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ALEXANDROV OF ANCIENT HELLAS

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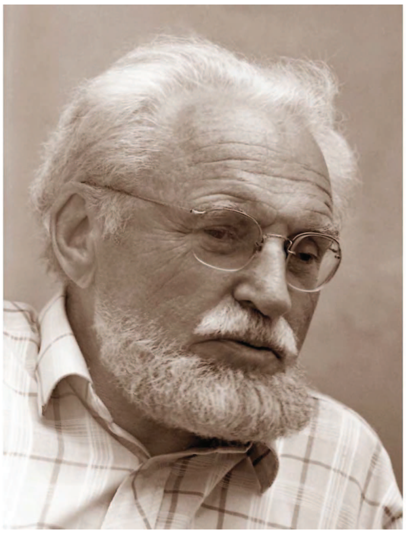
Abstract. This is a short overview of the life and contribution of Aleksandr Danilovich Alexandrov (1912–1999). Most attention is paid to his general outlook and ethical principles.



The photo by V. T. Novikov.

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Life's Signposts. Aleksandr Danilovich Alexandrov was born in the Volyn village of the Ryazan province on August 4, 1912. His parents were high school teachers. He entered the Physics Faculty of Leningrad State University in 1929 and graduated in 1933. His supervisors were Boris Delauney (1890–1980), a prominent geometer and algebraist, and Vladimir Fok (1898–1974), one of the outstanding theoretical physicists of the last century. The first articles by Alexandrov dealt with some problems of theoretical physics and mathematics. But geometry soon became his main speciality.

Alexandrov defended his PhD thesis in 1935 and his second doctorate thesis in 1937. He was elected to a vacancy of corresponding member of the Academy of Sciences of the USSR in 1946 and was promoted to full membership in 1964.

From 1952 to 1964 Alexandrov was Rector of Leningrad State University. These years he actively and effectively supported the struggle of biologists with lysenkoism. Genetics had been in the syllabus of LSU in the 1950s whereas this happened in the other domestic universities only in 1965. The name of Rector Alexandrov is connected with the uprise of the new areas of science such as sociology and mathematical economics which he backed up in the grim years. Alexandrov was greatly respected by established scholars as well as academic youth. "He led the University by moral authority rather than the force of direct order," so wrote Vladimir Smirnov (1887–1974) in the letter of commendation on the occasion of Alexandrov's retirement from the position of Rector.

In 1964 Mikhail Lavrentyev (1900–1980) invited Alexandrov to join the Siberian Division of the Academy of Sciences of the USSR. Alexandrov moved with his family to Novosibirsk where he found many faithful friends and students. By 1986 he headed a department of the Institute of Mathematics (now, the Sobolev Institute), lectured in Novosibirsk State University, and wrote new versions of geometry textbooks at the secondary school level. Alexandrov opened his soul and heart to Siberia, but was infected with tick-borne encephalitis which undermined his health seriously. From April of 1986 up to his death on July 27,1999, Alexandrov was on the staff of St. Petersburg Department of the Steklov Mathematical Institute.

Contribution to Science. Alexandrov's life business was geometry. The works of Alexandrov made tremendous progress in the theory of mixed volumes of convex figures. He proved some fundamental theorems on convex polyhedra that are celebrated alongside the theorems of Euler and Minkowski. While discovering a solution of the Weyl problem, Alexandrov suggested a new synthetic method for proving the theorems of existence. The results of this research ranked the name of Alexandrov alongside the names of Euclid and Cauchy.

Another outstanding contribution of Alexandrov to science is the creation of the intrinsic geometry of irregular surfaces. He suggested his amazingly visual and powerful method of cutting and gluing. This method enabled him to solve many extremal problems of the theory of manifolds of bounded curvature.

Alexandrov developed the theory of metric spaces with one-sided constraints on curvature. This gave rise to the class of metric spaces generalizing the Riemann spaces in the sense that these spaces are furnished with some curvature, the basic concept of Riemannian geometry. The research of Alexandrov into the theory of manifolds with bounded curvature prolongates and continues the traditions of Gauss, Lobachevsky, Poincaré, and Cartan.

The Mathematics Subject Classification, produced jointly by the editorial staffs of Mathematical Reviews and Zentralblatt für Mathematik in 2010, has Section 53C45 "Global surface theory (convex surfaces à la A. D. Aleksandrov)". None of the other Russian geometers, Lobachevsky inclusively, has this type of acknowledgement. Alexandrov became the fist and foremost Russian geometer of the twentieth century.

Sources of Geometry. It is impossible to grasp Alexandrov's outlook without turning to the roots of his cherished science. He wrote in 1981 that "the pathos of contemporary mathematics is the return to Ancient Hellas." His favorite slogan was "Retreat to Euclid!"

Geometry is part of the culture of the ancient world. The traces of any epoch transpire in its most abstract conceptions. It is impossible to grasp the elementary basics of nanotechnology of quantum logic out of the the modern cultural tradition. The hints of time are reflected in evolution of an arbitrary scientific system. Geometry was invented to meet various human needs. Its mystic, explorative, and economic sources coexisted in the common cultural environment of the man of the pre-Bible times. The strongest quest of geometry stemmed from the cadastral surveying aimed at regular taxation. The famous harpedonaptae of Egypt were tax agents who used ropes for measuring the tracts of land. The tricks and techniques of harpedonaptae were used in construction. Pyramids were erected long before the abstract definition of the geometrical form of a pyramid.

Bewildering is the history of the abstract geometric concepts of point, monad, figure, and solid which came from the remote ages. We are rarely aware of the fact the secondary school arithmetic and geometry are the finest gems of the intellectual legacy of our forefathers.

There is no literate who fails to recognize a triangle. However, just a few know an appropriate formal definition. This is not by chance at all, since the definition of triangle is absent in the Elements. Euclid spoke about three-lateral figures, emphasizing that "a figure is that which is contained by any boundary or boundaries." Clearly, his definitions remind us of the technology of cadastral surveying of his times. It is worth observing that the institution of property is much older than the art and science of geometry. To measure a tract of land from outside is legitimate whereas trespassing the borders is forbidden. The ancient rope stretchers had similar restrictions for measuring the constructions like pyramids. Clearly, the surveyors of the Kheops pyramid would mum every single word about the interiors of this building.

In the modern parlance, we say that Euclid considered convex figures and solid bodies. The concept of convexity seems quite elementary today. Some part of a plane or space is called convex provided that no straight line segment between any two points of this part lies within the object under consideration. If we drive three stakes in a tract of land and stretch a lasso whose loop surrounds the stakes, we will single out a triangle. The harpedonaptae did exactly the same, but the interior of the tract to be measured might be inaccessible to the surveyors without permission of the owner. Nowadays we also measure property and levy taxies but any unauthorized attempt to stretch a rope within somebody's property is still a felony of trespassing on land. The definitions of Euclid are listed among the immortal witnesses of the ancient economic relations.

Geometry as a Basis of Science. Geometry deals with the quantitative and qualitative properties of spatial forms and relations. The criteria for equality of triangles provide instances of qualitative geometric knowledge. Finding lengths, areas, and volumes exemplifies quantitative research.

The abstraction of a straight line in geometry can be attributed to intuitive perceptions. Any straight line is a "length without breadth" perceived as a whole. There are points on every straight line, and the straight line is complete, which is not postulated as obvious without much fuss or circumlocution. The reals of the ancients appeared as processes rather than completed figments of intuition. Each real is either a completed process of combining units/monads or an incomplete process of measuring noncommensurate quantities.

Science has confronted the problem of counting the continuum since remote ages. The incommensurability of the side and diagonal of a square became an outstanding discovery of Euclidean geometry.

When our ancestors had demonstrated the absence of any common measure of the side and diagonal of a square, they understood that rational numbers are scarce for practical purposes. It is worth recalling that the set of rational numbers is equipollent with the collection of natural numbers. This means that all rational numbers comprise a countable set, thus serving as an instance of the cardinal number that we use to express the size of the imaginary collection of all entries of the natural series. The most ancient idea of the potential infinity in the form of consecutive counting turned out insufficient for quantitative analysis in geometry.

The straight line segment has decomposed in points within the convergence theory of Fourier series. To measure parts of the segment with transfinite numbers is the problem of the continuum in the same sense in which the ancient tried to commensurate the diagonal and side of a square. The discovery that the side and diagonal of a square are incommensurable is the height of mathematics as awesome and ethereal as the independence of the fifth postulate, the axiom of choice, and the continuum hypothesis.

The incompleteness of the rationals led to no inconvenience prior to geometry. Humans had no inborn conceptions of the reals. Insufficiency of the rationals was revealed only in the practice of measurement. Geometry in the times of its onset was directly tied with the need of tax levying and cadastral surveying. The mathematics of harpedonaptae must possess the power of law. The requirement of standardized reports and universal measurement, rather than whatever a priori ideas, led to the construction of a complete collection of reals. The mathematical intuition of the ancients was based on the conception of a straight line segment as a judicially correct definition of a rope stretching taut between two stakes to be used as an etalon for measuring. Measure theory stems from geometry, the latter originated with the judicial procedures that required the extreme definiteness and unicity of application. The logic of Aristotle followed geometry and reflected the methodology of geometry.

Retreat to Euclid. Alexandrov accomplished the turnround to the ancient synthetic geometry in a much deeper and subtler sense than it is generally acknowledged today. The matter is not simply in transition from smooth local geometry to geometry in the large without diffentiability restrictions. In fact Alexandrov enriched the methods of differential geometry by the tools of functional analysis and measure theory, driving mathematics to its universal status of the epoch of Euclid. The mathematics of the ancients was geometry (there were no other instances of mathematics at all). Synthesizing geometry with the remaining areas of the today's mathematics, Alexandrov climbed to the antique ideal of the universal science incarnated in mathematics.

Alexandrov overcome many local obstacles and shortcomings of the differential geometry based on the infinitesimal methods and ideas by Newton, Leibniz, and Gauss. Moreover, he enriched geometry with the technique of functional analysis, measure theory, and partial differential equations. Return to the synthetic methods of *mathesis universalis* was inevitable and unavoidable as illustrated in geometry with the beautiful results of the students and descendants of Alexandrov like Misha Gromov, Grisha Perelman, Alexei Pogorelov (1919–2002), and Yuri Reshetnyak.

Geometry and Alexandrov's Outlook. Geometry appeared as a result of human activities. Geometry was invented to organize human's life and change it for the better. Human is the starting point, the creator, and the aim of life. The general outlook of Alexandrov was determined by his scientific views that were formed in studying geometry. It is not by chance that the ideas of Karl Marx's Theses on Feuerbach enchanted Alexandrov.

Alexandrov was not a man of the past, but he was not ashamed of the past. He was able to discern his own misconceptions and eliminate them. He never concealed his own mistakes but tried his best to repair them if possible. He was interested in what they had done rather then what they had been doing. He never made a vain boast and always hated meritocratism. His attitude to truth was dynamic and based on principle.

Everyone trusts themselves, whatever cirecumlo9cution notwithstanding. Alexandrov was capable of extending the practice of trust to the others, using the presumption of decency which acts up to the first infringement. Alexandrov himself was a man of honor whose statements deserved acceptance without proof in much the same way as one's own words. Alexandrov put trust higher that proof.

Alexandrov's Ethics. Synthesizing geometry with the other areas of mathematics, Alexandrov elevated to the antique ideals of the unique science and placed the scientific stance in the center of his ethical views.

Alexandrov's contemplations about morality are connected with opposing religious belief and scientific search. The genuine human with the earthly needs rather than an ideal abstraction occupies the center of Alexandrov's outlook. It is the human seeking for truth and creating the circumstances of life. The human who is the source and the aim of life, Alexandrov emphasized the openness of science as well as its principle refutation of all forms of dogmatism and subjectivism innate to belief.

Alexandrov hated all crooks, "marxism-borne" popes and inquisitors who used science for mean and greedy ends. There is a precipice of repulsion between science and power. Power confronts freedom which is the essence of mathematics. Alexandrov viewed science as the tool that liberates humans from material burdens and untether them intellectually. Geometry taught Alexandrov universal humanism. He liked the words of Paul the Apostle and repeated that "there is neither Greek, nor Jew" in geometry. Humanism, responsibility, and scientific stance are the ingredients of the perfect morality by Alexandrov. Human is the source and aim of everything. That is the essence of universal humanism. Human is responsible for everything. That is the meaning of responsibility. The scientific stance as human's statement free of subjectivism is that which makes the foundation of morality. Alexandrov's staunch principles made predictable and tragic his fate. The defense of truth is a heavy cross and a lonely service. Alexandrov often felt himself "red carpet clown." Misunderstanding and mockery are the rewards of an alive hero. Time shows all in due proportion. Alexandrov will remain in history as a noble knight of science.

Alexandrov and the Present Day. Alexandrov emphasized the criticism of science and its never-failing loyalty to truth. Science explains "how the thingummy's actually going on" with greatness and modesty, using experience, facts, and logic. The love and hatred to Alexandrov stem from the same sources. His reviews and opinions were welcome and appreciated, but his approaches and areas of research were silenced if not scorned. He was accused of zionism, but many bet and counted upon his antisemitism. His communistic beliefs were blasphemed obscenely, but he was humbly requested to write a letter or two to the Central Committee of the Communist Party of the USSR or the Party journal *The Communist*. His philosophical essays were spit upon furiously, but the same despisers required that their students used Alexandrov's popular writings at the final examinations in philosophy which were obligatory for admittance to the public maintenance of theses. The professorship of St. Petersburg is full of raptures about the palace, fountain, and park ensemble of Peterhof, but most of Alexandrov's colleagues will never forgive the sage decision of Rector Alexandrov who suggested to build a new university campus in Peterhof. During the years of Gorbi's *perestroika* Alexandrov was accused in confessing lysenkoism but decorated with the Order of the Red Banner of Labor for his efforts in safeguarding and propelling genetics and selection in the USSR. So were the scales of Alexandrov's personality.

Alexandrov's life spanned the rise and fall of the Soviet Union. Complicated if not paradoxical ideology of communism views the individual freedom as necessity understood within a collective. Collectivism tends to transform into the hegemony of standardization and totalitarianism in much the same way as individualism brings about the tyranny of absolutism and globalization. Dictatorship, as the simplest form of universal subordination, becomes the inevitable instrument of individualism and collectivism. Collectivism reveals itself as altruism in morality, generating mysticism in the realm of reasoning. The creed of individualism is egoism and rationality. Alexandrov's ideas oppose rational egoism, abstract objectivism, and mystical dogmatism. Humanization of science as the vector of its progress is the most attractive ingredient of Alexandrov's views of the future of science and society.

The universal humanism of the geometer Alexandrov, stemming from the heroes of antiquity, will always remain in the treasure-trove of the best memes of the humankind.

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