a prime ideal of a commutative ring R. Then the symbolic power of P for any $n \in \mathbb{N}$ is denoted by $P^{(n)}$ and is defined as

$$P^{(n)} = \{a \in R \; such\,that\,there\,exists\,some\; d \in R, d \notin P \; such\,that\; da \in P^n\}.$$

Also if I is an ideal of R, define as usual

$$\sqrt{I} = \{a \in R \; such\,that\; a^n \in I \; for\,some\; n \in \mathbb{N}\}.$$

Lemma 2. *Let R be a commutative Noetherian ring, and σ an automorphism of R. If P is a prime ideal of R such that $\sigma(P) = P$, then $\sigma(P^{(n)}) = P^{(n)}$ for all integers $n \geq 1$.*

Proof: See Lemma (2.10) of Bhat [6]. □

Lemma 3. *(Lemma (2.11) of Bhat [6]) Let R be a commutative Noetherian ring; σ and δ as usual. Let P be a prime ideal of R such that $\sigma(P) = P$ and $\delta(P) \subseteq P$. Then $\delta(P^{(k)}) \subseteq P^{(k)}$ for all integers $k \geq 1$.*

Proof: Let $a \in P^{(k)}$. Then there exists $d \notin P$ such that $da \in P^k$.
Let $da = p_1p_2...p_k$, $p_i \in P$.
Now $\delta(da) = \delta(p_1p_2...p_{k-1})\sigma(p_k) + p_1p_2...p_{k-1}\delta(p_k)$
$= \delta(p_1p_2...p_{k-2})\sigma(p_{k-1})\sigma(p_k) + p_1p_2...p_{k-2}\delta(p_{k-1})\sigma(p_k) + p_1p_2...p_{k-1}\delta(p_k)$
.
.
.
$= \delta(p_1)\sigma(p_2)...\sigma(p_k) + ... + p_1p_2...p_{k-2}\delta(p_{k-1})\sigma(p_k) + p_1p_2...p_{k-1}\delta(p_k)$
$\in P^k$ as $\sigma(P) = P$ and $\delta(P) \subseteq P$; i.e. $\sigma(d)\delta(a) + \delta(d)a \in P^k$. Now $a \in P^{(k)}$, and, therefore $\sigma(d)\delta(a) \in P^{(k)}$, which implies that there exists $d_1 \notin P$ such that $d_1\sigma(d)\delta(a) \in P^k$. Now $d_1\sigma(d) \notin P$ implies that $\delta(a) \in P^{(k)}$. □

Now we are ready to prove two main theorems of this work.

Theorem 5. *Let R be a commutative Noetherian weak σ-rigid ring, which is also an algebra over $\mathbb{Q}$, (σ an automorphism of R). Let δ be a σ-derivation of R. Then $O(R) = R[x;\sigma,\delta]$ is a Transparent ring.*

Proof: R is a commutative Noetherian $\mathbb{Q}$-algebra, therefore, the ideal (0) has a reduced primary decomposition. Let I_j, $1 \leq j \leq n$ be irreducible ideals of R such that $(0) = \cap_{j=1}^n I_j$. For this see Theorem (4) of Zariski and Samuel [15]. Let $\sqrt{I_j} = P_j$, where P_j is a prime ideal belonging to I_j. Now $P_j \in Ass(R_R)$, $1 \leq j \leq n$ by first uniqueness Theorem. Now by Theorem (23) of Zariski and Samuel [15] there exists a positive integer k such that $P_j^{(k)} \subseteq I_j$, $1 \leq j \leq n$. Therefore we have $\cap_{j=1}^n P_j^{(k)} = 0$. Now each P_j contains a minimal prime ideal U_j by Proposition (2.3) of Goodearl and Warfield [8], therefore $\cap_{j=1}^n U_j^{(k)} = 0$. Now R is commutative implies that R is 2-primal, and therefore, Proposition (2) implies that $\sigma(U_j) = U_j$ and $\delta(U_j) \subseteq U_j$, for all j, $1 \leq j \leq n$. Now Lemma (2) implies that $\sigma(U_j)^{(k)} = U_j^{(k)}$ and Lemma (3) implies that $\delta(U_j)^{(k)} \subseteq U_j^{(k)}$, for all j, $1 \leq j \leq n$ and for all $k \geq 1$. Therefore $O(U_j^{(k)})$ is an ideal of $O(R)$ and $\cap_{j=1}^n O(U_j^{(k)}) = 0$.

Now $R/U_j^{(k)}$ has an Artinian quotient ring, as it has no embedded primes, therefore $O(R)/O(U_j^{(k)})$ has also an Artinian quotient ring by Theorem (2.11) of Bhat [4]. Hence $O(R) = R[x;\sigma,\delta]$ is *Transparent ring.* □

Corollary 2. *Let R be a commutative Noetherian weak σ-rigid ring (σ an automorphism of R). Then $S(R) = R[x;\sigma]$ is a Transparent ring.*

Corollary 3. *Let R be a commutative Noetherian $\mathbb{Q}$-algebra and δ be a derivation of R. Then $D(R) = R[x;\delta]$ is a Transparent ring.*

We now prove the following, when R is not necessarily commutative.

Theorem 6. *Let R be a semiprime Noetherian weak σ-rigid ring, which is also an algebra over $\mathbb{Q}$, (σ an automorphism of R). Let δ be a σ-derivation of R. Then R is a Transparent ring and $R[x;\sigma,\delta]$ is also a Transparent ring.*

Proof: R is Noetherian, therefore, Theorem (2.4) of Goodearl and Warfield [8] implies that $Min.Spec(R)$ is finite. Also R is semiprime implies that $\cap_{P\in Min.Spec(R)}P = 0$. Now Proposition (2) implies that $\sigma(P) = P$ and $\delta(P) \subseteq P$, for all $P \in Min.Spec(R)$. Therefore $O(P) = P[x;\sigma,\delta]$ is an ideal of $O(R)$ and $\cap_{P\in Min.Spec(R)}O(P) = 0$. In fact $O(P)$ is a minimal prime ideal of $O(R)$ by Lemma (1). Now R/P has a right Artinian quotient ring by Theorem (5.12) of [8], and $\cap_{P\in Min.Spec(R)}P = 0$ implies that R is a *Transparent ring*. Now Theorem (2.11) of [4] implies that $O(R)/O(P)$ has a right Artinian quotient ring, and hence $\cap_{P\in Min.Spec(R)}O(P) = 0$ implies that $O(R) = R[x;\sigma,\delta]$ is a *Transparent ring*. $\square$

Question 1. Let R be a commutative Noetherian ring, which is also an algebra over $\mathbb{Q}$, (σ an automorphism of R) and δ a σ-derivation of R. Is $O(R) = R[x;\sigma,\delta]$ is a *Transparent ring*?

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